Vlacq's tables in Chinese

Introduction to Chinese and Japanese tables of logarithms, with a review of secondary sources

(second edition)

Denis Roegel

20 March 2012

Contents

1 The introduction of logarithms in China[1]

The first table of logarithms, Napier's *Descriptio*, was published in Edinburgh in 1614.[2] Twenty years later, the main tables had been published, by Briggs[3] and Vlacq,[4] and for 300 years most of the published tables of logarithms would in one way or another be based on them.

Logarithms were introduced in China not long afterwards, in 1653, by the Polish Jesuit Smogulecki (1611–1656) and adapted by Xue Fengzuo. Several tables of logarithms may have been published afterwards, perhaps by Verbiest. Emperor Kangxi [康熙, kāngxī] (1654–1722) became a patron of sciences and he was said to carry with him a small table of logarithms, perhaps Verbiest's. It was however only at the beginning of the 18th century, as a consequence of emperor Kangxi's effort to gather a new encyclopedia of mathematics, that logarithms became more widely publicized. This document focuses on the tables published at that moment.[5]

2 Vlacq's tables in Chinese

The tables annexed[6] to this document are reconstructions of two early 18th century Chinese tables of logarithms held in various libraries,[7] in particular in the libraries of the Paris observatory,[8] of

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[1]We draw the attention of the reader to the fact that the first version of this document did not clearly distinguish the two different sets of tables of logarithms published around 1720.

[2]John Napier, *Mirifici logarithmorum canonis descriptio*, Edinburgh, 1614.

[3]Henry Briggs, *Arithmetica logarithmica*, London, 1624, and *Trigonometria Britannica*, Gouda, 1633.

[4]Adriaan Vlacq, *Arithmetica logarithmica*, Gouda, 1628, and *Trigonometria artificialis*, Gouda, 1633.

[5]Smogulecki's tables, as edited by Xue Fengzuo, are described in: Denis Roegel, *A reconstruction of Smogulecki and Xue's table of logarithms of numbers (ca. 1653)*, 2011, and Denis Roegel, *A reconstruction of Smogulecki and Xue's table of trigonometrical logarithms (ca. 1653)*, 2011.

[6]The reconstructed tables are available as three separate documents.

[7]It is our pleasure to thank the curators of the libraries who arranged for us to consult the original tables. During the preparation of this survey and the associated reconstructions, we have also received help from 张荷花 and 龚艳琼 for decyphering some Chinese characters.

[8]Bibliothèque de l'observatoire de Paris, B2–17 and microfilms 345–346.

the *Institut de France* in Paris[9], of the city of Lyon,[10] of the Royal Society in London, and in the French National Library.[11]

Figures 1 and 2 show excerpts of the tables kept at the Paris observatory library. In order to avoid misunderstandings, we call this set of tables "V," as they are directly patterned on Vlacq's tables. These tables are in fact classical tables of logarithms and were merely obtained by using the digits 〇, 一, 二, 三, 四, 五, 六, 七, 八 and 九 instead of the Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The use of '〇' is not strictly Chinese but it is not proof of the Western origin of the tables, as was thought by Abel-Rémusat in 1857. In fact, '〇' had been used in China at least since the 13th century[12] and even five centuries before, soon after its introduction in India.[13] We should also observe that these tables use a left-to-right writing direction, which was not customary at that time.[14] The tables give the characteristic of the logarithm and no separating point.

It is not totally clear when the tables V were published and how they are related to the tables of the 數理精蘊 (Shuli Jingyun) which obviously contains expanded versions of these tables.[15] We will henceforth call the tables of the 數理精蘊 tables "S." Tables V were probably printed in 1721 or shortly before.[16] Faithful to Vlacq's

[9] Bibliothèque de l'Institut (Paris), Réserve 4° NS 5699.

[10] Bibliothèque municipale de Lyon, fonds chinois, Ms 92-94.

[11] Bibliothèque Nationale de France (Richelieu), CHINOIS 4882. We mention in passing that the library also contains other tables of logarithms, in particular CHINOIS 4883 which contains 7-place logarithms of the sines, cosines, tangents and cotangents for every minute of the quadrant, and written from left to right, and CHINOIS 4884 which contains a variety of smaller tables. Some of these tables may have been due to Verbiest.

[12] Jean-Claude Martzloff, *Recherches sur l'œuvre mathématique de Mei Wending* (1633–1721), 1981, p. 42.

[13] Yabuuti Kiyosi, *Une histoire des mathématiques chinoises*, 2000, p. 56.

[14] Smogulecki and Xue's tables of logarithms (ca. 1653) write numbers from right to left.

[15] Complete reconstructions of the tables of the 數理精蘊 were published by us in 2011, see Denis Roegel, *A reconstruction of the tables of the Shuli Jingyun (1713–1723)*.

[16] The four copies examined, at the Paris observatory, at the *Institut*, at the Lyon municipal library and at the French National Library are identical. The copy at the *Institut* has a handwritten note by Ignatius Kögler, dated in Beijing on February 27, 1721. This note reads: *Hunc magnum Canonem Logarithmorum, tum pro Sinibus ac Tangentibus ad Singula decem secunda, tum pro numeris absolutis ab unitate ad 100 000, typis sinensibus in Aula Pekinensi ius-*

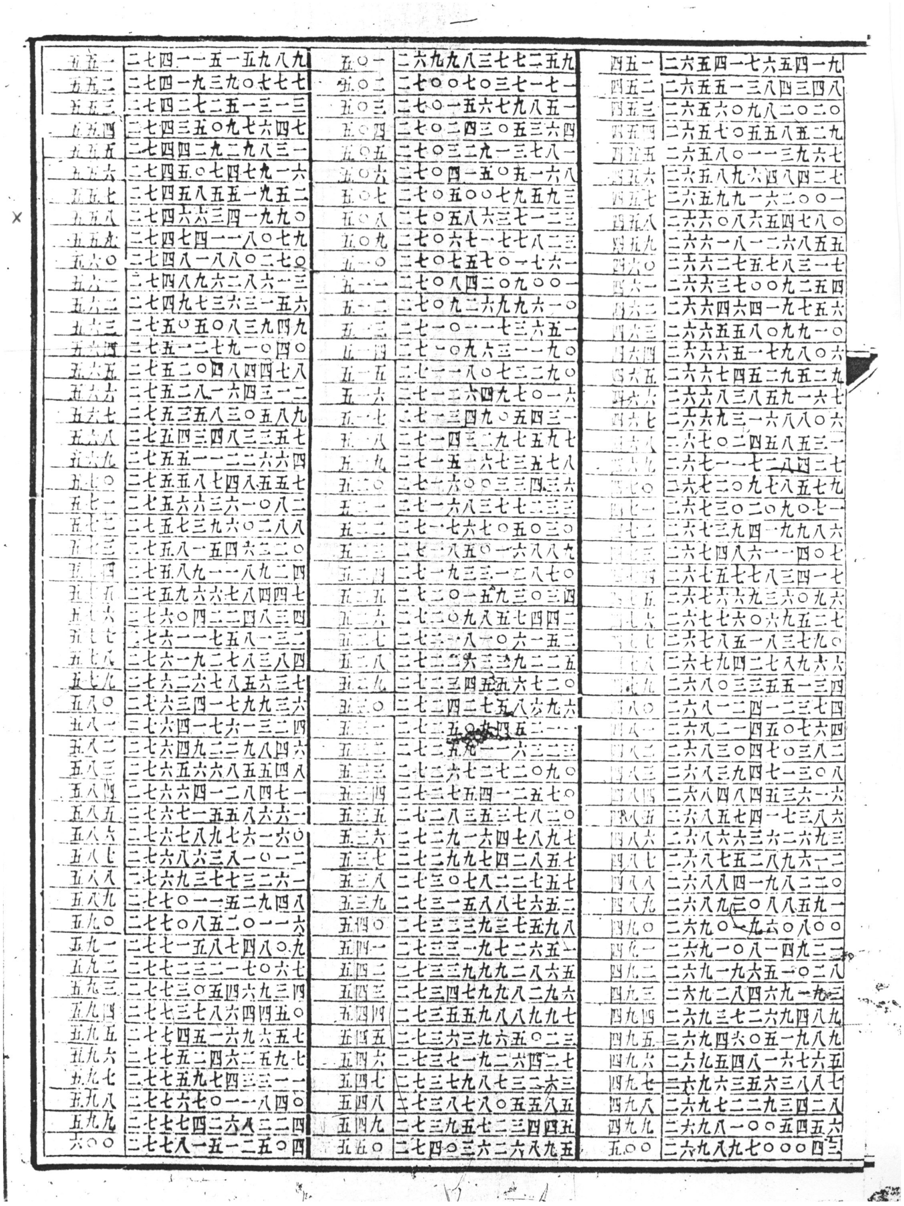

Figure 1: Excerpt of the Chinese table of logarithms of numbers (range 451–600). The layout follows that of Vlacq's 1628 tables, except that the differences are not given. Our mark on the left indicates one of many errors inherited from Vlacq's 1628 table: the last two digits of log 五五八 should be 89 and not 90. This table also gives the characteristic of the logarithm, like Vlacq's original table. (Source: Observatoire de Paris, B2–17)

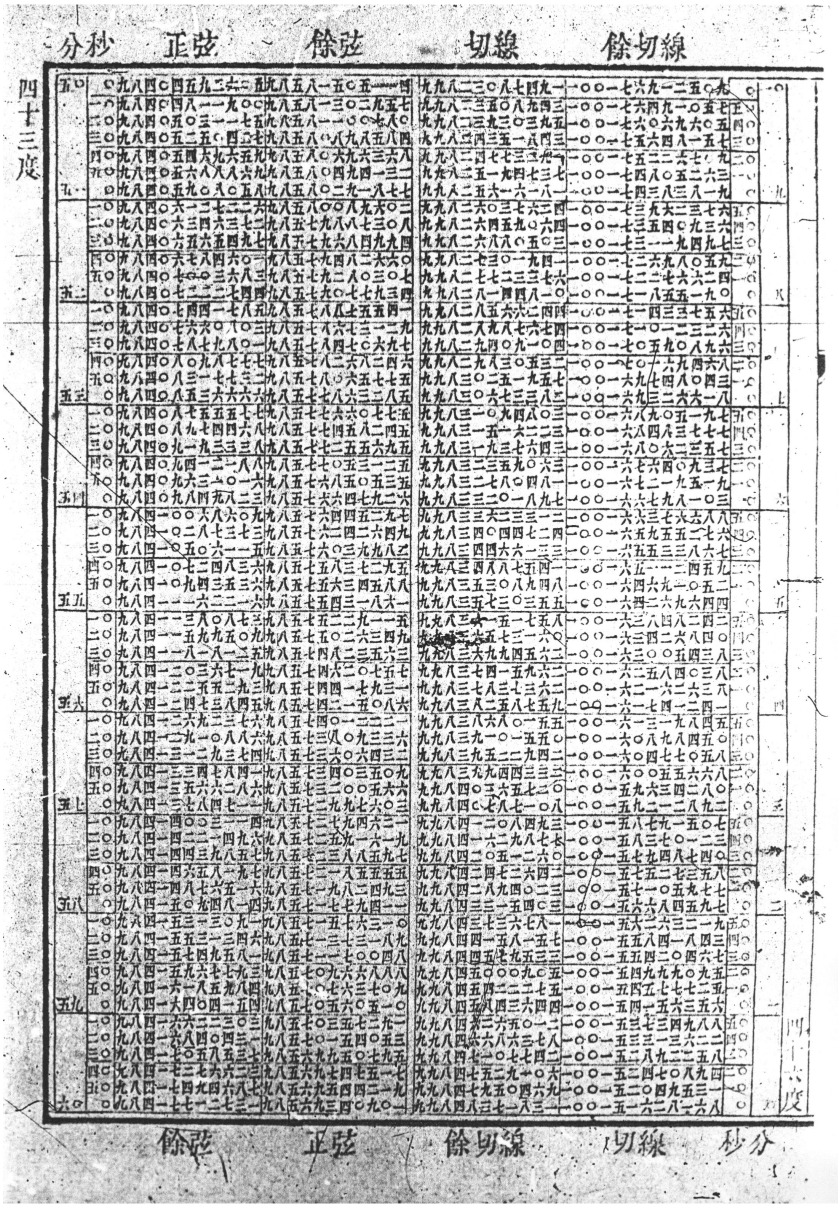

Figure 2: Excerpt of the Chinese table of logarithms of trigonometric functions (range 四三°五〇′–四四°). The layout follows that of Vlacq's 1633 table, and — like for the logarithms of numbers — differences are not given. (Source: Observatoire de Paris, B2–17)

tables, they may have been first stages in the making of the tables of the 數理精蘊. As we suggest below, they may have been produced by the Jesuits, and it was possibly their initiative which led to the tables being expanded and then included in the 數理精蘊.

2.1 The structure of the tables

The two types of trigonometric tables are not organized identically, and do not contain exactly the same information. Vlacq's tables, for instance, do not contain the logarithms of the secants and cosecants, and neither do tables V, but the expanded versions of the 數理精蘊 do.

Moreover, it should be observed that the Shuli Jingyun was later included in handwritten form in the Siku Quanshu collection, but that the tables were slightly adapted. The most conspicuous change concerns the logarithms of numbers which span three columns in the Shuli Jingyun, but only two in the Siku Quanshu. With this in mind, it appears that the two types of tables of logarithms of numbers are almost identical, except for the colored printing and the marginal titles. It is tempting to assume that the same woodcut was used for both tables of logarithms of numbers, but a comparison between the two tables held in Lyon shows many differences, so that it is most likely that the tables were totally recut. This excludes considering that the color version is a special version of the monochrome version using the same woodcut and supports the anteriority of these tables to those of the Shuli Jingyun.[17] We believe that the color versions

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su Imperatoris excusum. Reverenda Patri in Chrō P. Provinciali Societ. JESU per Germaniam Superiorem, debita submissionis, grati animi, ac memoria ergò venerabundi offert, indignus in Chrō filius, servus & exul Ignatius Kögler S.J., Pekini 27. Febr. 1721. Kögler was then president of the Board of astronomers. In the copy at the French National Library, the three volumes were bound together in the 19th century. The first volume bears the handwritten mention *Tables de Logarithmes chinois en 3 taos achetés 30 fr. à M. Picard, libraire rue du mail n°8 le 12 septembre 1816. J. (?) Langlet.* All copies except the one at the *Institut* bear the title 數表上 on the first volume, 數表下 on the second, and 度數表 on the last one. These titles are translated in Latin on the copy of Lyon. An additional handwritten note was inserted in the third volume of Lyon explaining the meaning of the headings for trigonometrical functions.

[17]On the history of xylography in China, see Jean-Pierre Drège, "Poirier et jujubier : la technique de la xylographie en Chine," *Le livre et l'historien : Études offertes en l'honneur du Professeur Henri-Jean Martin*, Genève, 1997,

were made by the Jesuits, perhaps as an aid to the Shuli Jingyun. The Shuli Jingyin appears to be monochrome.

2.2 Vlacq's source

In any case, whatever the real motivation and time of completion of the tables V, the tables of the logarithms of numbers were evidently patterned after the *Arithmetica logarithmica* computed by Ezechiel de Decker and published by Adriaan Vlacq in 1628, since the number of digits, layout and range are the same (three columns of 50 values per page). Moreover, the tables V contain the same errors as those found in Vlacq's table.[18] On the other hand, the table of logarithms of trigonometric functions is clearly based on Vlacq's *Trigonometria artificialis* published in 1633.[19]

Both tables give the logarithms to ten places,[20] as do Vlacq's. The original tables also contained differences (Δ), but they were omitted in both Chinese versions.

In 1827, as he was preparing his own table of logarithms (*Table of logarithms of the natural numbers, from 1 to 108000*, London, 1827), Charles Babbage identified the source of the tables V, based on the common errors in Vlacq's tables and in tables V. It should however be stressed that Babbage could have known the source without such a comparison, and perhaps did, since at least Vega's *Thesaurus* made notice of the source of tables V.

Tables V have apparently been translated by the Jesuit missionaries, or under their supervision, as it appears from the excerpts in section 3. The copy at the Royal Society was given by the French

pp. 85–93.

[18]One should also be careful, because some errors are common between Vlacq's and Briggs' tables, and it is sometimes tempting to conclude that the Chinese tables were copied from Briggs' tables. Briggs' error for $\log 80$, for instance, which is only a typographical error, was copied by Vlacq, and hence made it in the Chinese tables.

[19]The original tables of Decker and Vlacq are among a large set of tables which have been reconstructed by us in 2010.

[20]In his inventory of the Paris Observatory library, Bigourdan mistakenly writes that these tables give the logarithms with 8 places (Camille Guillaume Bigourdan, "Inventaire général et sommaire des manuscrits de la bibliothèque de l'observatoire de Paris," *Annales de l'Observatoire impérial de Paris*, 1895, p. F.20).

Jesuit missionary Antoine Gaubil, probably in 1755, through the president of the Russian Academy of Sciences. Gaubil was certainly also the source of the tables at the Paris observatory, since we know that Gaubil sent a copy of the tables for Cassini III in 1750 (Gaubil, *Correspondance de Pékin* 1722–1759, 1970). In addition, Father Dominique Parrenin, who arrived in China at the end of the 17th century, also sent tables of logarithms to Dortous de Mairan, probably around 1730 (Dortous de Mairan, *Lettres au R.P. Parrenin*, 1770).

2.3 The binding

The tables at the Paris observatory, at the library of the *Institut*, at the library of Lyon and at the French *Bibliothèque Nationale* use the traditional stitched binding, in which the sheets of paper have been printed on one side, then folded, the blank pages being concealed, and the leaves being gathered, with holes punched near the spine edge (opposite the folding), and stitched with a thin double silk cord.[21] Our reconstruction splits the tables in two parts, although we could have produced a larger table, leaving the burden of folding and binding the sheets to the reader.

All three volumes have the same size, approximately 17.5 cm wide and 26.4 cm tall and form an homogeneous set. This gives an original paper size of about 35 cm $\times$ 26.4 cm, prior to folding. The frames on each page are about 14.7 cm wide and 20.2 cm tall. The three volumes at the *Institut* are contained in an elaborate protecting box. The set at the Paris observatory is contained in a number of thick panels, which are less protective. There may have been an earlier box which has since been lost. The copy at the French *Bibliothèque Nationale* binds all three in one volume.

The volumes at the Paris observatory contain labelled inscriptions on the covers, stating that the work is by order of the emperor. These inscriptions do not appear on the three other copies that we have examined.

Some of the characters in the original tables are printed in red, and our reconstructions reproduce these features. The order of the

[21] On the details of the stitched binding technique, see Tsien Tsuen-Hsuin: *Paper and printing*, volume 5, chapter 32, of *Science and civilisation in China* (ed. Joseph Needham), 1985, Cambridge University Press, and especially pages 222–233.

pages follows the original order, and the tables start at the (western) end of the books. Each original folded page is numbered where the page is folded, and this feature is also found in the reconstruction, at the bottom left of each (westernly) even-numbered page.

According to Babbage, the Chinese tables comprise less space than equivalent Western tables, but there are several obvious reasons for that: first, the tables were printed without any differences, second, the paper used is very thin, and finally, the Chinese tables contain no introductions. The number of pages occupied by the tables is in fact the same in the Chinese tables and Vlacq's original volumes.

2.4 The logarithms of numbers

Volumes one and two represent a unique table giving the logarithms of numbers from 1 (一) to 100000 (一〇〇〇〇〇), and whose leaves are numbered from 1 (一) to 167 (一百六十七) for the numbers 1 to 50100 and from 168 (一百六十八) to 334 (三百三十四) for the numbers 50101 to 100000. The last leaf, numbered 三百三十四, is only partly filled, which is another feature we have reproduced.

Babbage checked for the following six errors in these volumes, errors which go back to Vlacq:

n	table	correct value
24626	四三九一三九三九七五一	四三九一三九三八七五一
38962	四五九〇六四一三四二〇	四五九〇六四一二四二〇
57628	四七六〇六三三五八七五	四七六〇六三三五四七五
57629	四七六〇六四一〇四三六	四七六〇六四一〇八三六
63747	四八〇四四五九七四一二	四八〇四四五九七五一二
67951	四八三二一九五八四二四	四八三二一九五八五二四

In addition, the Chinese tables contain some typographical errors. The last value of the first volume, for instance, is given as 五〇〇〇〇 instead of 五〇一〇〇.[22] This error does not occur in the Shuli Jingyun, since its first volume of logarithms of numbers only covers the range 1–50000.

[22]On the cover of the first volume in Lyon, a handwritten mention states incorrectly that the logarithms are given from 1 to 50000, but the second volume states correctly that the logarithms are given from 50101 to 100000.

2.5 The logarithms of trigonometrical functions

The third volume contains the logarithms of the trigonometric functions and its leaves are numbered from 1 (一) to 135 (一百三十五). The sines, cosines, tangents and cotangents are given every 10 seconds from 0° to 45°.

Here too, it is easy to compare the values with those of Vlacq's table. For instance, the last two digits of the logarithms of the sines from $0°3'$ to $0°57'$ are 66, 69, 05, 11, 81, 79, 09, 09, 62, 96, 72, 03, 85, 35, 84, 31, 28, 70, and 36, both in Vlacq's 1633 tables, and in the Chinese tables. In our reconstruction (last six pages), the correct values are 68, 71, 05, 13, 82, 79, 09, 11, 64, 97, 73, 03, 85, 36, 84, 32, 28, 72, and 35.

2.6 The reconstruction

In the reconstructions, the values of the logarithms were computed with the GNU `mpfr` library and the 10th place is correctly rounded.[23] The only exception concerns the values for $0°0'0''$, for which we intentionally copied the (incorrect) limit values given in the original tables. There should be three infinite values on the first line, but as it is not clear how these values should have been given in a Chinese table, we have sticked to the wrong values.

[23]See "MPFR: A multiple-precision binary floating-point library with correct rounding," *ACM Transactions on Mathematical Software*, 2007.

3 Secondary literature (excerpts)

This section lists the main Western sources mentioning Chinese and Japanese tables of logarithms. A few Chinese sources are also mentioned. The spellings of the original excerpts (between large horizontal brackets) have been preserved, and modern spellings have been added between small brackets. In some rare (but not all) cases where we were not able to print the traditional Chinese characters, we have used the simplified form of the characters.

3.1 Father de Mailla (1730)

In his *Histoire générale de la Chine*, tome 1, 1777, p. 168, Father Joseph-Anne-Marie de Moyrac de Mailla reproduces a letter he wrote to Nicolas Fréret from Beijing on September 27, 1730:

J'ai joint à ces six tomes un petit présent à votre adresse qui consiste, I.° en un grand livre Chinois, livre rare & précieux dont on ne trouve plus d'exemplaires ; il sera bon pour la bibliothèque de Lyon. 2.° Un autre petit de trois pouces, un peu plus de haut, sur deux environ de large, que l'empereur Kang-hi [康熙, kāngxī] portoit toujours pendu à sa ceinture, je l'ai eu d'un eunuque du palais ; il contient en Chinois les tables des *Sinus* & des *Logarithmes.* (...)[24]

3.2 Father Gaubil (1742)

In a letter to the mathematician Dortous de Mairan, written from Beijing on October 6, 1742, the Jesuit Antoine Gaubil mentions in passing that "[L]es logarithmes chinois que vous avez ont été dirigés par les Européans ici." (Gaubil, *Correspondance de Pékin 1722–1759*, 1970, p. 554). These logarithms were apparently sent to him by Father Dominique Parrenin, as it appears from Dortous de Mairan's correspondence.

[24](Editor) The French *Bibliothèque Nationale* contains a small volume of logarithms and other functions which might well be de Mailla's book, or a similar one.

3.3 Father Gaubil (1750)

On October 26, 1750, Antoine Gaubil writes again to Dortous de Mairan "[J]e vous ai adressé pour Mr de Thiry[25] les tables chinoises des Sinus et Logarithmes, et ceux des nombres." (Gaubil, *Correspondance de Pékin 1722–1759*, 1970, p. 620).

3.4 Father Gaubil (1755)

On April 28, 1755, Antoine Gaubil writes to the count Kirill Razumovsky, president of the Russian Academy of Sciences, and sends him the tables of logarithms for Thomas Birch, who was then secretary of the Royal Society in London: "[J]e prens la liberté d'addresser, par votre voie, à M. Birch, Secrétaire de la Soc. royale d'Angleterre, un gros rouleau de cartes avec le plan de Péking ; c'est pour la Soc. royale. J'ajoute pour ce secrétaire un billet de monnoye chinoise et les logarithmes en caractères chinois." (Gaubil, *Correspondance de Pékin 1722–1759*, 1970, pp. 808, 817–818, 837–838).

3.5 Dortous de Mairan (1770)

In his *Lettres au R.P. Parrenin, Jésuite, Missionnaire à Pékin, etc.* (1770), p. 56, Dortous de Mairan recounts that Father Parrenin had sent him tables of logarithms, probably around 1720 or 1730:

> Le *P. Parrenin* m'avoit envoyé encore en différens temps, plusieurs autres curiosités de cette espèce, plusieurs volumes de calcul, de Tables de sinus, tangentes & sécantes, & de Logarithmes, construites & imprimées ou estampées à la Chine, depuis l'arrivée de nos Missionnaires, & d'après eux ; avec deux grands volumes d'Architecture, de Perspectives & de Machines, dans le même goût, dont les Figures ou les Planches sont gravées en bois, & très-proprement exécutées.

[25](Editor) This is César-François Cassini de Thury, 1714–1784, who was director of the Paris observatory from 1756 to 1784.

3.6 Voltaire (1776)

In *Lettres chinoises, indiennes et tartares*, Genève, 1776, pp. 60–61 (reproduced in *Œuvres de Voltaire*, tome 48, Paris, 1832, p. 220–221), Voltaire comments about the Jesuits in China:

> Mais, monsieur, si les Chinois aiment tant les bons mathématiciens, pourquoi ne le sont-ils pas devenus eux-mêmes ? Pourquoi ayant vu nos éphémérides ne se sont-ils pas avisés d'en faire ? pourquoi sont-ils toujours obligés de s'en rapporter à nous ? Le gouvernement met toujours sa gloire à faire recevoir ses almanachs par ses voisins, et il ne sait pas encore en faire. Ce ridicule honteux n'est-il pas l'effet de leur éducation ? Les Chinois apprennent long-temps à lire et à écrire, et à répéter des leçons de morale ; aucun d'eux n'apprend de bonne heure les mathématiques. On peut parvenir à se bien conduire soi-même, à bien gouverner les autres, à maintenir une excellente police, à faire fleurir tous les arts, sans connaître la table des sinus et les logarithmes. Il n'y a peut-être pas un secrétaire d'état en Europe qui sût prédire une éclipse. Les lettrés de la Chine n'en savent pas plus que nos ministres et que nos rois.

3.7 Joseph-Marie Amiot (1788)

In the *Mémoires concernant l'histoire, les sciences, les arts, les mœurs, les usages, &c. des Chinois*, tome 13, Paris, 1788, p. 132, Joseph-Marie Amiot writes, imagining what future historians might conclude about the scientific knowledge of the Chinese:

> "On sait aussi que ce n'est qu'au commencement du dix-septieme siecle que les logarithmes ont été mis au grand jour par un Baron Ecossois, nommé *Jean Néper*. Comment auroit-il pu arriver que les Chinois & les Européens eussent eu les mêmes vues dans le même genre ? Il est clair comme le jour, que l'un de ces deux Peuples est enté l'un sur l'autre. Si quelqu'un en doutoit, il n'auroit, pour dissiper entiérement tous ses doutes, qu'à confronter les Tables que nos

anciens Européens du dix-septieme siecle désignoient sous le nom général de Tables d'*Ulacq* [Vlacq], avec les Tables chinoises de ce tems-là. C'est précisément le même ordre, c'est la même forme, ce sont les mêmes usages ; il n'y a entre elles aucune différence, ce sont les mêmes Tables : quoi de plus frappant ?"

3.8 Vega (1794)

In his *Thesaurus logarithmorum completus*, 1794, Georg Vega writes on page III:

Es ist merkwürdig, daß weder von der genannten Arithmetica logarithmica, noch auch von dem eben so seltenen Werke des nämlichen Adrian Vlacq unter dem Titel : *Trigonometria artificialis etc. Goudae* 1633 bis jetzt in Europa eine neue Auflage gemacht wurde, da doch zu Pekin in China zu Anfange dieses Jahrhunderts beyde genannte Werke neu aufgelegt wurden. Vor ohngefähr zwey Jahren wurde mir diese Chinesische Auflage des Vlacq zum Kaufen angeboten. Der Titel dieser Auflage ist : Magnus Canon Logarithmorum, tum pro Sinibus ac Tangentibus, ad singula decem secunda ; tum pro Numeris absolutis ab unitate ad 100000. Typis sinensibus in Aula Pekinensi jussu Imperatoris excusus 1721. Drey Foliobände, auf chinesisches Papier gedruckt.

3.9 Montucla and Lalande (1802)

In the third volume of Montucla's *Histoire des mathématiques*, 1802, J. F. Montucla and Jérôme de Lalande write on page 358:

Une anecdote particulière que nous apprend la préface de M. Vega, c'est que tandis qu'en Europe les deux ouvrages de Vlacq, les plus complets en ce genre, devenus très-rares, sollicitoient une nouvelle impression, que personne n'osoit entreprendre, ces deux ouvrages furent réimprimés à la Chine et dans le palais impérial, en caractères chinois et sous ce titre, rendu en latin : *Magnus canon logarithmorum, tùm pro sinibus ac tangentibus ad singula dena secunda, tùm pro numeris absolutis ab unitate ad 100000. Typis sinensibus in aula Pekinensi, jussu imperatoris (Kang-hi) excusus, 1721.* (*in-fol.* trois vol. en pap. chinois). M. Vega en a vu un exemplaire à Vienne. Nous avons, au reste, déjà remarqué que cet empereur étoit un grand admirateur, et de la sévérité des démonstrations d'Euclide et de l'invention des logarithms ; et ce que peut-être alors aucun prince européan n'auroit su faire, il calculoit très-facilement un triangle.

3.10 John Barrow (1805)

In his *Travels in China, etc.*, Philadelphia, 1805, p. 195, John Barrow writes:

Had the missionaries been disposed to confer a real service on the Chinese, instead of misleading the world by their strange and wonderful accounts of this people, instead of bestowing so much time in translating into Chinese a set of logarithm tables for the use of Kaung-shee [康熙, kāngxī], the second emperor of the present dynasty, of which they pretend he was so fond that he always carried them about with him suspended to his girdle, they rather have taught them the use and the convenience of the Arabic numbers, of whose combinations and results their own language is not capable, and have instructed a few of their youth in the principles of arithmetic and the mathematics. For such an omission, however, human nature can readily find an excuse. It would be too great an instance of self-denial to relinquish the advantages and the credit which their superior skill had gained them over a vast empire, by making the individuals of that empire participate in their knowledge.

3.11 John Barrow (1807)

In *Some account of the public life, and a selection from the unpublished writings, of the Earl of Macartney*, London, 1807, John Barrow writes on page 483 of the second volume:

> A table of logarithms has been published in the Chinese character; but this admirable assistant to arithmetical calculations was introduced by the Jesuits in the reign of *Cang-shee*.

3.12 Code pénal de la Chine (1812)

In *Ta-Tsing-Leu-Lée, ou les lois fondamentales du code pénal de la Chine*, tome I, Paris, 1812, within a section about the criminal offences of astronomers, p. 48, we can read:

> Il est aussi à observer qu'avec la permission de l'Empereur éclairé *Kang-Hee* [康熙, kāngxī], les missionnaires Européens ont imprimé et publié, à Pékin, en caractères chinois, nombre d'ouvrages utiles sur cette science, parmi lesquels on distingue une superbe édition de Tables de Logarithmes, qui est à présent dans la bibliothèque de la société royale (de Londres).

3.13 Manuscrits de la bibliothèque de Lyon (1812)

In the *Manuscrits de la bibliothèque de Lyon*, tome premier, 1812, pp. 142–143, Antoine François Delandine gives the following description of two tables of logarithms owned by the Lyon library:[26]

35. Ou piao[27] (*numerorum tabulæ*) en chinois. — 3 tom. en un *tao*, form. *in*-4°.

Ces tables numériques, supérieurement imprimées, sont divisées en trois volumes ; le premier comprend les logarithmes, depuis 1 jusqu'à 50,000 ; le second s'étend depuis 50,000 jusqu'à 100,000 ; le troisième présente des tables de sinus et de tangentes. Chaque page des deux premiers contient six colonnes de chiffres, dont trois sont en couleur rouage, et les trois autres en noir. Chaque page du dernier volume offre dix minutes, et chaque minute, renfermée dans sa case, donne les logarithmes pour 10, 20, 30, 40 et 50 secondes. Le papier de l'ouvrage est très-blanc, la couverture est en étoffe de soie violette ; ce qui est la couleur spécialement attribuée à la famille impériale régnante. Les étiquettes marginales sont en soie jaune ; le *tao* est recouvert de la même étoffe violette que la couverture.

36. Tables de logarithmes pour les calculs astronomiques (en chinois).[28] — 4 tom. en un *tao*, form. gr. *in*-4°.

Cet ouvrage est l'un des chefs-d'œuvre de la typographie chinoise. Il est divisé en sept colonnes, offrant les chiffres et calculs, et sort de l'imprimerie impériale de Pékin. Il est couvert de papier de soie jaune, et parsemé de sable d'or. L'étiquette marginale est en étoffe de soie blanche.

[26](Editor) These two sets of tables are now available at the Lyon library under call numbers Ms 92–94 and Ms 95–98.

[27](Editor) In fact 數表 (shu biao). This set contains the tables described in the introduction.

[28](Editor) This set contains volumes 5 to 8 of the tables of the Shuli Jingyun.

3.14 Aubin-Louis Millin (1814)

In the *Magazin encyclopédique, ou journal des sciences, des lettres et des arts*, tome IV, 1814, in an article on the Chinese language, p. 281, Aubin-Louis Millin writes:

> Les Jésuites ont refait dernièrement les tables astronomiques des Chinois sur celles de Tycho-Brahé. Ils ont également publié, en langue chinoise, les tables de logarithmes et un planisphère céleste : si dans la suite, on découvre ces tables, et ce planisphère, et qu'on en reconnoisse la justesse, on pourra se croire fondé à soutenir que les Chinois ont été, de tout temps, de grands calculateurs ou d'habiles astronomes.

3.15 Charles Babbage (1827)

In the *Memoirs of the astronomical society of London*, vol. III., Part I., 1827, pp. 65–67, Charles Babbage wrote:

> As the library of the Royal Society contains a table of logarithms printed in the Chinese character, which contains no indication or acknowledgement of its being copied from any other work, I was curious to compare it with European tables in the six cases alluded to. I found on examination that precisely the same errors occurred in the Chinese tables as those which I have noticed as being found in the tables of VLACQ. The conclusion from this fact is irresistible:— The Chinese tables were copied from those of VLACQ.
>
> As these tables are of rare occurrence, a short account of them may not be uninteresting. The copy in the library of the Royal Society was presented to the Society by PERE GAUBIL in 1750.[29] It consists of two thin volumes; each containing the logarithms of 50,000 numbers; each page contains three columns of natural numbers, and

[29] (Editor) According to Gaubil's correspondence, the copy was probably given to the Royal Society in 1755.

three columns of their corresponding logarithms; each column consisting of fifty lines. The numbers begin at the end of the volume, and the first column is on the right hand side of the page: but, unlike the usual mode of writing the Chinese character, the figures read from left to right. There is no column of differences, nor any running title; and the volumes are without title-page, preface, or instructions. They are printed on a beautiful thin yellow paper which is double, as is usual in Chinese books. The natural numbers are printed in red ink, not at the same time as the logarithms which are in black ink, but probably after them, as they occasionally fall upon and interfere with the black rules. The length of the printed part of each page is very nearly eight inches, and its breadth five and three quarters.

Another volume presented by the same donor accompanies these; it contains the logarithmic sines, cosines, tangents, and cotangents, to every ten seconds. It is printed on the same kind of paper as the former, and resembles it in many respects. It has a heading in red ink; the length of the printed part of the page is $8\frac{5}{8}$ inches, and its breadth $5\frac{6}{8}$. It is probably copied from the *Trigonometria Artificialis* of VLACQ, which was printed in 1633. These tables do not appear to have been printed with much care, since I noticed two errors in the short examination I made of them.

These three volumes if bound together would form one moderate sized volume in royal octavo, and are comprised in a smaller space than any tables of an equal number of figures which have been printed in Europe.

3.16 Bulletin des sciences mathématiques (1827)

In the *Bulletin des sciences mathématiques, astronomiques, physiques et chimiques*, tome septième, Paris, 1827, p. 340, we read:

9 mars. — On lit une Notice sur les erreurs communes à beaucoup de tables de logarithmes, par M. Babbage. Ce géomètre ayant dernièrement publié des tables de logarithmes stéréotypées pour les

nombres naturels, à l'usage du levé trigonométrique de l'Irlande, les a comparées avec beaucoup d'autres tables. Les siennes, celles de Véga et les dernières de Callet sont les seules qu'il ait trouvées exemptes de fautes, dans les logarithmes des sept nombres suivans : (24626), (38962), (57628), (57629), (63747), (67951). Et ce qu'il y a de curieux, c'est que toutes ces fautes ont été commises par des copies successives à partir des tables de Vlacq. Des tables en caractères chinois, et déposées à la librairie de la Société royale, avaient précisément les mêmes fautes ; preuve que les tables chinoises avaient été copiées sur des tables européennes.

3.17 Mechanics magazine (1827)

In the *Mechanics magazine*, volume 6, 1827, pp. 313–314, we can read:

The Chinese have been represented, by some of the French Missionaries, as profound astronomers, at a time when all Europe was in a state of barbarism; as being able to calculate the recurrence of eclipses, to adjust the irregular motions of the sun and moon, to measure the distance of the planets, &c. But if even the Chinese did possess such knowledge, of which there is no proof whatever, it must long since have been extinguished. When Adam Schaal, one of the earliest Jesuists, made his way to Pekin, he found that the Chinese knew so little of astronomical calculations, that they had introduced an intercalary month into the wrong year! At a later period, the emperor Kaung-hee [康熙, kāngxī], brought the president of his imperial board of astronomy to trial, because he could not calculate the length of shadow which a gnomon would throw, but which was immediately done by Father Verbiest. This sensible prince put himself under the tuition of the Jesuits, who made a quadrant for him, and translated into the Chinese language, a set of logarithm tables, which were printed, and a copy of which is now in the library of the Royal Society of London; a very beautiful specimen of Chinese typography. Kaung-hee [康熙, kāngxī] carried these tables and his quadrant suspended from his girdle, and when in Tartary, is said to have constantly amused himself in taking angles and measuring the height of mountains.

3.18 Polytechnisches Journal (1827)

In the 25th volume of the *Polytechnisches Journal*, published in 1827, page 85, we read:

Logarithmen-Tafeln.

Hr. Babbage verglich neulich mit seinen Logarithmen-Tafeln die Tafeln früherer Herausgeber von Vlacq 1628 bis auf Hutton 1822. Vega's Tafeln und Callet's (in den letzten Ausgaben) fand er allein fehlerfrei : die übrigen sind an vielen, und fast alle an denselben Zahlen fehlerhaft, zum deutlichen Beweise, daß ein Herausgeber den anderen (die meisten Vlacq'n) copirten. Er verglich eine chinesische Ausgabe, und fand sie dort fehlerhaft, wo die Vlacq'sche gefehlt ist, zum deutlichen Beweise, daß die chinesischen Logarithmen europäischen Ursprunges sind. (Vergl. *London Journ. of Arts.* Mai 1827. S. 173. und *Philosoph. Magaz.* Mai S. 353.)

3.19 George Thomas Staunton (1828)

In the second volume of his *Miscellaneous notices relating to China, and our commercial intercourse with that country*, 1828, George Thomas Staunton writes on page 74:

It is also to be observed, that in the beginning of the last century, under the patronage of the enlightened Emperor, *Kang-hee*, the European missionaries at Pekin printed and published in the Chinese character several useful works connected with this science, some of which, particularly a beautiful edition of a table of logarithms, are at present in the library of the Royal Society.

3.20 The Foreign Quarterly Review (1829)

In *The Foreign Quarterly Review*, volume III, 1829, in a review of Wronski's *Canons de Logarithms*, the reviewer writes on page 309:

> Prior to the invention of logarithms, to carry on any very complicated numerical computations was almost an impracticable task, and the well-known facility with which it is now performed, while it is the highest eulogium they can receive, supersedes the necessity of our enlargin upon the subject. From the first appearance of Napier's *Mirifici Logarithmorum Canonis Descriptio* in 1614, the constant and steady demand for these tables has led to very many publications of them; but as each succeeding editor has for the most part blindly copied from those who preceded him, there has been a pretty regular transmission of error, which has enabled an ingenious countryman of our own to expose the plagiarism even of the Chinese, who, having with their usual ingenuity, transcribed the figures, with their accustomed effrontery laid claim to the invention.

3.21 The Edinburgh Encyclopædia (1830)

In the thirteenth volume of the *Edinburgh Encyclopædia*, 1830, page 67, in a discussion of Vlacq's tables, we read:

> It is a remarkable fact, that while, in Europe, the tables of Vlacq, the most complete of their kind, had become very scarce, and no one would run the risk of bringing out a new edition, they were reprinted in the royal palace of the Emperor of China, with Chinese characters. The title, rendered into Latin, was, *Magnus Canon Logarithmorum,*

tum pro sinibus ac tangentibus ad singula dena secunda, tum pro numeris absolutis ab unitate ad 100000. Typis sinensibus in aula Pekinensi jussu imperatoris (Kanghi) excusus, 1721. The work extended to three volumes folio, and was printed on Chinese paper.

3.22 Monthly Notices of the RAS (1831)

In the *Monthly notices of the astronomical society of London*, vol. I., 1831, page 9, we have:

Mr. Babbage knowing that there was in the Library of the Royal Society a table of logarithms printed in the Chinese character, and which exhibits no indication or acknowledgment of its being copied from another work, was naturally desirous to compare it with European tables. On doing so, he found that in the *six* cases above noted, errors occurred precisely as in the European tables; this furnishing an irresistible proof that the Chinese tables have an European origin.

3.23 Biot (1831)

In the *Journal des savans*, 1831, p. 494, reviewing the *Memoirs of the astronomical Society of London*, Biot writes:

On trouve, dans les mémoires mêmes que nous analysons, un exemple singulier de cette transmission faite d'infidélités. M. Babbage, l'un des membres les plus distingués de la Société astronomique, faisant réimprimer des tables de logarithmes, et conférant nécessairement pour ce dessein toutes les éditions précédentes les plus complètes, avoit trouvé plusieurs erreurs de chiffres dans les tables de Vlacq. Il apprit qu'il existoit dans la bibliothèque de la Société royale un exemplaire de tables de logarithmes imprimé en caractères chinois, que le célèbre père Gaubil avoit adressé à cette société en 1750.[30] Ce sont deux petits volumes contenant les logarithmes de 50,000 nombres, lesquels sont disposés de manière à occuper moins de place qu'aucune table européenne qui renfermeroit autant de chiffres. Mais il n'y a de chinois que la forme, car M. Babbage y trouva exactement les mêmes erreurs de chiffres qu'il avoit découvertes dans les tables de Vlacq. M. Abel-Rémusat m'a fait voir que la Bibliothèque royale de Paris possède un exemplaire de ces mêmes tables chinoises, reconnoissable aux mêmes caractères de forme et aux mêmes erreurs. Outre ces preuves d'identité, M. Rémusat remarque que leur date moderne est rendue évidente par l'emploi du zéro, qui n'est pas conforme à l'usage chinois.

3.24 George Lillie Craik (1831)

In *The pursuit of knowledge under difficulties*, volume 2, London, 1831, p. 54, George Lillie Craik wrote:

Napier did not expound the process by which he constructed his logarithms in his first publication. This appeared only in a second work, published at Edinburgh in 1619, after the death of the author, by his third son, Robert. In this work also the logarithmic tables appeared in the improved form in which, however, they had previously been published at London, by Mr. Briggs, in 1617. They have since then been printed in numberless editions, in every country of Europe. Nay, in the year 1721, a magnificent edition of them, in their most complete form, issued from the imperial press of Pekin, in China, in three volumes, folio, in the Chinese language and character.

3.25 Dionysius Lardner (1834)

In the article "Babbage's Calculating Engines," *The Edinburgh review*, volume 59, 1834, p. 277–278, Dionysius Lardner writes:

[30](Editor) According to Gaubil's correspondence, this was probably more likely in 1755.

One of the tests most frequently resorted to for the detection of errors in numerical tables, has been the comparison of tables of the same kind, published by different authors. It has been generally considered that those numbers in which they are found to agree must be correct; inasmuch as the chances are supposed to be very considerable against two or more independent computers falling into precisely the same errors. How far this coincidence may be safely assumed as a test of accuracy we shall presently see.

A few years ago, it was found desirable to compute some very accurate logarithmic tables for the use of the great national survey of Ireland, which was then, and still is in progress; and on that occasion a careful comparison of various logarithmic tables was made. Six remarkable errors were detected, which were found to be common to several apparently independent sets of tables. This singular coincidence led to an unusually extensive examination of the logarithmic tables published both in England and in other countries; by which it appeared that thirteen sets of tables, published in London between the years 1633 and 1822, all agreed in these six errors. Upon extending the enquiry to foreign tables, it appeared that two sets of tables published at Paris, one at Gouda, one at Avignon, one at Berlin, and one at Florence, were infected by exactly the same six errors. The only tables which were found free from them were those of Vega, and the more recent impressions of Callet. It happened that the Royal Society possessed a set of tables of logarithms printed in the Chinese character, and on Chinese paper, consisting of two volumes: these volumes contained no indication or acknowledgment of being copied from any other work. They were examined; and the result was the detection in them of the same six errors.

It is quite apparent that this remarkable coincidence of error must have arisen from the various tables being copied successively one from another. The earliest work in which they appeared was Vlacq's Logarithms, (folio, Gouda, 1628); and from it, doubtless, those which immediately succeeded it in point of time were copied; from which the same errors were subsequently transcribed into all the other, including the Chinese logarithms.

3.26 Jeremiah Day (1836)

In *The teacher's assistant in the "course of mathematics adapted to the method of instruction in the American colleges"*, New Haven, 1836, p. 228, Jeremiah Day writes:

Note. For a more full history of Logarithms, Hutton's introduction to his tables may be consulted with advantage. See also Rees' and the Edinburgh Encyclopædia, and especially Maseres' Scriptores Logarithmici, in 6 vols. quarto, in which is contained a vast collection of interesting and useful matter. The early editions of Logarithmic tables which are valuable have become scarce. Some that have been mentioned in the history are highly valuable for their correctness, especially the Arithmetica Logarithmica, 1624, and the subsequent edition of Vlaq. The best modern tables are Hutton's 8vo., Taylor's 4to, and Callet's Tables Portatives, Paris, 1795. The editions of Gardiner are generally very incorrect. It will, perhaps, be interesting to some, to know that while the copies of Vlaq, the most complete of their kind, had become very scarce in Europe, and no one would run the risk of bringing out a new edition, they were reprinted in the royal palace of the Emperor of China, in 3 vols. folio, with Chinese characters. We said that Gardiner's tables were incorrect. We add in explanation, that the remark is not to be applied to all the editions. The octavo edition published at London in 1706, and especially the one published in 1742, may be recommended with entire confidence. But most of the subsequent editions, and especially the 5th, published in 1787, is so incorrect, that no dependence can be placed upon it. More may be learned of them from Hutton's Introduction.

3.27 An historical and descriptive account of China (1836)

In *An historical and descriptive account of China, etc.*, volume 3, Edinburgh, 1836, p. 229–231, the authors write (based on Babbage's description):

It is, however, a curious fact, that they have adopted one of the greatest improvements that ever was made in Europe, namely, the application of logarithms to calculation. In the library of the Royal Society of London, there is a table of these printed in the Chinese character. It consists of two thin volumes, each containing the logarithms of 50,000 numbers; every page contains three columns of natural numbers, and three columns of their corresponding logarithms; each column extends to fifty lines. The numbers begin at the end of the volume, and the first column is on the right-hand side of the page; but, unlike the usual mode of writing the Chinese character, the figures read from left to right. There is no column of differences, nor any running-title, and the volumes are without titlepage, preface, or instructions. They are printed on a beautiful thin yellow paper, which is double, as is usual in their books. The natural numbers are given in red ink, and the logarithms in black. This curious work was presented to the Royal Society by PÈRE GAUBIL in 1750;[31] also another volume, which contains the logarithmic sines, cosines, tangents, and cotangents, to every ten seconds. It has a heading of red ink, is executed on the same kind of paper as the former, and resembles it in many respects. The length of the printed part of the page is $8\frac{5}{8}$ inches, and its breadth $5\frac{6}{8}$. A question naturally occurs with regard to these tables: Have they been copied from others computed and printed in Europe? or have they been constructed in China? This question can be answered with perfect certainty. Mr Babbage, in preparing a table of logarithms, with a view to accuracy, compared them with those published by others at different times and in different forms; and he discovered that six errors were common to almost all the tables,—a proof that they were copied from each other, or from some original one that contained these errors. This remarkable fact induced him to examine whether these mistakes were in the Chinese table; and he found that it contained the very identical six. Thus, it was evident that their table was copied from some one of those printed in Europe; it was, in fact, from that of Vlacq, the original source of the inaccuracy,—a work which came out in 1628. The Chinese trigonometrical table was probably copied from

[31] (Editor) According to Gaubil's correspondence, this probably took place in 1755 and not in 1750.

the *Trigonometria Artificialis* of VLACQ, which was printed in 1633. These three Chinese volumes, united, would form a moderately-sized royal octavo, and are comprised in a smaller space than any tables of an equal number of figures published in this part of the world.

The logarithmic tables must have been introduced into China by the missionaries, no doubt with a hope of exciting a taste for mathematical science, to which they afford such powerful aid. We have no evidence, however, of their having produced such a happy effect.

3.28 John Francis Davis (1836)

In *The Chinese: A general description of the Empire of China and its inhabitants*, volume 1, London, 1836, p. 30, John Francis Davis writes:

Permission was given to the Jesuits to build two churches at Peking, and new labourers were allowed to enter the country: among these, Ferdinand Verbiest, another German Jesuit, and a man of distinguished science, became the coadjutor of Adam Schaal. On the accession of Kanghy [康熙, kāngxī], then a boy of eight or nine years of age, under the tutorship of four Tartars, the disputes which ensued with the intolerant Dominicans produced an unfavourable impression on the minds of the rules of China. Accusations were preferred against the missionaries, and their zeal to make converts was condemned as dangerous. It is said that Schaal died of chagrin, and that Verbiest was compelled for some time to abscond. When Kanghy [康熙, kāngxī], however, a monarch of enlarged and liberal mind, came to exercise the government in his own person, Verbiest was made President of the Astronomers; and through his influence the expelled missionaries were allowed to return to their churches. By the aid of Verbiest, the Emperor was enabled to cast guns, and to compose a mathematical work, with tables of logarithms.

In volume 2, p. 282, he writes:

> Proceed we now from medicine to another subject. In the science of numbers, and in geometry, the Chinese have, as usual, nothing to teach us; being, on the contrary, indebted for a good deal to Europe, as may be seen from the logarithmic tables and other works prepared for the Emperor Kâng-hy [康熙, kāngxī] by the Jesuits. Their arithmetic, as well as their weights and measures, proceed universally on the decimal scale; and decimal fractions are their *vulgar* fractions, or those in common use. It is remarkable that the single exception to this consists in their *kin*, or marketing pound weight, which, like ours, is divided into sixteen parts. It is most probable that both originated in the facilities afforded by the binary division into halves, quarters, eighths, and sixteenths. The sexagesimal division of the great circle was early borrowed by the Chinese from the Arabians, and of course used by the missionaries in the construction of their trigonometrical map of the empire. No algebraic knowledge is to be found in China, while it is certain that the *Hindoo* attainments in algebra were much superior to their astronomical science, and bear, besides, all the features of originality, which the latter does not.

And on page 288:

> The work, in one hundred Chinese volumes, composed and translated by Matthew Ricci and other missionaries, by desire of Kâng-hy [康熙, kāngxī], is a remarkable production. It is executed in the best style of native books, and being now very scarce and expensive, cannot be procured under sixty or eighty Spanish dollars, which is quite a *fancy price* for a Chinese work. It treats of spherical trigonometry, geometry, astronomy, and music, and contains also tables of logarithms, which were merely turned into native figures, and not calculated by the missionaries. The diagrams in geometry are accurately and neatly cut, and the whole is a very respectable specimen

of printing, worthy of the Emperor's patronage. The title means in English, "The profound sources of numbers—by imperial authority."

3.29 Robert Napier (1839)

In his edition of John Napier's *De arte logistica*, Edinburgh, 1839, Robert Napier wrote on page LXXVI:

> The Chinese are said to have laid clam to the invention; but the splendid copy of the Logarithms which issued from the imperial press of Pekin, contains certain errors which have been recently discovered in the European tables, previously published.

3.30 Charles William Wall (1841)

In the second part of *An examination of the ancient orthography of the jews and of the original state of the text of the Hebrew bible*, volume 3, London, 1841, p. 92–96, Charles William Wall summarizes what he calls the *logarithmic theft of the Chinese*:

> In connexion with the arithmetic of the Chinese, their logarithmic tables, though not derived from India, deserve some notice; as they afford an instance of an expedient, forming part of the artful system of contrivances resorted to by this people, for the purpose of stealing imperceptibly into the credit of having made inventions which never originated with them. For the exposure of their attempt in this case we are indebted to a very interesting paper of Mr. Babbage's, in the third volume of the Transactions of the Astronomical Society of London, respecting some errors that are common to many tables of logarithms. These errors the author detected by comparing the principal European editions of logarithms with his own; and traced them to VLACQ's tables, printed at Gouda in 1628, as their

source; their propagation through subsequent tables having arisen from the universal practice of copying which prevails in the drawing up of such works. After detailing the particulars of six of those errors, and enumerating the editions in which in common they are to be found, Mr. Babbage proceeds as follows. "As the library of the Royal Society contains a table of logarithms printed in the Chinese character, *which contains no indication or acknowledgment of its being copied from any other work*, I was curious to compare it with European tables in the six cases alluded to. I found, on examination, that precisely the same errors occurred in the Chinese tables, as those which I have noticed as being found in the tables of VLACQ. The conclusion from this fact is irresistible; —the Chinese tables were copied from those of VLACQ. As these tables are of rare occurrence, a short account of them may not be uninteresting. The copy in the library of the Royal Society was presented to the Society by *Pere Gaubil* in 1750.[32] It consists of two thin volumes; each containing the logarithms of 50,000 numbers; each page contains three columns of natural numbers, and three columns of their corresponding logarithms; each column consisting of fifty lines. The numbers begin at the end of the volume, and the first column is on the right hand side of the page; but, unlike the usual mode of writing the Chinese character, the figures read from left to right. There is no column of differences, not any running title; *and the volumes are without title-page, preface, or instructions."—Memoires of Astron. Soc. of London*, vol. iii. p. 66.

This description serves to give us some insight into the mode, practised by the Chinese, of preparing the way for their laying claim to the merit of discoveries not really made by them, but which they had learned, and, in general, very imperfectly learned, from strangers. A foreign invention is first communicated to the public, without any notice or acknowledgment of the source from which it was derived; and when its external origin is forgotten, it is then, in the most positive and barefaced manner, asserted to be a production of native talent. In reference to the sequel of this practice, I have further to observe, that—before their historic documents were, through transcriptions into alphabetic writing, as well known to foreigners as they now are,—the mandarins, there is the strongest reason to

[32](Editor) Probably rather in 1755, see Gaubil's correspondence.

believe, as soon as ever a claim of the kind was set up by them in the manner just described, had, with a view to its support, an account of the invention which was the subject of it, surreptitiously introduced into some early part of their annals. Their eagerness to establish thus the priority of discovery in favour of their own nation, leads to the exposure of the artifice to which they have, for such purpose, resorted. For several acquisitions in science and the arts, evincing a degree of skill that could be only the result of a long course of experience and gradual improvement, have been placed by them in ages before such experience could have been obtained, and when, consequently, it is absolutely impossible that its fruits could have been reaped by either the Chinese or any other people.

(...) Had Lord Napier lived a few centuries earlier than he did, and had his invention reached China three or four hundred years ago, we should now find a chapter in the *Chu-king* [書經] describing *Yao*, or *Yu*, or some such fictitious emperor, ordering his astronomers to invent logarithms, and those very able savans inventing them accordingly, in immediate fulfilment of his orders; and still farther we should, no doubt, meet with the celebration of so wonderful an effort of Chinese sagacity in the veracious annals of China. But the mandarins can no longer play these tricks with their historic documents; they have quite sufficient employment on their hands to conceal or defend those they have already played. For any claim, therefore, which they may make to the invention of their logarithmic tables, they must content themselves with the negative proof, of which they have laid the foundation in the contrivance, above described, for burying the foreign origin of those tables in oblivion. Such proof may, perhaps, impose upon a very ignorant and prejudiced population in the Celestial Empire, but it can hardly succeed elsewhere; and, at all events, the Chinese pretensions, under this head, are wholly set at rest in Europe, by the ingenious discovery of Mr. Babbage.

With respect to the tables in question, I have only to add, that the omission in them of the column of differences, shows how very little the use of logarithms is understood by the Chinese; and that even this imperfect knowledge of the subject is extended to but a small proportion of their savans, is evident from the very clumsy application of the Indian principle of numeric notation which still prevails in China. It is impossible but that men who had learned to read even a single line of those tables, must have ever after been aware of the total inutility of expressing numbers by two rows of figures; as the values of those in the principal row, which depend upon their places, are sufficiently indicated by their respective distances from the place of units, without the addition of the second row.

3.31 Crawford (1844)

In a report on the emigration of Chinese people, *Revue de l'Orient*, volume 5, 1844, Crawford writes on page 294:

> Presque tous peuvent lire et écrire, et plusieurs sont habiles arithméticiens avec l'aide du *sannpann*[33] (logarithme chinois).

Crawford's report was also published in the *Annales maritimes et coloniales*, volume 4, pp. 317–321, 1844.

3.32 Auguste Haussmann (1848)

In his *Voyage en Chine, Cochinchine, Inde et Malaisie*, volume 2, (1848), page 204, Auguste Haussmann writes:

> Ils doivent aux missionnaires chrétiens la connaissance des logarithmes ; leurs mathématiciens en ont fait imprimer, dit-on, des tables assez complètes.

[33](Editor) This is actually the *suanpan* 算盘, which is the Chinese abacus, not related with logarithms.

3.33 Samuel Wells Williams (1848)

In *The middle Kingdom; a survey of the geography, government, education, social life, arts, religion, &c., of the Chinese Empire and its inhabitants*, New York, 1848, volume 2, p. 147, Samuel Wells Williams writes:

(...) Both these compilations derive most of their value from the mathematical writings of the Romish missionaries; it is stated in the latter work that "the western scholar, John Napier, made logarithms." The knowledge of mathematics even among learned men is very small, and the common people study it only as far as their business requires; the cumbersome notation, and the little aid such studies give in the examinations, doubtless discourage men from pursuing what they seem to have no taste for as a people. A curious fact regarding the existence of six errors in these tables discovered by Babbage to have been perpetuated in most of the European logarithmic tables since the publication of the Trigonometria Artificialis of Vlacq in 1633, proves the source whence the Chinese derived them, and their imitative fidelity in copying them. Mathematical treatises on plane and spherical trigonometry and geometry exist in the language, based upon the works of foreigners, but mixed up with some crude notions of the compilers. Chinese authors readily acknowledge the superiority of western mathematicians, and generally ascribe their advances in the exact sciences to them.

3.34 Penny cyclopædia (1851)

In *The supplement to the Penny cyclopædia of the society for the diffusion of useful knowledge*, volume 2, London, 1851, in a chronology of logarithmic tables, p. 602, we read:

1721. In this year was printed at Pekin, by command of the Emperor Kang-Hi [康熙, kāngxī], in Chinese type and in three folio volumes, Vlacq's logarithmic tables of sines, &c., to ten seconds, and of numbers to 100,000. (Brunet, from Vega, who had seen it at Vienna.)

3.35 Colburn's United Service Magazine (1852)

In a series of articles on the origin of gunpowder published in *Colburn's United Service Magazine and Naval and Military Journal*, 1852, part I, London, p. 177, the author writes:

> The Chinese are good hands at claiming inventions; and Mr. Babbage's account of the Chinese logarithmic tables is worth recording. A copy of logarithmic tables, in two volumes, was presented, in 1750, by le Père Gaubil.[34] Mr. Babbage was unable to discover whether it was an original compilation or a copy, because it had neither date, title page, nor any other mark to show its origin. Comparing the tables with those of several European authors, he discovered that they contained certain errors which were found in Vlacq's tables, printed at Gonda in 1628. The conclusion was decisive: the Chinese tables had been copied from Vlacq's.

3.36 Karl Biernatzki (1856)

Karl L. Biernatzki's "Die Arithmetik der Chinesen," *Journal für die reine und angewandte Mathematik*, volume 52, 1856, was translated in French in 1863 (see below) and we do not reproduce the original article here.

3.37 Abel-Rémusat (1857)

In his *Élémens de la grammaire chinoise, etc.*, Paris, 1857, p. 23, writing about the writing direction of the Chinese, Abel-Rémusat writes:

[34](Editor) Probably in 1755 instead of 1750, see Gaubil's correspondence.

Dans les tables de logarithmes imprimées en chinois, on a adopté l'ordre horizontal, en commençant par la gauche ; mais cet arrangement est calqué sur celui des tables originales, et ne se retrouve nulle part ailleurs.

3.38 Désiré De Haerne (1858)

In *La Belgique*, tome sixième, Bruxelles, 1858, p. 370, Désiré De Haerne wrote:

Lorsque lord Macartney était à Pékin, un portugais qui se nommait évêque de Pékin, et qui était peu versé dans les mathématiques et dans l'astronomie, présidait au département, où se traitaient les questions relatives au calendrier, &c. L'Empereur Khang-hi [康熙, kāngxī] fut émerveillé de la science des Pères Verbiest, Adam Schaal et autres, qui lui construisirent un gnomon, et, chose plus difficile, lui traduisirent en chinois les tables des logarithmes ; c'est là une preuve évidente de l'ignorance de la nation en cette matière. Une copie de ces tables se trouve aujourd'hui dans la bibliothèque de la société royale de Londres. C'est un bel échantillon de typographie chinoise. L'Empereur porta ses tables et son quadran solaire suspendus à sa ceinture, comme des objets précieux, et l'on dit que, lorsqu'il était en Tartarie, il s'amusait constamment à mesurer la hauteur des montagnes par la résolution trigonométrique des triangles.

3.39 Almanach de l'Illustration (1860)

In the *Almanach-annuaire de l'Illustration contenant la concordance des cinq calendriers grégorien, julien, israélite, mulsulman, chinois*, Paris, 1860, a note on the Chinese calendar writes:

> Au reste, les connaissances des Chinois sur la théorie du calendrier sont restées longtemps fort incomplètes, et ce n'est guère que sous l'empereur Kang-hi [康熙, kāngxī], contemporain de Louis XIV, qu'ils ont appris des Jésuites, installés à la cour de Pékin, à calculer avec une exactitude suffisante les mouvements célestes qui déterminent les périodes annuelles. Et encore, s'il fallait en croire certains récits, les astronomes actuels de la cour auraient perdu l'habitude des tables des logarithmes à tel point qu'ils auraient recours, tous les ans, à l'obligeance des Russes pour avoir un calendrier dont ils puissent être parfaitement sûrs.

3.40 Jules Mohl (1861)

In the *Journal asiatique*, cinquième série, tome 18, Paris, 1861, p. 128–129, Jules Mohl writes:

> (...) C'est d'autant plus à regretter que les Chinois ont montré une aptitude singulière pour les mathématiques, et que sans aucun doute les sciences naturelles auraient fleuri également, si cet amour exclusif des lettres ne les avait fait négliger. M. Wylie, à Shanghaï, s'est mis en communication avec les mathématiciens du pays et travaille avec eux, et c'est par lui que nous recevons de temps en temps quelques données sur l'histoire, mal connue, des mathématiques chinoises anciennes et modernes. Il a trouvé une école de mathématiciens, qui, depuis l'invasion des Mantchous, s'est tenue éloignée des affaires, a refusé d'adopter les méthodes importées par les Jésuites ; elle a travaillé sur son vieux fonds de savoir, le perfectionnant par des méthodes à elle et restant quelquefois en arrière des découvertes européennes, quelquefois les devançant. Une autre école a accepté

l'enseignement des Jésuites, mais a cherché depuis à les perfectionner par son propre travail et avec une certaine jalousie nationale contre les étrangers. Je vois dans une notice sur des ouvrages chinois tout récents, que M. Wylie a insérée dans un journal à Shanghaï, que le libraire qui publie un ouvrage qui contient une méthode nouvelle en Chine pour calculer les logarithmes, dit dans une note que, « si ce livre arrive à la connaissance des disciples de Napier, ils gémiront de n'avoir pas fait cette découverte. » On lit dans une autre notice, que M. Wylie a mise à la tête de sa traduction chinoise de l'algèbre de Morgan, qu'en algèbre les Chinois avaient été en avance sur nous pendant plusieurs siècles et jusqu'aux quarante dernières années, mais que leur notation était plus incommode que la nôtre. (...)

3.41 Sinibaldo de Mas (1861)

In *La Chine et les puissances chrétiennes*, tome second, Paris: Hachette, 1861, p. 322, Sinibaldo de Mas wrote:

(...) Cet ouvrage fut suivi, en 1859, de la traduction par M. Wylie de la « Géométrie algébrique ou analytique, et du calcul différentiel et intégral de Loomis. » ...

« La promptitude avec laquelle les Chinois ont adopté les éléments de géométrie d'Euclide, le calcul par logarithmes et d'autres nouveautés d'importation européenne, ne saurait être oubliée. Un esprit de recherche se répand parmi les Chinois, et une classe considérable d'étudiants reçoit avec avidité l'instruction sur des sujets scientifiques venant de l'Occident. De simples essais superficiels et des résumés populaires sont loin de suffire à de tels indigènes désireux d'apprendre.

3.42 Karl Biernatzki (1863)

In his "Arithmétique et algèbre des chinois," *Nouvelles annales de mathématiques*, deuxième série, tome deuxième, Paris, 1863, pages 538–540, translated from the above, Karl L. Biernatzki writes:

Vers la fin du XVII^{e} siècle les missionnaires composèrent une Algèbre en chinois sous le titre : *Tseay-kang-fang* [借根方, jiègēnfāng], et la présentèrent à l'empereur Kang [康熙, kāngxī]. C'est à cette occasion que cet empereur ordonna la confection de la célèbre encyclopédie dont il révisa chaque feuille. Le titre chinois est : *Leuh-lei-yuen-yuen* [律历渊源, lǜlìyuānyuán] (Sources secrètes de l'harmonie et des nombres). La troisième partie de cet excellent ouvrage, intitulée : *Suh-li-tsing-wang* (Dépôt des finesses des règles arithmétiques), traite des sciences exactes et sert encore au collége d'astronomie de Péking. Il est divisé en deux sections principales : la première traite de l'origine des nombres ; on raconte comment Fohi vit sortir de la rivière Jaune un dragon portant sur son dos le système décimal. Un dessin reproduit cet événement, suivi d'un autre dessin qui représente une tortue sortant du fleuve Lo et sur la carapace est figuré ce système décimal, qui se montre à l'éminent philosophe Yu. Cette première partie est terminée par l'ouvrage *Tschan-pi*, mentionné ci-dessus. Les trois parties suivantes sont en XII livres avec une introduction à la géométrie, mais moins claire, moins *solide* que celle d'Euclide. On y expose ce qui est nécessaire sur les surfaces et sur les corps de diverses formes. Dans le dernier livre on parle des proportions et on donne des plans et des projections pour la confection des coupes et dessins. La cinquième partie comprend ce qu'on pourrait appeler l'*arithmétique en figures* ; la théorie des calculs est exposée par principes et éclaircie par des figures et des exemples. La seconde section principale traite en quarante chapitres de l'application de l'arithmétique et contient cinq divisions. La première, en deux chapitres, servant d'introduction, contient des tables de poids et mesures, des règles pour les quatre opérations et les fractions. La seconde division, en huit chapitres, traite des lignes, des proportions, des progressions, de la règle d'alliage, de la règle de société, des profits et pertes et des équations. La troisième, en huit chapitres,[35] s'occupe de calculs de la surface des corps, de l'extraction des racines carrées, de l'ancienne et de la nouvelle trigonométrie, de l'usage des huit lignes trigonométriques, de la méthode pour déterminer les côtés d'un triangle, de la mesure des figures rectilignes ou curvilignes, des segments circulaires et des polygones réguliers. La quatrième section, en huit chapitres, contient ce qui concerne les valeurs, l'extraction de la racine cubique, la mesure des polyèdres et des surfaces courbes, des sphères et des segments sphériques, les poids de diverses substances du règne animal, végétal ou minéral ; enfin les Tas. La cinquième division, en dix chapitres, comprend des dissertations sur l'algèbre, sur diverses équations y relatives, sur les logarithmes et l'usage des secteurs ; il y a, en outre, huit volumes supplémentaires avec des Tables.

Les deux premiers volumes donnent le calcul des sinus, cosinus, tangentes, cotangentes, jusqu'au 90^e^ degré. Le troisième et le quatrième volume contiennent les diviseurs de tous les nombres de 1 à 100000, pour faciliter le calcul par logarithmes. A la fin de chaque série de dix mille, on donne la liste des nombres premiers. Le cinquième et le sixième volume contiennent les logarithmes des nombres de 1 à 100000 avec dix décimales qui sont évidemment une copie des Tables de Vlacq, imprimées en Hollande en 1628. A la fin on trouve des règles pour calculer les logarithmes des nombres plus grands que 100000 et une Table des pesanteurs spécifiques de diverses substances. Le septième et le huitième volume sont des Tables de logarithmes de sinus, cosinus, tangentes, cotangentes, sécantes, cosécantes de 0° à 90°.

Le style de cette encyclopédie est clair, populaire, et destiné à être lu et compris par tous les Chinois instruits.

Les Chinois s'attribuent la découverte des logarithmes. Du moins un mathématicien, nommé Le-scheu-lan [李善蘭, Li Shanlan], vivant aujourd'hui à Shang-haï, dans son ouvrage *Tay-suh-tan-yuen* [对数探源, duìshùtànyuán] (Découverte de l'origine des logarithmes), dit qu'il possède une méthode pour calculer les logarithmes par des considérations géométriques et qui n'est pas connue des Européens. Un mandarin, nommé Ta-heu, est aussi occupé à Hang-tschau à pu-

[35](Editor) This is probably wrong, and this division certainly contains twelve chapters, and not eight. The total number of chapters is then forty.

blier une nouvelle manière de calculer les logarithmes.

(*A summary of Biernatzki's study by J. Bertrand also appears in the* Journal des savants, *1869*)

3.43 Alexander Wylie (1867)

In his *Notes on Chinese literature*, 1867, Alexander Wylie writes:

(pp. 89–90) The 天步真原 [tiānbùzhēnyuán] *T'ëen poō chin yuên* is a small treatise on the calcuation [sic] of eclipses according to the European method, written about the commencement of the present dynasty, by 薛鳳祚 [xuēfèngzuò] Sëĕ Fung-tsoó, who had been initiated into the western theory, by Nicolas Smogolenski 穆尼各 *Muh Ne kō*, then resident at Nanking. This is the first book in which logarithms are introduced.

(pp. 96–97) In 1713, the same year that the *Leih sëáng k'aòu ching* [歷象考成, lìxiàngkǎochéng] was completed, a companion work from the same source also appeared, containing the mathematical processes initiatory to the astronomical formulæ in the above. This gives a comprehensive detail of the science of arithmetic as it then stood, embracing all the recent European introductions, under the title 數理精蘊 *Soó le tsing yun* [shùlǐjīngyùn], and is divided into three parts. The first part in 5 books is discursive and theoretical, in which the origin of numeration is traced up to the ancient sages of China, and the nucleus of the *Chow pe* is given with a commentary. Next is a treatise on Geometry, giving the theory of linear measurements, which is followed by a demonstration of the theory of numbers. The second part in 40 books is practical being divided into 5 sections, the first of which gives Weights, Measures, Notation, and the initial rules of arithmetic; the second section treats of linear measurements in all its varieties; the third is on surfaces, with their relative proportions; the fourth is on solids of every kind plane and curved. The last section contains the earliest record we have of the process of European Algebra, which had been introduced into China by some of the missionaries, under the title 借根方 *Tsēáy kàn fang* [jiègēnfāng]. The native algebra *T'ëen yuên* does not seem to have been known by the compilers, as it is not even mentioned. This section also gives the earliest complete treatise on Logarithms, which is followed by details on the use of the sector. The third part contains 8 books of tables;— first the 8 lines of the trigonometrical canon for every 10 seconds; next is a table of factors of numbers up to 100,000, with a catalogue of prime numbers at the end; then follows a table of logarithms of natural numbers up to 100,000, which appears to be a transcript of Vlacq's table published in Holland in 1628, as it contains the six errors of that table faithfully copied; the last two books are a table of the logarithms of the 8 lines of the trigonometrical canon for every 10 seconds.

The above publication with the *Leih sëáng k'aòu ching* [lì xiàng kǎo chéng, 歷象考成], and a third work on music, entiled [sic] 律吕正義 *Leŭh leu ching ê* [lǜlǚzhèngyì], together constitute the grand thesaurus of the exact sciences, known as the 律歷淵源 *Leŭh leīh yuen yuên* [lǜlìyuānyuán], drawn up under direct imperial superintendence, commenced during the years of K'ang-he [康熙, kāngxī], completed in those of Yung-chíng, [雍正, yōngzhēng] and published early in the Kēen-lung [乾隆, gānlōng] period.

(pp. 101–103) The 翠薇山房算學 *T'sûy wei shan fâng swân hëŏ* [cuìwēishānfángsuànxué] is a mathematical compendium published in the earlier part of the Taòu-kwang [道光, dàoguāng] period, by 張作楠 Chang Tsŏ-nan [zhāngzuònán], in 38 books, consisting of 15 parts, on—Solid mensuration, including a chapter on European algebra, Additional rules for plane mensuration, Supplementary section on Solid and plane mensuration, which treats of the *T'ëen yuên* algebra, Tables of the eight lines of the canon, Logarithmic tables of the eight lines, Problems of spherical trigonometry, Chief points in spherical trigonometry, Tables of terrestrial longitude and latitude, Latitude and solar tables, Tables of altitude throughout the year, Maps and tables of the fixed stars, Maps and tables of the meridian stars, Tables of meridian stars according to several watches, Tables of meridian stars according to the several hours, and Formulæ for calculating eclipses. This appears to be a compilation from various sourcesm with nothing original; there is a want of uniformity also, the numbers in some of the tables being read from right to left, and in others from left to right; it is useful however as a book of reference.

(...)

The 算法大成 *Swân fă ta chîng* [suànfădàchéng], in 21 books, is a compendium of mathematics of recent date, by 陳杰 Ch'în K'ēē [chénjié], in two parts, the first of which was published in 1843, and contains the common rules of Arithmetic, Logarithms, and Plane and spherical trigonometry; the second part, which appears to be still in manuscript, treats of Mathematical chronology, and Practical rules regarding Agriculture and Military service. The author states it to be his object merely to give simple and useful rules, and consequently omits all notice of the *T'ëen yuên* and kindred processes, which he regards as rather curious than edifying. For the mechanical part of calculation, he prefers the abacus as the most convenient, after which he places Napier's rods, and considers pencil calculation as the least advantageous of all.

(...) About the year 1845, 李善蘭 Lè Shén-lân a self-taught student issues a small treatise, entitled 方員闡幽 *Fang yuên ch'én yew* [fāngyuánchănyōu], in which he shews by a differential process, that the excess of the square over its contained circle, is equal to the aggregate of an infinite series of pyramids. In another treatise entitled 弧矢啓秘 *Hoo shè k'è pé* [húshĭqĭbì], he gives new rules for deducing the several lines from each other, especially the arc from the secant and vice versa, which had not been given in any previous native work. A few years later another work of Lè's, the 對數探源 *Túy soó t'án yuên* [duìshùtànyuán] appeared, being an investigation of the theory of Logarithms, in which by an original train of thought, he has arrived at something like the same result as Gregory St. Vincent, when he discovered the Quadrature of the Hyperbola in the 17th century.

The 對數簡法 *Tûy soó këèn fà* [duìshùjiănfă] is a Ready method for computing Logarithms, by 戴煦 Taé Heu [dàixù], in which he discovers as he thinks for the first time an intermediate table for facilitating the calculation of common logarithms. This intermediate table appears as the same as Napier's system of logarithms, though there is every reason to believe that this author was unaware that he had been already forestalled. In a supplement to the same work he gives a further refinement of his process, making great use of the Napierian modulus, which he arrives at in the course of his operations.

(p. 104) The contributions of foreigners in recent times to works of this class, have not been extensive. In 1849, Dr. Hobson published a popular digest of modern European Astronomy, with the title 天文略論 *T'ëen wān lēŏ lûn* [tiānwénlüèlún]. This gives a plain view of the solar system, referring the motions of the orbs to the influence of gravitation, and pointing to God as the author of all the stupendous works of creation. In 1859, a translation of Herschel's 侯失勒 *How shih lŏ* [hóushīlè], "Outlines of Astronomy," in 18 books, was published at Shanghae, with the title 談天 *T'an t'een* [tántiān]. In 1853, the 數學啓蒙 *Soó heò k'e mûng* [shùxuéqǐmēng] appeared, which is a compendium of arithmetical rules including logarithms, with a table of the latter up to 10,000.

3.44 Paul Dalloz (1868)

In *Les mondes — Revue hebdomadaire des sciences et de leurs applications aux arts et à l'industrie*, sixième année, janvier-avril, tome 16, 1868, p. 156, reviewing a book by Fédor Thoman, Paul Dalloz writes:

Adrien Vlacq, géomètre et libraire à Gauda [sic], détermina, à l'aide des tables de Briggs, les logarithmes des sinus et des tangentes qui se trouvaient dans l'ouvrage de Rheticus (*Opus Palatinum*). Sa *Trigonometria artificialis*, publiée en 1663 [sic], contient les logarithmes des sinus et tangentes de dix en dix secondes et les vingt premières chiliades des logarithmes de Briggs, le tout avec dix décimales. La table de Vlacq a produit toutes les tables existantes pour la division sexagésimale du cercle ; en 1721 elle a été réimprimée à Pékin en caractères chinois, par ordre de l'empereur de Chine.

3.45 Alexander Wylie (1897)

In his book *Chinese researches*, published in 1897, Alexander Wylie wrote on page 192, when describing the *Shuli Jingyun* [數理精蘊, shùlǐjīngyùn]:

> The concluding section in 10 volumes treats on Algebra, Miscellaneous problems not embraced in the preceding rules, Logarithms, and 2 volumes in explanation of the use of the Sector. Besides the above, there are 8 supplementary volumes of tables, 2 containing the sines, cosines, tangents, cotangents, secants and cosecants, for every 10 seconds throughout 90 degrees. The 2 next give a table of the divisors of numbers from 1 up to 100,000 to facilitate the computation of logarithms. At the end of every 10th thousand, a list of prime numbers is also given. The 2 next volumes contain a table of the logarithms of natural numbers from 1 to 100,000 calculated to 10 places of figures. This is apparently a transcript of the table that was published in Holland by Adrian Vlacq in 1628. At the end is a catalogue of the logarithms of the ratios of a great number of mathematical terms, and the relative gravity of various substances. The last 2 volumes have a table of the logarithms of sines, cosines, tangents, cotangents, secants and cosecants for every 10 seconds from zero up to 90 degrees.

On pages 193–194, he writes:

> Although it has been a predominant feeling with most native writers to trace everything of a scientific character to a remote native origin, yet they seem never to have questioned the right of John Napier to the invention of Logarithms; nor have they on that account undervalued the discovery; and even at the present day we find some applying themselves with great zeal to the subject, ignorant of the advances that have been made in the West, since the days that it was first brought before the Chinese. There is one 李善蘭 Li Shen-lan, a relative of Li Juy above mentioned, now residing in Shanghai, who

has recently published a small work called 对数探源 *Tuy-soo-tan-yuen* [duìshùtànyuán], "Discovery of the Source of Logarithms," in which he details an entirely new method for their computation, based upon geometrical formulæ; which he says in his introduction is "ten thousand times easier than the methods used by the Europeans," and that "although *they* can just calculate the numbers, yet they are ignorant of the principle." This small indication of self-satisfaction may be very well overlooked, as quite pardonable in one who has had no better aid than that afforded by the *Leuh-lih-yuen-yuen*, and who has here given us, as the result of four years' thought, a theorem, which in the days of Briggs and Napier, would have been sufficient to raise him to distinction.

A mandarin at Hangchow, named 戴煦 Tai Heu, has also been engaged for some time past on a new method of obtaining logarithms; the blocks for which are now being cut, and it will be before the public in a very short time.

3.46 Louis VanHée (1913)

Louis VanHée, in his note "Tables de logarithmes," *T'oung Pao* (通報), volume 14, 1913, p. 495, writes:

L'Empereur K'ANG-HI [康熙, kāngxī] portait souvent sur lui un petit livre joliment imprimé où se trouvaient des tables de logarithmes. Ce précieux imprimé-bijou fut envoyé plus tard aux jésuites français d'Europe, probablement à Lyon. A la dispersion de l'Ordre, les bibliothèques des savants religieux furent malheureusement livrées à des mains peu scrupuleuses ou du moins maladroites. Existe-t-il une trace quelconque de ces tables de logarithmes employées par le grand empereur de la dynastie TS'ING [清, qīng] ?

3.47 Yoshio Mikami (1913)

In *The development of mathematics in China and Japan*, New York: Chelsea Publishing Company, 1913, Yoshio Mikami wrote on pages 115–116:

Logarithms were considered for the first time in China by the missionary Mu Ni-ko [穆尼閣, Smogulecki], who had been at Chiangming during the Shunchih Era [順治, Shùnzhì] (1644–1660). His publication bore the title *T'ien-pu Chên-yüan* [天步真原, tiānbùzhēnyuán].

Describing the work published in 1713 under the guidance of the Emperor Kangxi [康熙, kāngxī], Mikami wrote on page 117:

The second part of *Su-li Ching-yün* [數理精蘊, shùlǐjīngyùn] begins with the very rudiments of arithmetic, the proportions, percentage partition, simultaneous equations, extraction of square and cube roots, together with those where additional members are present in these operations, areas of rectilinear and curved figures, volumes of solids, summation of progressions of advanced kinds, the *chieh-ken-fang* [借根方, jiègēnfāng], and logarithms, etc., whereto are added some books on the tables of trigonometrical lines and of common logarithms, the latter being given for 11 decimal places[36]. These logarithmic tables are said to be the same as those published by Adrian Vlacq in 1628 in Holland.

About Li Shan-lan, Mikami writes on pages 126–128:

Some of his writings were lost during the long war of the Taiping [太平, tàipíng] Rebellion, but the others were published under

[36](Editor) These 11 places must include the characteristic and there are actually therefore only 10 fractional decimal places.

the title *T'so-ku-chi Suan-hsiao*, which contained thirteen separate articles in 24 books. He treated therein of triogonometrical [sic] functions, logarithms, ellipse, summation of power progressions, etc.

(...)

As we have said, logarithmic tables were published in the *Su-li Ching-yün* [數理精蘊, shùlǐjīngyùn] of 1713. The Chinese mathematicians widely employed these tables and became perfectly convinced of the conveniency of them. But there were no treatises in China, in which the construction of such tables was explained. The publication of algebraical treatises containing the theory of logarithms is only of a recent date. The Chinese mathematicians, who were delighted with logarithmic calculation, could not be satisfied without the theory. They did not however seek for explanations in European works; on the contrary they attempted to make the matter clear themselves. Their studies were crowned with some success. They at last arrived at the natural logarithms in their explanations. Tai Hsü [戴煦], Li Shan-lan [李善蘭], Hsü Yu-t'ing, etc., obtained solutions. Although their ways of attack were different in some points their results all agreed.

Tai's article on the subject was noticed by the English savant Ai Yüeh-so, who was exceedingly struck with it. This fact is mentioned by A. Wylie in the preface to the *Tai-wei-chi Shih-chi*. Ai Yüeh-so desired to see the author, pay him due respect and talk with him about mathematics. So he went to Hang Chou, where Tai lived, on a visit in 1854. But the Chinese mathematician refused to see the stranger. Ai Yüeh-so translated afterwards, it is said, the book of Tai and published it in England.[37] This passage is recorded by Chinese writers as an incident highly interesting.

Following these three masters, K'u Kuan-kuang [顾观光, gùguāngguāng] (1799–1862), Tou Pai-chi, Hsia Luan-hsiang, and others also carried on studies on the subject of logarithms.

Finally Ting Ch'ü-chung, together with Tsêng Chi-hung, wrote the *Tsui-su Hsiang-hsieh* or "Logarithms Explained in Detail", 5 books, where the explanations are given in the algebraical way of the Europeans.

[37] Chu's "Third Part of Biographical Collections", Book 4.

3.48 Smith and Mikami (1914)

In *A history of Japanese mathematics*, Chicago, 1914, p. 268–270, David Eugene Smith and Yoshio Mikami write:

The Japanese first learned of logarithms through the Chinese work, the *Su-li Ching-yün* [數理精蘊, shùlǐjīngyùn], printed at Peking in 1713. This was not the only Chinese publication of the subject, however, for it is a curious fact that no complete edition of Vlacq's tables[38] appeared in Europe after his death, and that the next publication[39] thereafter was in Peking in 1721,[40] a monument to Jesuit learning. The effect of these Chinese works was not marked, however. Ajima,[41] who died in 1798, was one of the first Japanese mathematicians to employ logarithms in practical calculation, and his manuscript upon the subject was used by Kusaka in writing the *Fukyū Sampō* (1798), but the tables were not printed. A page from an anonymous table in an undated manuscript entitled *Tai shin Rio su hio*, giving the logarithms to seven places is shown in the illustration (figure 3). The first printed work to suggest the actual use of the tables was Book XII of Sakabe's *Sampō Tenzan Shinanroku* (Treatise on Tenzan Algebra), published in 1810–1815. Speaking of them he says: "Although these tables save much labor, they are but little known for the reason that they have never been printed in our country. If anyone who cares to copy them will apply to me I shall be glad to lend them to him and to give him detailed information as to their use." He gave the logarithms of the numbers 1–130 to seven decimal places, by way of illustration. He may possibly have had some Dutch work on the subject, since he knew the word "logarithm," or possibly he had the Peking tables of 1713 and 1721.

Sakabe further says: "The ratios involved in spherical triangles, as given in the *Li-suan Ch'uan-shu* [歷算全書, lìsuànquánshū], are

[38] His *Logarithmica Arithmetica* appeared at Gouda in 1628.

[39] They had been reprinted in part in George Miller's *Logarithmicall Arithmetike*, London, 1631.

[40] *Magnus Canon Logarithmorum ... Typis sinensibus in Aula Pekinensi jussu Imperatoris*, 1721.

[41] (Editor) On Ajima and others, see Helaine Selin (ed.), *Encyclopaedia of the history of science, technology, and medecine in non-western cultures*, Dordrecht: Kluwer, 1997.

Figure 3: An anonymous manuscript table of logarithms. The top line reads 1 to 30 from right to left. The third line reads 64 to 93. The fifth line reads 127 to 156. The even lines are the values of the logarithms, to seven places. For instance, $\log 127 = 2.1038037$ and $\log 130 = 2.1139434$. The characteristic is given every ten values, except when it changes (for instance when going from 9 to 10).

so numerous that it is tedious to handle them. Since addition and subtraction are easier than multiplication and division, Europeans require their calculations involving the eight trigonometric lines[42] to be made by means of adding and subtracting logarithms. They do not know, however, how to obtain the angles when the three sides are given, or how to get the sides from the three angles,[43] by the use of logarithms alone."

The first extensive logarithmic table was printed by Koide Shūki (1797–1865) in 1844. Another one was published by Yegawa Keishi in 1857, in which the logarithms were given up to 10,000,[44] and in the same year an extensive table of natural trigonometric functions was published by Okumura and Mori Masakado, in their *Katsu-yen Hio*.

Although the tables were used more or less in the first half of the nineteenth century, the theory of logarithms remained unknown for a long time after it was understood in China. Ajima, Aida, Ishiguro, and Uchida Gokan seem to have been the first to pay any attention to the nature of these numbers, but few explanations were put in print until Takemura Kōkaku published his work in 1854. Since Uchida used only logarithms to the base 10, his theory as to developing them is very complicated.[45]

3.49 Louis VanHée (1914)

Louis VanHée, in his article "Première mention des logarithmes en Chine," *T'oung Pao* (通報), volume 15, 1914, pp. 454–457, writes:

Le premier livre chinois, dans lequel il est parlé de logarithmes est dû à Sié Fong-Tsou [薛鳳祚, xuēfèngzuò], élève du jésuite polonais Smogolenski. Jean Nicolas Smogolenski (1611–1656), noble polonais, renonça à son palatinat de Nekelse, en faveur de son frère, pour

[42] I.e., the six common functions together with the versed sine and the coversed sine.

[43] Of a spherical triangle.

[44] Endō, Book III, p. 135.

[45] Endō, Book III, p. 143.

entrer en 1635 chez les jésuites. Après avoir brillamment achevé ses études, dont deux années de droit, au collège romain, il eut l'honneur d'y soutenir « le grand acte de philosophie ».

Parti pour la Chine en 1644, il se trouvait en 1646, au Kiang-Nan [江南, jiāngnán]. De 1647 à 1651 nous le voyons au Tché-Kiang [浙江, zhèjiāng], soit avec le célèbre P. ALENI, soit avec le P. da CUNHA.

(...)

SIÉ Fong-tsou, [薛鳳祚, xuēfèngzuò] *Yi-fou*, naquit à Tse-tch'ouen [四川, sìchuān] [Chantong, 山东, shāndōng]. Dans sa jeunesse il suivit WEI Wen-Kw'ei [魏文魁, Wèiwénkuí] et restait partisan des vieilles méthodes. Dans la période Choen-tche [順治, Shùnzhì] (1644–1661) après avoir causé mathématiques avec l'Européen Smogolenski, il changea d'opinion et ne fit plus que décrire le système étranger. De là une dizaine d'ouvrages sur *l'Harmonie de la science Européenne et chinoise* ; son titre pour *les logarithmes toei-chou pi-lié*, correspond au terme technique des Européens *Kia-chou* ; dans les *4 lignes trigonométriques à la chinoise*, comme la division sexagésimale du degré employé par les Européens n'est pas commode, il réduisit les tables en notation centésimale d'après le vieux système chinois, et il ne donna que les 4 lignes sinus et cosinus, tangente et cotangente ; (...)

3.50 David Eugene Smith (1925)

In the second volume of his *History of mathematics*, 1925, David Eugene Smith writes (on page 518 of the 1958 edition which reprints Smith's last edition):[46]

Holland was the third Continental country to recognize the work of Napier and Briggs. In 1626 there was published a work by Adriaen Vlacq, assisted by Ezechiel de Decker.[47] In 1628 Vlacq republished Briggs's tables, filling the gap from 20,000 to 90,000. The tables in

[46](Editor) We have dropped all but the last original footnotes.

[47](Editor) For details on the collaboration of Vlacq and De Decker, see our reconstruction of the 1628 tables.

this work were reprinted in London by George Miller in 1631. It is interesting to note that the next complete edition of Vlacq's tables appeared in China.[48]

Later, on page 523, he wrote:

Logarithms in the Orient. Logarithms found their way into China through the influence of the Jesuits. The first treatise upon the subject published in that country was a work by one Sié Fong-tsu, a pupil of the Polish Jesuit John Nicolas Smogolenski (1611–1656). This treatise was published about 1650, although Smogolenski had already mentioned the theory in one of his works.[49] Vlacq's tables (1628) were reprinted in Peking, as already stated, in 1713.

3.51 James Henderson (1926)

In his *Bibliotheca tabularum mathematicarum*, 1926, page 78, James Henderson gives the following description of the Chinese tables:

Magnus Canon Logarithmorum, tum pro Sinibus ac Tangentibus, ad singula decem secunda, tum pro Numeris absolutis ab unitate ad 100000. Typis sinensibus in Aula Pekinensi jussu Imperatoris excusus 1721. (Pekin. 3 folio volumes printed on Chinese paper.)

I have not seen this work and the title is taken from p. III of the Introduction to Vega's *Thesaurus*, where he states that a copy had been offered to him for purchase in Vienna two years before (1792). This work evidently contains all the logarithms of numbers from the *Arithmetica Logarithmica* of 1628, and the log.-trigonometrical functions as given in the *Trigonometria Artificialis* of 1633. According to

[48] *Magnus Canon Logarithmorum ... Typis Sinensibus in Aula Pekinensi ...*, 1721.

[49] The *T'ien-pu Chên-yüan*, as stated in Volume I, page 436.

Montucla (*Histoire*, vol. III, p. 358) the Emperor, by whose command the tables were printed, was Kang-hi. Rogg (*Handbuch*, p. 408) confirms Vega as regards the title and has inserted the Emperor's name in parentheses. Rogg took the title from Brunet's *Manuel du Libraire*. It will be noticed that Vlacq's name is not given on the title-page.[50]

3.52 Louis VanHée (1926)

Louis VanHée, in his article "The great treasure house of Chinese and European mathematics," *The American Mathematical Monthly*, volume 33, number 10, 1926, pp. 502–506, writes on page 505:

> 19. Logarithms. This feature was introduced into China by the Polish Jesuit Smogolenski[51] (1611–1656) who, knowing that the Chinese of that period were not prepared for the theory, gave only the tables[52] and the mechanical rules for using them. In the 19th century the influence of the west led to a demand for the underlying principles, and these were adapted from English works, but with an absurd claim for originality for three Chinese writers, Ku, Li, and Tsou.

and on page 506:

> 25. Logarithmic tables. The Jesuits had, as stated under 19, introduced logarithms in the 17th century. So remarkable did this invention seem that the emperor, K'ang-Hsi, had a small table prepared for his own use. The compilation known as the *Shu-li Tsing-*

[50] See De Morgan, *English Cyclopaedia*, vol. VII, p. 998.
[51] In Chinese, Mu Ni-ko. This is derived from the original Polish form—Nicolas Smogulecki, *Ni-ko* being from *Nico*las, and *Mu* being from the syllable *Smo*.
[52] Described in his *T'ien-pu Chen-yüan*.

yün, a part of the *Lü-li Yüan-yüan* printed at Peking in 1713, reproduced the Vlacq tables of 1628.[53]

3.53 Cyrus Peake (1934)

In the article "Some aspects of the Introduction of Modern Science into China," *Isis*, volume 32, number 1, December 1934, pages 173–219, Cyrus Peake writes on page 176:

> Nevertheless, works published during and after this period of Catholic missionary activity, particularly in the field of mathematics, disclose marked western influence. For example, in the *Shu Li Ching Yün*[54] [數理精蘊, shùlǐjīngyùn] published in 1723 by Imperial order, there is a comprehensive summary of mathematics, incorporating such Western developments as had found their way to China. It contains the earliest description of European algebra and the earliest complete treatise on logarithms in Chinese.[55]

[53] Smith, I, 436; II, 524.

[54] (Editor) Peake's article does not give the Chinese characters, and instead gives the character numbers in Giles' *Chinese-English dictionary* (Shanghai, 1912). We have not reproduced these numbers here.

[55] Peake, C. H., *Nationalism and Education in Modern China*, (New York, 1932) pp. 215, 216. Also see Van Hée, Louis S. J., "Première mention des logarithmes en Chine," *T'oung Pao*, Vol. 15 (1914), pp. 454–457 and Wylie, *op. cit.*, pp. 89, 90. The first mention of logarithms appeared in the book *T'ien Pu Chên Yüan* [天步真原, tiānbùzhēnyuán] about 1650 by Hsieh Fêng-Tso [薛鳳祚, xuēfèngzuò], a student of the Polish Jesuit Smogolenski.

3.54 Henri Bernard-Maître (1945)

In his bibliography of Chinese adaptations of European works (*Monumenta Serica, volume 10, pp. 1–57, 309–388*), Henri Bernard-Maître gives the following entry on p. 380:

518. (Verbiest), "Tables de logarithmes", Xy[56] ca 1681 pour l'usage de K'ang-hi in-12 ou in-24 Cou 4882-4884.

3.55 Pierre Huard (1956)

In a summary of Chinese science based on the works of Needham and Sarton, ("Panorama de la Science chinoise et de quelques-unes de ses disciplines," *Revue de synthèse*, 1956, pp. 419–518), Pierre Huard gives the following short notices:

(p. 456) 1600–1750. Diffusion des notions élémentaires de géométrie, de trigonométrie et d'algèbre occidentale (*Tsié Ken Fang*). Dans leur ignorance du passé, les Chinois collaborateurs des Jésuites créent une terminologie nouvelle. A cette période appartiennent Sie Fong-Tso († 1680) [薛鳳祚, xuēfèngzuò], auteur du premier exposé chinois des logarithmes (*Kia Chou*) et de trigonométrie sphérique, issu de sa collaboration avec le R.P. Smogolenski (1611–56).

(p. 498) Sie Fong-Tso (vers 1650) [薛鳳祚, xuēfèngzuò]. Elève de Wei Wen-kouei [魏文魁, Wèiwénkuí] puis du R.P. Smogolenski. A le premier exposé en chinois des logarithmes (*kia chou*). [Cf. R.P. Van Heee, *T'oung Pao*, 1914, p. 454–456).]

(p. 507) Smogolenski (R.P. Nicolas), alias Mou Ni-ko [穆尼閣] (1611–56). Maître et ami de Sié Fong-tsou [薛鳳祚, xuēfèngzuò].

[56](Editor) Xy = Xylography. The attribution to Verbiest is possible, but not certain, given that Bernard-Maître refers to Maurice Courant's *Catalogue des livres chinois, etc.* (volume 2, 1910), and that one of the three references given by Courant (4882) is the one of interest here, and therefore not attributable to Verbiest.

Celui-ci a écrit un traité sur le calcul des éclipses (1650) qui est le premier livre chinois dans lequel il est fait usage des logarithmes, notion introduite en Chine par le R.P. Smogolenski.

3.56 Karl Menninger (1957)

The table given by Menninger in his *Number words and number symbols: a cultural history of numbers* (1992, first German edition in 1957), pages 461–462 (figure 4), appears nearly identical to those of the Paris observatory and the *Institut* in Paris. Menninger claims that this table is dated 1713, but the date is probably only inferred. According to Menninger, this table was not printed in movable type, but each page was obtained using a unique wooden plate.

3.57 Chesneaux and Needham (1958)

In the chapter "Les sciences en Extrême-Orient du XVI^e aux XVIII^e siècle," *Histoire générale des sciences* (ed. R. Taton), volume 2, Paris, 1958, p. 683, Jean Chesneaux and Joseph Needham write:

> Le P. Rho publie en 1628 un traité d'analyse népérienne, que Smogulecki [Mu Ni-Ko, 穆尼閣] et Sie Fong-tsou [薛鳳祚, xuēfèngzuò] complètent en 1654 par de nouvelles tables de logarithmes.

And on page 686:

> (...) on a vu qu'on doit ainsi à Sie Fong-tsou [薛鳳祚, xuēfèngzuò], élève du jésuite Smogulecki [Mu Ni-Ko, 穆尼閣], le premier ouvrage purement chinois utilisant les logarithmes népériens : c'est un traité de calcul des éclipses, publié en 1650.
>
> (...)

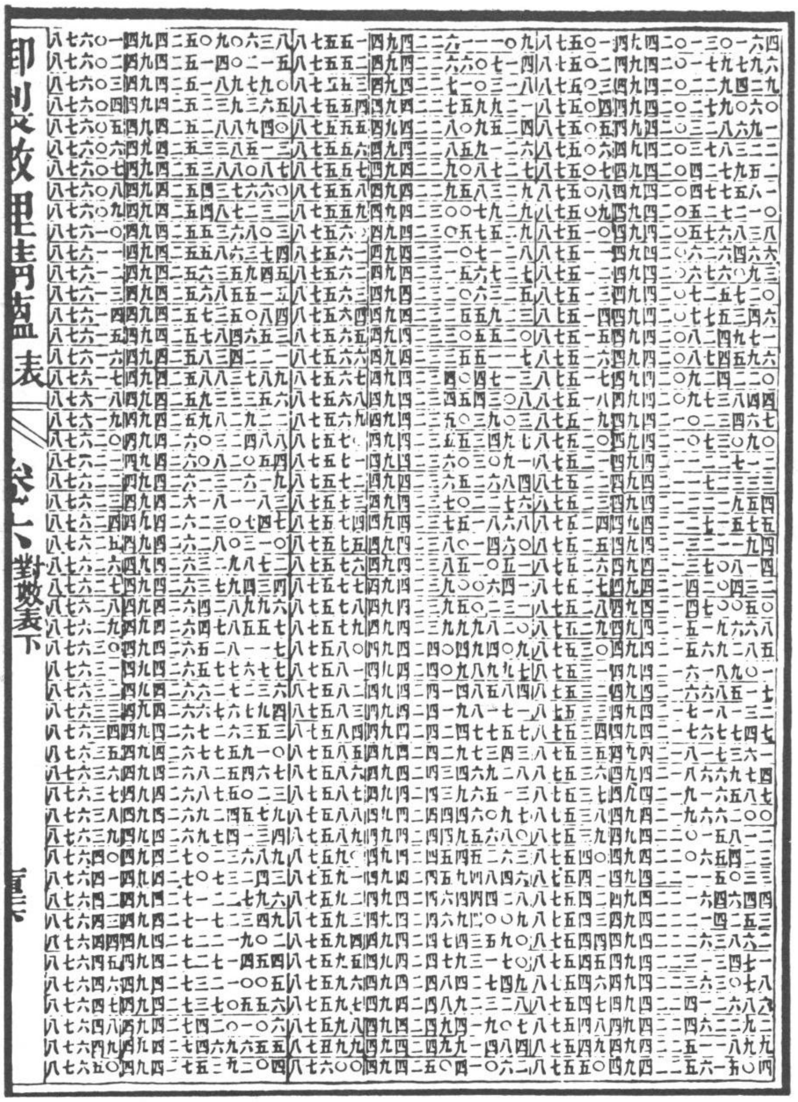

Figure 4: Menninger's table. The text on the left is partly covered but indicates that the table is part of the 數理精蘊 encyclopædia.

Mais c'est surtout dans les sciences mathématiques qu'on peut saisir combien la fusion fut incomplète entre science moderne et science chinoise traditionnelle. La grande encyclopédie scientifique publiée en 1723, sur l'ordre même de K'ang-hi [康熙, kāngxī], le *Liu-li yuan-yuan* (« Océan des calculs du calendrier ») [律歷淵源, lǜlìyuānyuán], due aux lettrés Ho Kouo-tsong et Mei Kou-tch'eng ainsi qu'à leurs nombreux collaborateurs, fait certes la plus large place aux travaux modernes ; la seconde partie en est consacrée aux progressions, opérations arithmétiques, racines carrées, calculs trigonométriques, logarithmes, exposés à l'européenne ; la troisième partie, relative à la théorie de la musique et aux instruments de musique chinois et occidentaux, est directement l'œuvre des PP. Pereira (jésuite) et Pedrini (lazariste).

And on page 694, about Japan:

[l]es logarithmes pénètrent au Japon en 1767, par l'édition japonaise d'un traité chinois de logarithmes.

3.58 Joseph Needham (1959)

In *Science and Civilisation in China*, volume III, Cambridge University Press, 1959, pp. 52–53, Joseph Needham writes:

With the coming of the Jesuits to Peking at the beginning of the 17th century, the period of what may be called 'indigenous mathematics' for the purpose of the present book comes to an end. Those who wish to study the period of collaboration between Chinese scholars and the Jesuit mathematicians will find a careful chapter in Mikami on the subject. The extent of the appreciation with which Matteo Ricci and his companions were received may be gauged when we remember that they were almost the only persons of foreign birth who ever attained the distinction of having their biographies admitted to the Chinese official histories. The translation of the first six books of Euclid into Chinese was undertaken by Ricci (Li Ma-Tou) [利玛窦] and Hsü Kuang-Chhi [徐光启, Xu Guangqi]; it was completed in +1607. The *Thung Wên Suan Chih* [同文算指, tóngwénsuànzhǐ] (Treatise on European Arithmetic) was dictated by Ricci and recorded by Li Chih-Tsao [李之藻, Li Zhizao]; it was printed in +1614. These were followed by books on more advanced geometry and surveying. Later in the century there came (+1669) the *Hsin Fa Shuan Shu* [新法算书, xīnfǎsuànshū] (Mathematical Methods of the New Calendrical System) which had been compiled by Adam Schall von Bell (Thang Jo-Wang [汤若望, tāngruòwàng]) and other Jesuits before +1635. Logarithms first appear in the *Thien Pu Chen Yuan* [天步真原, tiānbùzhēnyuán] (True Course of Celestial Motions), a treatise on eclipses by Nicholas Smogulecki (Mu Ni-Ko [穆尼閣]); and his pupil Hsüeh Fêng-Tsu [薛鳳祚, xuēfèngzuò] produced in +1653 the first Chinese logarithmic tables together with a discussion of them.[57] After the beginning of the 18th century, compendia of mathematics were compiled and issued by imperial order in the Khang-Hsi [康熙, kāngxī] reign period; there was the *Lü Li Yuan Yuan* [律历渊源, lǜlìyuānyuán] (Ocean of Calendar Calculations) of +1713 (in which van Vlacq's logarithm tables of +1628 were reprinted in Chinese form), and the *Shu Li Ching Yün* [數理

[57](Editor) This book was the *Bili duishu biao* [比例对数表] (tables of the logarithms of proportion), 对数 being the Chinese word for logarithm.

精蘊, shùlǐjīngyùn] (Collected Basic Principles of Mathematics) of +1722. From this time onwards the Chinese mathematical literature becomes voluminous, but, though still somewhat isolated, it is part of the world literature.

3.59 Wong (1963)

In his article on "China's opposition to Western science during the late Ming and early Ch'ing," *Isis*, volume 54, part 1, 1963, pp. 29–49, George H. C. Wong wrote on page 37:

The use of logarithms was described and explained in Joan Nicolas Smogulecki's (1611–1658) *T'ien-pu chên-yüan* (1644?), particularly in the section entitled *Pi-li tsui-shu piao* (Table of Logarithms) (in 10 *chüan*). The Jesuits introduced algebra into China during the seventeenth century using the title *A-êrh-je-pa-ta* (or A-êrh-je-pa-la) commonly known to the Chinese as *Tung-lai-fa* (Method from the East) or *Chieh-ken fang-fa*. Even though no formal book existed the main algebraic ideas were expressed in the *Shu-li ching-yun* (*chüan* 31–36), a compendium on Western and Chinese mathematics compiled with the collaboration of the Jesuits under Imperial auspices between 1723 and 1756.

3.60 Charles Naux (1971)

In his *Histoire des logarithmes de Neper à Euler*, volume 2, Paris: A. Blanchard, p. 30, Charles Naux writes:

La théorie des logarithmes a été introduite en Chine, au cours de la seconde moitié du XVII[e] siècle, par des missionnaires, des jésuites en particulier, venus dans l'espoir de répandre leur religion. La puissance de la tradition leur a interdit de réaliser leurs desseins, tandis que, grâce à un heureux concours de circonstances, la présence d'un empereur, Khang-Hi [康熙, kāngxī], aimant les mathématiques, leur a permis de diffuser la culture scientifique latine dans le monde des mandarins pourtant bien peu perméable au désengagement de leur culture.

La science chinoise accordait beaucoup d'intérêt à l'astronomie, non par amour désintéressé de la science, mais parce qu'elle permettait de prédire les éclipses, et que celles-ci étaient considérées comme jouant un rôle déterminant dans le cours des événements. Or, un jésuite belge, le Père Verbiest (arrivé en Chine en 1659, mort à Pékin en 1688), avait capté la confiance de Khang-Hi [康熙, kāngxī], en lui montrant la supériorité des astronomes européens sur ceux de la Chine, et la grande exactitude de leurs prévisions des éclipses. Dans un jeu qui fut parfois très périlleux, et demanda des prodiges de diplomatie, les Pères Jésuites eurent la satisfaction de diffuser leurs connaissances scientifiques avec une facilité et une ampleur auxquelles ils n'avaient jamais osé croire. Et c'est dans leur programme d'éducation que fut introduit l'usage du calcul logarithmique dont le succès fut si brillant qu'il fallut composer des tables en langage et en symboles numériques chinois.

D'après les derniers missionnaires revenus de Chine, il en existerait encore des exemplaires qui furent d'abord réunis à Khazan.

3.61 Yabuuti Kiyosi (1974)

In his book "中国の数学" (Chinese mathematics), Yabuuti wrote:[58]

Les logarithmes, inventés par le mathématicien anglais John Neper au début du XVII^e^ siècle, furent introduits en Chine peu après le *Livre du calendrier de l'ère Chongzhen.* Le jésuite polonais Nikolaus Smogulecki (en chinois Mu Nige), missionnaire à Nankin au début de la dynastie Qing, semble avoir initié certains Chinois à la théorie héliocentrique. Il s'était lié d'amitié avec des savants comme Xue Fengzuo, Fang Yizhi et son fils Fang Zhongtong. Après la mort de

[58](Editor) We quote the French translation published in 2000.

Smogulecki en 1656, Xue Fengzuo, qui avait été son élève, publia la *Somme de la science calendaire*, en douze sections. Cet ouvrage contient un corpus de connaissances en mathématiques, pharmacologie, physique, hydraulique et sur les armes à feu ; il comporte notamment trois sections intitulées respectivement « Table de proportions des logarithmes », « Nouvelle table de proportion des quatre lignes » et « Méthode de calcul du triangle ». La première donne les logarithmes de base 10 des nombres de 1 à 20 000, avec six chiffres après la virgule. La deuxième donne les logarithmes de quatre lignes trigonométriques : le sinus, le cosinus, la tangente et la cotangente ; elle s'appuie sur les travaux du mathématicien anglais Henry Briggs. La troisième donne entre autres choses les formules de l'angle moitié et les formules d'analogie de Neper. L'invention des logarithmes a révolutionné les techniques de calcul. Ils simplifiaient en effet considérablement les calculs les plus compliqués ; Kepler fut le premier à les utiliser pour les calculs astronomiques.

3.62 Leo Miller (1979)

In hist study on "Milton and Vlacq," *Papers of the Bibliographical Society of America*, volume 73, 1979, 145–208, Leo Miller writes on pp. 162–163:

Vlacq's larger volumes continued long in use, but apparently the demand for copies was satisfied by the first editions. His tables found their way to the Far East, appearing in a Chinese text in 1713; and in 1721 his work is said to have received the extraordinary accolade of a Peking imprint, by order of the Emperor K'ang-Hsi, in three volumes folio, *Magnus Canon Logarithmorum, tum pro Sinibus ac Tangentibus, ad singula decem secunda; tum pro Numeris absolutis ab unitate ad 100000. Typis sinensibus in Aula Pekinensi jussu Imperatoris excusus 1721.*

By 1792 or so a copy of this Peking set had drifted back to Germany, and is there acquired by Georg Vega, major in the artillery of the Holy Roman Empire and professor of mathematics, corresponding member of the British Royal Society. In 1794 Vega brings out in German and Latin a *Thesaurus Logarithmorum Completus, ex Arithmetica Logarithmica, et ex Trigonometria Artificiali Adriani Vlacci Collectus, Plurimis Erroribus Purgatus, In Novum Ordinem Redactus | Vollständige Sammlung grösserer logarithmisch-trigonometrischer Tafeln, nach Adrian Vlack's Arithmetica Logarithmica und Trigonometria Artificialis, verbessert, neu geordnet und vermehrt*, published by Weidmann at Leipzig in two volumes folio. Vega's edition became standard thereafter. It has been reproduced in facsimile as *10 Place Logarithms* by G. E. Steichert & Co. in 1923 on poor quality paper, and in reduced size facsimile in good format by Hafner Publishing Co., New York, 1958.[59]

3.63 Ho Peng Yoke (1985)

In the book *Li, Qi and Shu: An Introduction to Science and Civilization in China*, 1985, Ho Peng Yoke writes on page 108:

(...) Jean Nicolas Smogolenski (1611–56), known in China as Mu Nige 穆尼閣, together with his pupil Xue Fengzu 薛鳳祚 (?–1680) completed a treatise on eclipses entitled *Tianbu zhenyuan* 天步真原 (True Course of Celestial Motions) in 1653, which included in it a book called *Sanjue suanfa* 三角算法 (Methods of Trigonometry). It

[59] We know the 1721 Peking edition only from Vega's *Thesaurus*, where the Latin title is given in the *Einleitung*, page III, with the spelling *Sinibus*. I have so far not located any copy. The Vatican and the National Library of Peking report none. Joseph Needham, *Science and Civilization in China*, Cambridge, England, volume 3, p. 53, says that Vlacq's 1628 tables were printed by imperial order in Chinese in the *Lū Li Yuan Yuan* ("Ocean of Calendar Calculations") in 1713. Professor Spence, Yale University, helpfully writes me, 17 March 1978, that *Lū-li yuān-yüan* was ordered in 1713, published in 1723, and the tables of logarithms are included in its second section *Yū-ting shu-li ching yūn*. (Anyone with capability both in mathematics and in Chinese script can find these at Kent Hall, Columbia University, in elegant brush strokes on thousands of tissue paper pages sewn together with basting stitches in eighteen clothbound cartons secured by none latches.) It is possible that Vlacq's name does not appear in the 1713, 1721, or 1723 publications.

showed the use of the logarithmic tables and how to use logarithms of trigonometrical functions, and brought such tables to China for the first time.

On page 110:

The eighteenth century was also a period for the collection of mathematical texts for inclusion in compendia. There was *Luli yuanyuan*, which included the three works *Lixiang kaocheng* 曆象考成 (Compendium of Calendrical Science and Astronomy), based on the work of Cassini and Flamsteed and published by Imperial Order in 1713; *Lulu zhengyi* 律呂正義 (Collected Basic Principles of Music), written in 1713; and *Shuli jingyun* 數理精蘊 (Collected Basic Principles of Mathematics), written in 1721. They were published together under a single title in 1723.

3.64 Lǐ Yǎn and Dù Shí rán (1987)

In the book *Chinese mathematics: a concise history*, 1987, the authors write on page 191:

The first phase of Western mathematics passing into China centred on the emendation of the calendar. This phase lasted from Matteo Ricci's arrival in China (1582 AD) until the Yōng Zhēng (雍正) reign of the Qīng (that is, the beginning of the 18th century): in all more than than 150 years. The main historical events during this phase of the entry of Western mathematics into the East were:

(a) in the initial stages translation into Chinese of two books: Euclid's *Elements of Geometry* and Clavius' *Epitome of Practical Airthmetic*;

(b) during the end of the Míng and the beginning of the Qīng availability of the mathematical knowledge for the computation of the Western calendar;

(c) the mathematical research of Méi Wéndǐng (梅文鼎, 1633–1721 AD); and

(d) The editing of the *Collected Basic Principles of Mathematics* (數理精蘊, Shù lǐ jīng yùn) under Emperor Kāng Xī (康熙).

In the initial phase the most important elements of the mathematics entering China were Euclid's geometry, the techniques of calculating with pen and paper, (plane and spherical) trigonometry, and logarithms.

On pages 204–205, they write:

While the Polish missionary Nicolas Smogulecki was evangelizing in Nánjīng, Xuē Fèngzuò (薛鳳祚, ?-1680 AD) and Fāng Zhōngtōng (方中通, 1633–1698 AD) studied under him. Xuē Fèngzuò also published a collection of books called *Understanding Calendar Making* (曆學會通, Lì xué huì tōng) jointly with him. (There were prefaces written in 1652 and 1654.) Among these the most important were two volumes introducing logarithms: *New Tables for Four Logarithmic Trigonometric Functions* (比例四线新表, Bǐ lì sì xiàn xìn biǎo) one chapter, and *Logarithm Tables with Explanations* (比例對數表, Bǐ lì duì shù biǎo) one chapter.

On pages 208–209:

Smogulecki and Xuē Fèngzuò's works also brought in the methods for calculating the sides and angles of triangles by using logarithms, so they also introduced logarithmic trigonometric functions (details below).

(...)

In 1653 AD the missionary Smogulecki taught Xuē Fèngzuò logarithms. There is a brief introduction to logarithms in their joint work *Logarithm Tables with Explanations* (比例對數表, Bǐ lì duì shù biǎo). At that time they were not called 'logarithms' but 'corresponding numbers' (比例数, Bǐ lì shù) or 'power numbers' (假數, jià shù). In Chapter 12 of the book *Logarithm Tables with Explanations*, six-figure tables of logarithms for 1 to 10 000 are given, for example:

Number	Corresponding number (logarithm)
1	0.000000
2	0.301030
3	0.477121
4	0.602060
...	...

Logarithms were originally brought in for their convenience in astronomical calculations. The calculations in various books written by Smogulecki and Xuē Fèngzuò were done using logarithms. They said: 'Change multiplications and divisions into additions and subtractions' and 'it saves six or seven tenths of the work compared with the earlier procedures and in addition to that there is no worrying about errors in multiplications and divisions'. In the book *Essentials of Trigonometry* (三角法要, Sān jiǎo fǎ yào) they introduced general methods for various types of logarithmic trigonometric calculations. For example, the sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C},$$

was changed to

$$\log b = \log a + \log \sin B - \log \sin A$$

for calculation, etc.

Later on, in the *Collected Basic Principles of Mathematics* (1723 AD) edited by Emperor Kāng Xī, there were more detailed discussions of logarithms. Before this book came out logarithms had no great influence on Chinese mathematics and people using logarithms were rare.

On pages 220–221:

Most of the various sorts of calculation in the *Collected Basic Principles of Mathematics* do not go beyond the limits of the various mathematics books compiled by Méi Wéndǐng or the *Treatise on European Arithmetic*, but towards the end of the 'Conclusion' there are some new items that had not previously appeared. These are the method of computing tables of logarithms and the introduction of the completing the square method of solution of quadratic equations.

As mentioned above, the use of logarithms had already been introduced by the work of the missionary Smogulecki and Xuē Fèngzuò. However the method for computing these tables of logarithms was recorded for the first time in the *Collected Basic Principles of Mathematics.*

In chapter 38 of the *Collected Basic Principles of Mathematics, Logarithms* it says: '1 is the beginning of number, when used to multiply or divide, the number does not change, so the logarithm of 1 is defined to be 0. The logarithm of 10 is 1, the logarithm of 100 is 2, the logarithm of 1000 is 3, the logarithm of 10 000 is 4, ... similarly going on to myriads of myriads just add on 1 each time. This is the key to logarithms.' This is to say, first of all set

$$\begin{aligned} \log 1 &= 0, \\ \log 10 &= 1, \\ \log 100 &= 2, \\ \log 1000 &= 3, \\ &\dots \end{aligned}$$

But for the intermediate values between 1 and 10, 10 and 100 etc., how are the logarithms to be computed? Three types of method were presented in the *Collected Basic Principles of Mathematics.* This is the first time the computation of logarithms was presented in China. Although in the Europe of that time Gregory and N. Mercator (1668 AD) had already used the method of series to compute such tables, nevertheless this advanced method of getting the tables did not pass into China. Of the three methods in the *Collected Basic Principles of Mathematics*, the two that were more practicable both required taking successive square roots and were very complicated. The method of series was only eventually adopted by Dài

Xǔ (戴煦) and Lǐ Shànlán (李善蘭). However their methods were not influenced by the West, they were derived through their own independent thought and research (details in the next chapter).

On pages 252–253:

It is also worth selecting some important points from the investigations with the technique of 'cones' by Lǐ Shànlán, Dài Xǔ's work on logarithms and Lǐ and Dài's work in the area of the expansion of trigonometric functions in order to introduce them.

In the three books *Explanation of the Square and the Circle* (方圓闡幽, Fāng yuán chǎn yōu), *Unveiling the Secrets of Arc and Sagitta* (弧矢啓秘, Hú shǐ qǐ mì) and *Seeking the Source of Logarithms* (對數探源, Duì shù tàn yuán) (completed before 1846 AD), Lǐ Shànlán used his unique technique of 'circular cones' to bring out a lot of significant results.

(...)

In his book *Seeking the Source of Logarithms* Lǐ Shànlán also used this type of technique of 'cones' to obtain a formula for the natural logarithm, $\log_e n$, of a number n as:

$$\log_e n = \left(\frac{n-1}{n}\right) + \frac{1}{2}\left(\frac{n-1}{n}\right)^2 + \frac{1}{3}\left(\frac{n-1}{n}\right)^3 + \frac{1}{4}\left(\frac{n-1}{n}\right)^4 + \dots$$

Dài Xǔ used another method in his *Concise Technique of Logarithms* (對數簡法, Duì shù jiān fà, 1846 AD), where he also derived a formula for $\log_e n$. Using this formula to compute logarithmic tables is much simpler and faster than the method used in the *Collected Basic Principles of Mathematics*.

3.65 Han Qi (1992)

In his article 《數理精蘊》對數造表法與戴煦的二項展開式研究 ("Briggs' Method for Calculating Common Logarithm in Shu Li Jing Yun and Its Influence on Dai Xu's Study of Binomial Theorem,"), *Studies In The History Of Natural Sciences*, Vol. 11(2), pp. 109–119 (1992), Han Qi (韓琦) explains the methods used to compute logarithms in China, and in particular in the Shuli Jingyun.[60] On page 111, he uses the example of $\log 80$ in order to support a theory according to which the first tables in three volumes and in red and black were based on Briggs' table, and that Briggs' errors were corrected in the Shuli Jingyun, and that this was the reason for the new versions. However, this conclusion cannot be drawn on this sole example, because Briggs' error for $\log 80$ was also copied by Vlacq, although apparently many copies of Vlacq's 1628 table had it corrected by hand. Therefore, it is likely that Vlacq's tables were the only tables used for the red and black tables, and the Shuli Jingyun. It is possible that the authors of the two sets of tables used different versions of Vlacq's tables, one with the hand correction, and the other without.

3.66 Du Shi-ran and Han Qi (1992)

In "The Contribution of French Jesuits to Chinese science in the seventeenth and eighteenth centuries," *Impact of science on society*, volume 167, 1992, pp. 265–275, Du Shi-ran and Han Qi write on pages 272–273:

The new mathematical knowledge in *Shu li jing yun* [數理精蘊, shùlǐjīngyùn] included Pardies' *Elemens de Geometrie*,[61] *Suan fa yuan ben* (i.e. the seventh volume of Euclid's *Elements*),[62] an English mathematician, H. Briggs' method for calculating logarithm

[60] (Editor) We do not, for the moment, provide an excerpt of this article.

[61] Most of this book was translated into Manchu and Chinese.

[62] Han Qi, The Introduction of Western Mathematics during the Kangxi's Reign and Its Influence on Chinese Mathematics. Doctoral Dissertation, Beijing, 1991 (Institute for the History of Natural Sciences, Academia Sinica), pp. 29–31.

published in his *Arithmatica Logarithmetica* [sic] in 1624,[63] a Dutch mathematician, A. Vlacq's logarithm table,[64] and the Newton-Raphson and other methods for the solution of equations of higher degree. According to the French Jesuits' records, Bouvet and his confrères translated Pardies' *Elemens de Geometrie*, whilst Foucquet used Vlacq's book when he taught the Emperor.[65]

(...) After *Shu li jing yun* was published in 1723, no new mathematical knowledge was translated during the remainder of the eighteenth century. So it was that *Shu li jing yun* remained a standard textbook into the following century, and the methods included in it were taken as mathematical models for Chinese mathematicians in the nineteenth century.

The influence of *Shu li jing yun* has three aspects: firstly, the method of *Lian bi li* (continued proportions) for calculating infinite series; secondly, the *Jie gen fang* (symbolic algebra) [借根方, jiègēnfāng] and the rules for calculating polynomials; and lastly, the Briggs' method for calculating logarithms.[66]

3.67 Wann-Sheng Horng (1992)

In his article on "19th century Chinese mathematics" (*Philosophy and conceptual history of science in Taiwan*, 1992), Wann-Sheng Horng wrote on page 172:

As far as mathematical knowledge is concerned, the *Xi Yang Xin Fa Li Shu* includes a basic survey of the methods of (Western) plane and spherical trigonometry. In addition, Napier's so-called

[63] Han Qi, H. Briggs Method in the Shu li jing yun and the Chinese Mathematician Dai Xu's Study of Binomial Theorem. *Studies in the History of Natural Sciences*, vol. 11, no. 2, 1992.

[64] *ibid.*

[65] Witek, J. *Controversial ideas in China and in Europe: a biography of J. F. Foucquet (1665–1741).* Roma, 1982, p. 188.

[66] Han Qi, H. Briggs Method in the Shu li jing yun and the Chinese Mathematician Dai Xu's Study of Binomial Theorem. *Studies in the History of Natural Sciences*, vol. 11, no. 2, 1992.

calculation 'bones', Galilean proportional dividers and several other calculating devices were also introduced in these books. On the other hand, Xue Fengzuo published a collection of books in 1656 called *Li Xue Hui Tong* ('Understanding Calendar Making') based upon techniques he had learned from the Polish missionary Nicolas Smogulescki (sic) in Nanjing. Among these, the most important were two volumes introducing logarithms: the *Bi Li Si Xian Xin Biao* ('New Tables for Four Logarithmic Trigonometric Functions') and the *Bi Li Dui Shu Biao* ('Logarithmic Tables with Explanations'). These tables, invented by John Napier (1550–1617) and refined by Henry Briggs (1556–1630), were so indispensable to the Western method of computing calenders that 18th- and 19th-century Chinese mathematicians could no longer ignore their theoretical foundations (Li and Du, 1987, p. 205).

Xue Fengzuo (ca. 1620–1680), who came to Nanjing in 1653, further studied trigonometry and logarithms with Smogulescki.

3.68 Jean-Claude Martzloff (1994)

In his article "A glimpse of the post-Verbiest period: Jean-François Foucquet's *Lifa wenda* (Dialogues on calendrical techniques) and the modernization of Chinese astronomy or Urania's feet unbound," in *Ferdinand Verbiest (1623–1688): Jesuit missionary, scientist, engineer and diplomat*, 1994, Jean-Claude Martzloff wrote on page 526:

(...) Foucquet's works were conscientiously studied by the Kangxi emperor's third son, Yinzhi, and his team of astronomers:

> (Les manuscrits de Foucquet) sont aujourd'hui entre les mains du troisième Prince qui pour se passer des Jésuites fait calculer actuellement par des Chinois toutes nos tables du Ciel, des sinus, des logarithmes, etc. (Foucquet) a donné aux Chinois malgré nous toutes les nouvelles tables de la Hire réduites à ce méridien aux époques chinoises.

3.69 Agustín Udías (1994)

In his article "Jesuit Astronomers in Beijing," *Quarterly Journal of the Royal Astronomical Society*, 1994, volume 35, pages 463–478, Agustín Udías writes on page 473:

> In 1720, Ignaz Kögler [戴进贤] (1680–1746) born in Landsberg, Germany, was named president [of the Board of Astronomy]. (...) His main scientific work consisted in the renewal of the astronomical instruments. (...) He was also responsible for the publication in Chinese of astronomical and mathematical works such as tables of logarithms, catalogues of eclipses and observations of Jupiter's satellites.

3.70 Eli Maor (1994)

In *e: the story of a number*, 1994, Eli Maor writes:[67]

> Interestingly enough, the next country to embrace the new invention was China, where in 1653 there appeared a treatise on logarithms by Xue Fengzuo, a disciple of the Polish Jesuit John Nicholas Smoguleçki (1611–1656). Vlacq's tables were reprinted in Beijing in 1713 in the *Lü-Li Yuan Yuan* (Ocean of calendar calculations). A later work, *Shu Li Ching Yün* (Collected basic principles of mathematics), was published in Beijing in 1722 and eventually reached Japan. All of this activity was a result of the Jesuits' presence in China and their commitment to the spread of Western science.

[67](Editor)The author does not seem to be aware that the *Shuli Jingyun* is part of the *Lü-Li Yuan Yuan*.

3.71 Wang Yusheng (1996)

In "Li Shanlan: forerunner of modern science in China,", in Fan Dainian and Robert S. Cohen (ed.), *Chinese studies in the history and philosophy of science and technology*, Kluwer Academic Publishers, 1996, p. 350, Wang Yusheng writes:

Gu Guangguang [顾观光, gùguānguāng] wrote the preface to Li's *A search for the Origins of Logarithms (Dui Shu Tan Yuan)* (1845) and other books on 'the art of pointed cones.'[68]

(...)

During that time, Li Shanlan [李善蘭]'s most important research achievement was the 'art of pointed cones' (1845). Western mathematicians developed the theoretical foundations for calculus in the last half of the seventeenth century. However, because the Qing government had for a long time kept the country closed to international communication, Chinese mathematicians knew nothing about it, except for a very few expansions of the infinite series of trigonometric functions and the method of calculating logarithms which the missionaries had brought to China. Under such circumstances, Li suddenly came forward and, through diligent study, invented the art of pointed cones based on the art of figurate numbers and the method of limits in traditional Chinese mathematics.

(...)

It is especially noteworthy that Li Shanlan [李善蘭]'s logarithms, which were based on the art of pointed cones, have their own characteristic features which have been praised by both Chinese and foreign scholars. The British missionary A. Wylie (1815–1887) said,

> By an original train of thought, he [Li Shanlan] has arrived at something like the same result as Gregory St. Vincent, when he discovered the Quadrature of the Hyperbola in the 17th century... He has here given us a theorem, which in the days of Briggs (1556–1631) and Napier (1550–1617), would have been sufficient to raise him to distinction.

[68](Editor) On this method, and on the 垛積 [duǒjī] method in general, see Tian Miao, "The Westernization of Chinese Mathematics: A case study of the *duoji* method and its development," *East Asian Science, Technology, and Medicine*, volume 20, 2003, pp. 45–72.

Gu Guanguang [顾观光, gùguānguāng] found that Li's method of searching for logarithms was better and faster than the method the missionaries brought to China. Gu, who believed that foreigners "deliberately presented a complicated calculating method to bewilder the public," praised "Li [Shanlan], Dai [Xi][69] and others who could grasp its essence and develop it," loudly proclaiming "so as to tell those Chinese who are deceived and not yet awake."

[69](Editor) Probably Dai Xu, 戴煦, 1805–1860.

3.72 Peter Engelfriet (1998)

In his book *Euclid in China*, 1998, Peter Engelfriet writes, page 352:

> An important exception to this pattern was the Polish Jesuit Smogulecki, who resided in Nanjing from 1651–3, and who was responsible for the introduction of logarithms into China. His most prominent pupil was Xue Fengzuo, who became, together with Mei Wending and Wang Xishan, the most capable astronomer of the period. In Chinese sources, Smogulecki is praised for having been willing to discuss scientific matters without trying to convert people.

3.73 Catherine Jami (1998)

In "Traductions et synthèses : les mathématiques occidentales en Chine, 1607–1782," *L'Océan Indien au carrefour des mathématiques arabes, chinoises, européennes et indiennes* (ed. D. Tournès), pages 117–126, 1998, Catherine Jami writes page 119:

> Dans l'œuvre de Xue Fengzuo [薛鳳祚, xuēfèngzuò] (1600–1680), les mathématiques se présentent un peu comme dans le *Chongzhen lishu* [崇祯历书, chóngzhēnlìshū] : des connaissances incluses dans un corpus centré sur un autre intérêt. Ce qui lui vaut d'être mentionnéici est la présence de tables logarithmiques, les premières en Chine. Xue Fengzuo est l'un des premiers savants qui, sous les Qing, entreprirent des recherches en mathématiques et en astronomie indépendamment des institutions impériales. Il tenait les tables logarithmiques du jésuite Nicolas Smogulecki (1611–1656), dont l'enseignement scientifique dans la ville de Nankin apparaît comme marginal à côté du travail fait par ses confrères de Pékin, alors que, paradoxalement, Nankin restait un grand centre intellectuel.

3.74 Andrea Bréard (2001)

In the chapter "On mathematical terminology: culture crossing in nineteenth-century China," in *New terms for new ideas: Western knowledge and lexical change in late Imperial China*, Leiden, 2001, Andrea Bréard writes about Li Shanlan, p. 317:

> Published in Nanjing in 1867, the *Zeguxizhai suanxue* 則古昔齋算學 (Mathematics from the *Zeguxi*-Studio) contain essays inspired by Western mathematics on the basis of Jesuit works (logarithms, conic sections, infinite series, prime numbers, etc.) as well as independently developed essays on traditional themes.

and later on page 322, citing Alexander Wylie:

> Although this is the first time that the principles of Algebraic Geometry have been placed before the Chinese (so far as the translator is aware), in their own idiom, yet there is little doubt that this branch of the science will commend itself to native mathematicians, in consideration of its obvious utility; especially when we remember the readiness with which they adopted Euclidâs Elements of Geometry, Computation by Logarithms, and other novelties of European introduction. A spirit of inquiry is abroad among the Chinese, and there is a class of students on the empire, by no means small in number, who receive with avidity instruction on scientific matters from the West.

3.75 Han Qi (2003a)

In "L'enseignement des sciences mathématiques sous le règne de Kangxi (1662–1722) et son contexte social," *Éducation et instruction en Chine*, vol. II (ed. Christine Nguyen Tri and Catherine Despeux), 2003, pp. 69–88, Han Qi writes on p. 80:

Toutefois, la *Somme des principes mathématiques essentiels* [數理精蘊, shùlǐjīngyùn] rapporte que les nouvelles connaissances mathématiques qui y furent introduites étaient la méthode de calcul des logarithmes décrite par le mathématicien anglais Henry Briggs dans son ouvrage *Arithmetica Logarithmetica* [sic][70] (1624) ainsi que la table logarithmique du mathématicien néerlandais A. Vlacq, méthode de résolution des équations du plus haut degré.

3.76 Han Qi (2003b)

In an article on "Antoine Thomas, SJ, and his mathematical activities in China," published in *The history of the relations between the low countries and China in the Qing era (1644–1911)*, (ed. W.F. Vande Walle), Leuven, 2003, Han Qi writes p. 108 that Thomas (1644–1709) compiled some Chinese tables of sines, cosines, tangents and logarithms, and that a few of these tables are found in the Palace Museum of Beijing.

3.77 Benjamin A. Elman (2003)

In "*The Jesuit Role as "Experts" in High Qing Cartography and Technology*," 2003, pp. 223–250, Benjamin A. Elman writes:[71]

Xue Fengzuo 薛鳳祚 (1600–1680) studied astronomy and mathematics under the Jesuit Jean-Nicholas Smogolenski. Xue then applied the techniques of spherical trigonometry and logarithms to surveying, which was appreciated by the Qianlong Imperial Library editors in their review of Xue's *Compendium on the Yellow River and Grand Canal* (兩河清彙). The editors noted that Xue's mathematical expertise was an invaluable aid in analyzing problems related

[70](Editor) The correct title is *Arithmetica logarithmica*.

[71]This article is a preliminary version of a chapter in Elman's book, see http://www.press.ntu.edu.tw/ejournal/files/history\200306\31_07.pdf.

to flood control and canal upkeep. His use of European trigonometry was recognized as a clear improvement over the native forms of trigonometry known as "double application of proportions" (*chongcha* 重差, i.e., properties of right triangles expressioned as a function of angles), which had dominated Chinese surveying techniques until that time.

3.78 Kangxi exhibition catalogue (2004)

In 2004, an exhibition was devoted to the Kangxi emperor at the Castle of Versailles. The catalogue of the exhibition *Kangxi, empereur de Chine : 1662–1722, La Cité interdite à Versailles*, Paris, 2004, contains several articles of interest.

In the chapter "Chronique de l'empereur au fil des jours," Jean-Paul Desroches writes page 46:

En 1681, Verbiest produit également une nouvelle table de logarithmes.[72]

In the chapter "Les sciences sous le règne de Kangxi," Catherine Jami writes page 76:

Pour enseigner à l'empereur, les jésuites rédigèrent et utilisèrent des traités. Un certain nombre d'entre eux, d'abord écrits en mandchou puis traduits en chinois, furent insérés dans une encyclopédie de mathématiques, d'astronomie et d'harmonie musicale qui parut

[72](Editor) This information seems to be extracted from Henri Bernard-Maître's bibliography of Chinese adaptations of European works (*Monumenta Serica, volume 10, pp. 1–57, 309–388*), but Bernard-Maître refers explicitely to the three tables given in Maurice Courant's *Catalogue des livres chinois, etc.* (volume 2, 1910), and we know that at least one of these tables is the one adapted from Vlacq and related to the Shuli Jingyun. This casts some doubt to the attribution of the other two tables to Verbiest, or at least it requires additional analysis.

en 1723, le *Lüli yuanyuan* [律历渊源, lǜlìyuānyuán], « Source du calendrier et de l'harmonie ». L'ouvrage fut compilé à partir de 1713 par des savants chinois travaillant sous patronage impérial. Une nouvelle institution, l'Office de mathématiques (*Suanxue guan*) avait été créée pour abriter ce projet. Aucun jésuite n'apparaît sur la liste des auteurs de l'ouvrage. À la fin de son règne, Kangxi a cherché à assurer l'appropriation autant que la diffusion des connaissances acquises auprès des jésuites. La partie mathématique du *Lüli yuanyuan* repose en grande partie sur les cours des jésuites ; elle est cependant présentée comme une synthèse des connaissances de la tradition chinoise et des « études occidentales ».

Finally, on page 266, there is a long description of the *Shuli Jingyun*:

Cet ouvrage composé de cinquante-trois volumes reliés, édité en 1717 par Mei Juecheng, assisté de Chen Houyao, Ming Antu et *al.*, avec la collaboration des jésuites Jean-François Gerbillon (1654–1707) et Joachim Bouvet (1656–1730) qui fournirent des traductions, fut imprimé à la cour à l'aide de caractères de cuivre mobiles pendant le règne de Kangxi.[73] Les demi-feuillets, bordés de quatre marges blanches délimitées par un double trait, comportent neuf lignes de vingt caractères chacune, suivies d'une illustration et de sa légende sur des lignes de quinze caractères.

Kangxi, qui aimait les sciences depuis l'enfance, avait une prédilection pour l'astronomie et les mathématiques. Cherchant à améliorer sa politique (*qiu zhi*), il invita à la cour des missionnaires occidentaux versés dans les sciences et techniques, à qui il attribua des postes importants. Son intérêt pour la question le conduisit à étudier assidûment l'astronomie et les méthodes de calcul occidentales de l'époque et, suivant la suggestion du mathématicien Chen Houyao de « publier tous les livres relatifs au mouvement des astres pour le bénéfice de l'empire », à fonder un Bureau d'Initiation à

[73] On the printing with movable copper type in the Shuli Jingyun, see Fan Jingzhong (范景中), "铜活字套印本《御制数理精蕴》," 歷史文獻, 2, 1999, pp. 263–277.

la science (*Mengyang zhai*) où furent compilés des ouvrages savants sous son patronage. Le *Yuzhi shuli jingyun* est un ouvrage de mathématiques vérifié par l'empereur Kangxi lui-même. L'ouvrage est constitué de deux parties, une première partie intitulée *Ligang mingti* (*Principes et systèmes*) en cinq volumes, qui contient les principes fondamentaux de mathématiques, le *Hetu* (*Plan du fleuve Jaune*) et le *Luoshu* (*Rouleaux d'écriture de la rivière Luo*), le *Zhoubi jingjie*, les principes fondamentaux de géométrie et de calcul, et une seconde partie intitulée *Fentiao zhiyong* (*Utilisation des différents systèmes*) qui traite de calcul, de géométrie plane, de trigonométrie plane, de géométrie dans l'espace et d'algèbre.

À cela s'ajoutent en annexe quatre tables de mathématiques en huit volumes dont la table des huit lignes (*baxian biao*) et la table des logarithmes (*duishu biao*), ainsi que les principes de la règle à calcul occidentale (*jisuan chi*). L'ouvrage regroupe donc des connaissances mathématiques occidentales et les résultats des recherches des savants du début de la dynastie des Qing, examine les différences entre sources chinoises et occidentales, fait le point sur les avantages et les inconvénients des mathématiques du passé et du présent, et aborde toutes les branches des mathématiques de l'époque, ce qui a valu le nom d'« encyclopédie mathématique ». Peu de temps après, cet ouvrage fut regroupé avec le *Lixiang kaocheng* (*Examen des signes de l'astronomie*) et le *Lülü zhengyi* (*Commentaire vérifié des traités d'acoustique et de musique*) dans un livre unique, le *Yuzhi lüli yuanyuan* (*Étude complète de la musique et du calendrier éditée sur ordre impérial*), synthèse des échanges scientifiques entre Chine et Occident, qui exerça une grande influence.

3.79 Benjamin A. Elman (2005)

In *On their own terms: science in China, 1550–1900*, Harvard University Press, 2005, Benjamin A. Elman writes:

(p. 100) To enhance the accuracy of the Chinese computus, Xu Guangqi [徐光启, xúguāngqǐ] recognized how important European astronomical tables, maps of the heavens, and instruments were for the reform. All of these accompanied the treatises presented to the throne from 1631 to 1635. For his astronomical tables, Rho had used early seventeenth-century calendars. Others used the 1551 Copernican Prutenic tables by Reinhold, which the Gregorian reformers had also employed. Kepler's Rudolphine Tables of 1627 were not available in China until a full set was sent to Beijing in 1646. The Chinese created correspondence tables between European calculation tables and Chinese measurements through a synthesis (*huitong*, lit., "combining and interpenetrating").

In addition, the Jesuits introduced trigonometric tables in the *Mathematical Astronomy of the Chongzhen Reign* and translated the Englishman Henry Briggs's (1556–1630) *Arithmetica Logarithmica*, which was published in Chinese in 1624.[74] Briggs was a professor of Gresham College, the original site of the British Royal Society until 1710, and later professor of geometry at Oxford. Nikolaus Smogulecki (Jean Nicholas, 1611–1656) and his collaborator Xue Fengzuo [薛鳳祚, xuēfèngzuò] (1600–1680) introduced trigonometric logarithms[75] in the early Qing. In an astronomical work, circa 1656, which pioneered spherical trigonometry and logarithms in China, Smogulecki introduced the latest European method for calculating eclipses. The use of both arithmetical and trigonometric logarithms enhanced the ease of astronomical calculations. For example, when Verbiest was asked by the Kangxi [康熙, kāngxī] emperor to prepare the groundwork for the Eternal Calendar (*yongnian li*), which was completed in 1678, he had to calculate its accuracy for two thousand years in the future. To accomplish this task, the Astro-calendric Bureau had to go through an enormous number of calculations that were made easier using trigonometric logarithms.

(...)

(p. 101) Xu Guangqi and Adam Schall completed a remarkable star atlas in 1631 while also presenting the throne with a treatise on fixed stars. The latter followed Brahe's methods and other works on positional astronomy. Schall had to transform the two coordinate systems, equatorial in China and ecliptic in Europe, even though

[74](Editor) In fact, it was published in England in 1624.

[75](Editor) The author means logarithms of trigonometric functions, which he opposes to "arithmetical logarithms," his name for the common logarithms of numbers.

Brahe, like late Europeans, chose equatorially mounted instruments and recorded in equatorial coordinates rather than ecliptic. Because trigonometric logarithms were not introduced until the early Qing by Smogulecki and Xue Fengzuo, the system of equatorial coordinates used in the Jesuit star atlas was classically Chinese and became universal in Europe only after Brahe's time.

About the mathematician Mei Wending [梅文鼎] (1633–1721), he writes:

(p. 153) As a youth, Mei studied astrology and mathematical astronomy (*tianwen lixue*) along with the Classics and Histories. When he came to Beijing for the 1666 capital region examination at the age of thirty-three *sui* (roughly thirty-two years old), which he failed, he acquired several books on European mathematical astronomy. From 1669 to 1678, Mei met Fang Zhongtong [方中通, fāngzhōngtōng] four times in Nanjing when Mei and Fang took the provincial examinations. Mei used to collect books on astronomy and mathematics. While in Nanjing in 1675, for instance, Mei located works on logarithms of trigonometric functions and celestial motions by Smogulecki and Xue Fengzuo. By 1675, he had also seen works from the *Mathematical Astronomy of the Chengzhen Reign* collectanea and thus had access to much of the mathematical tool kit introduced by the Jesuit mission. In 1678, Mei met the famous book collector Huang Yuji [虞稷, huángyújì] (1629–1691) and obtained a copy of the seminal *Computational Methods in Nine Chapters*, which although a mathematical classic was by then hard to come by in its complete form.

(...)

(p. 156) Mei's goal was to synthesize calculating techniques (*suanshu* [算術], i.e., arithmetic) and methods of measurement (*liangfa*, i.e., geometry) into a unified mathematics (*shuxue*) [数学]. Unlike Fang Zhongtong [方中通, fāngzhōngtōng], who reduced European mathematics to the ancient Chinese mathematical classics, Mei sought to rehabilitate traditional mathematics. In addition, he never denigrated what the Jesuits introduced because he recognized the limitations of traditional methods for solving problems in mathematical astronomy. He sought to use European achievements to advance traditional techniques, now reinvigorated with logarithms and trigonometry. In the late eighteenth century, this approach climaxed with the rehabilitation of the Ten Mathematical Classics.

(...)

(p. 159) Mei Wending was the first to explicitly use the traditional term for determining quantities (*jihe*) [几何] to mean geometry. In addition, Mei used trigonometry to apply the equivalent of spherical geometry, which he regarded as a simplification of traditional methods for transforming equatorial to ecliptic coordinates, etc. His writings on Napier's bones and the proportional compass argued that the new calculating devices developed by Napier and Galileo were essential aids for computational astronomy. Mei tied them to traditional counting rods and the late Ming use of logarithms.

When his grandson compiled Mei's collected works in 1761, forty years after his death, Mei Wending was recognized as the leading literatus who had mastered all the fields of European mathematics introduced to China.

Concerning the compendium of mathematics begun in 1713, he writes:

(p. 179) In 1712, Chen [Chen Houyao, 厚耀, chénhòuyào] proposed a new compendium of European mathematics to replace the Ming *Mathematical Astronomy of the Chongzhen Reign*. The result was the *Sources of Musical Harmonics and Mathematical Astronomy* (*Lüli yuanyuan*) [律历渊源, lǜlìyuānyuán] collectanea, which included the *Treasury of Mathematics*. In 1713, the Kangxi [康熙, kāngxī] emperor charged Mei Juecheng and Chen Houyao with supervising He Guozong [何國宗, héguózōng], Ming Antu[76] [明安圖, míngāntú], and others to complete the project. The *Sources* was printed in 1723. This special group of mathematical and astronomical specialists included Wei Tingzhen and those Mei Wending had helped

[76](Editor) Elman incorrectly writes Minggatu.

train before he died in 1721. All had benefited from Li Guangdi's court patronage.

Altogether the emperor recruited more that [sic] one hundred scholars—regardless of their civil examination status—to join the Academy of Mathematics. Mei Juecheng was chief and Minggatu assistant editor for preparation of the *Treasury*. In addition to those in the Academy of Mathematics who studied mathematics, astronomy, and music, a large number of instrument makers were hired for the technical needs of the new academy. A team of fifteen calculators verified the computations based on the theoretical notions, mathematical techniques and applications, and numerical tables in the first part of the *Treasury*.

Patterned after mathematical textbooks used in Jesuit colleges, the *Treasury* introduced European algebra. The last part had a section on logarithms to the base 10 and drew on the methods used in Briggs's 1624 *Arithmetica Logarithmica* to compute decimal logarithms. Although Briggs's work had been introduced in 1653, the *Treasury* explained the use of logarithms in greater detail, and it also included tables for sines, cosines, tangents, cotangents, secants, and cosecants for every ten seconds up to ninety degrees, as well as a list of prime numbers and a log table of integers from 1 to 100,000 calculated to ten decimal places. The emperor had the *Treasury* distributed empirewide. It was reprinted in 1875, 1882, 1888, 1896, and 1911, which suggests a more limited audience until the nineteenth century.

(...) The French Jesuits in the court significantly influenced the compilation of the *Sources of Musical Harmonics and Mathematical Astronomy* and its staff of over one hundred working under Wang Lansheng [王生] and Mei Juecheng.

About Xue Fengzuo, he writes on page 195:

We saw in Chapter 3 that Xue Fengzuo studied astronony and mathematics under the Jesuit Jean-Nicholas Smogulecki. Xue then applied the techniques of spherical trigonometry and logarithms to surveying, which was appreciated by the Qianlong Imperial Library editors in their review of Xue's compendium on the Yellow River and Grand Canal. The editors noted that Xue's mathematical expertise was an invaluable aid in analyzing problems related to flood control and canal upkeep. His use of European trigonometry was recognized as a clear improvement over the native forms of trigonometry known as "double application of proportions" (*chongcha* [重差, chóngchā], i.e., solving right triangles by using the proportions of their sides), which dominated traditional Chinese surveying techniques.

3.80 Eugen Gabowitsch (2005)

In the chapter "Ages in Chaos" in the second volume of Anatoly Fomenko's *History: Fiction or Science*, 2005, Eugen Gabowitsch writes on page XV:

One must also recollect the alleged invention of the logarithms in China that took place 500 years before they were invented in the Netherlands. The comparison of two publications, European and Chinese, demonstrates that *a misprint* from Napier's table of natural logarithms (first published in 1620) *was repeated* in a Chinese book that is presumed to be 500 years older. Is that the natural way of making history, one wonders?[77]

3.81 Jean-Claude Martzloff (2006)

In *A history of Chinese Mathematics*, Springer, 2006, p. 165, discussing the *Shuli Jingyun* [數理精蘊, shùlǐjīngyùn] encyclopedia published in 1723, Jean-Claude Martzloff writes:

As shown recently by a young Chinese historian of mathematics, Han Qi, the section on logarithms is an almost complete translation

[77](Editor) This excerpt was included here because it differs so much from the other sources, and it casts a doubt on the content of the whole book.

of the method used by H. Briggs (1556–1630) to compute decimal logarithms. Although logarithms were first introduced in China as early as 1653, long before the publication of the *Shuli Jingyun* [數理精蘊, shùlǐjīngyùn], the translation of Briggs's treatise surpassed what had been previously published in China on the same subject. In this section of the book, everything is explained at length with an almost unimaginable wealth of detail and numerical examples; even arithmetical operations are often fully worked out. This aspect of the compilation of the *Shuli Jingyun* [數理精蘊, shùlǐjīngyùn] is moreover characteristic of the whole book and the contrast with Chinese mathematical books published before the 17th century is striking.

Finally, the *Shuli Jingyun* [數理精蘊, shùlǐjīngyùn] also has eight chapters of tables for the sines, cosines, tangents, cotangents, secants and cosecants for every ten seconds up to 90 degrees. A list of prime numbers is also given as well as a table of logarithms of integers from 1 to 100000 calculated to ten decimal places and perhaps borrowed from Vlacq.

And, on page 174:

In 1852, Li Shanlan [李善蘭] took refuge in Shanghai in order to escape the Taiping revolt. There he met Alexander Wylie (1815–1887) from the London Missionary Society. Wylie was fluent in Chinese (he later became an outstanding sinologist) but not particularly versed in scientific subjects. However, like many other protestant missionaries, he rapidly gained an ability to face the problem of diffusion of fundamental scientific knowledge into the Chinese society which most Protestant missionaries considered essential for the Christianisation of China. But he also learnt about native mathematics and was favourably impressed by a little treatise on logarithms composed by Li Shanlan in 1846:

> Li Shanlan [...] now residing in Shanghai [...] has recently published a small work called *Tuy-soo-tan-yuen* [对数探源, duìshùtànyuán] in which he details an entirely new method for their computation, based upon ge-

> ometrical formulae [...] [he] has had no better aid than that afforded by the *Leuh-lih-yuen-yuen* [律历渊源, lǜlì-yuānyuán] and [...] has here given us as a result of four years' thought a theorem which, in the days of Briggs and Napier, would have been sufficient to raise him to distinction.

3.82 Zhang Xiping (2006)

In his book *Following the steps of Matteo Ricci to China*, 2006, Zhang Xiping wrote on page 71:

> Other books such as *Bili Duishu Biao* (Logarithm Table) jointly written by Nicolas Motel (sic) and Xue Fengzuo were collected in the books *Shuli Jingyun* (Collected Basic Principles of Mathematics) and *Lixiang Kaocheng* (Complete Studies on Astronomy and Calendar).

3.83 Zhang Xiping (2007)

In "Conversations between China and the West: The missionaries in early Qing Dynasty and their researches on the *Book of Changes*," *Frontiers of History in China*, 2(4), 2007, pages 469–492, Zhang Xiping writes on page 477:

> On June 17 [1713] that year, there is a report from He Su to Kangxi:
>
> Petrus Jartoux, Bernard-Kilian Stumpf, Franciscus Tilisch and Jean-Francoise [sic] Foucquet have translated the logarithm table into Chinese and named it *Catechism of Mathematic Table*, this version is better than the previous one, which they have sent one copy to

me. Stumpf said: "We tried our best to do the calculation and translation work, but it cannot be guaranteed that there is no mistake in our work. We are looking forward to Your Majesty's correction and suggestions, with which we will be able to continue our work. After the whole task is finishedm the new edition of the book should contain six or seven volumes."

On page 478, he writes:

There are also some documents in the Vatican Library recording Kangxi's interest in algebra:

This message should be sent to Wang Daohua. Recently, I have been studying the new theories of algebra with my sons since early morning. I could not understand why it is said that they were easier than the old ones. I feel the new theories are much more difficult, and have more mistakes than the old ones. I recommended the old ones to the European men in Beijing to help them edit the logarithm table. You should send this decree with the logarithm table to Beijing and have them studied carefully by the European men. The incorrect points should be deleted. A man told me that a number multiply the same number is unavailable for calculation, which was completely ridiculous and showed his ignorance in mathematics.

3.84 Shi Yunli (2008)

In his article on "Nikolaus Smogulecki and Xue Fengzuo's *Tianbu Zhenyuan* 天步真原: Its production, publication, and reception," *East Asian Science, Technology and Medicine*, 27, 2008, pp. 63–126, Shi Yunli writes:

According to Xue Fengzuo, twenty years after he studied traditional Chinese astronomy and mathematics from Wei Wenkui 魏

文魁, an astronomer known for his anti-Western stance during Xu Guangqi's astronomical reform, he arrived at Nanjing in the early spring of the ninth year of Shunzhi reign-period (9 Feb. 1652 — 28 Jan. 1653) and began to learn trigonometry, logarithm and the table of logarithmic trigonometry from Smogulecki. In less than two years, they co-authored a number of books and treatises, including the *Bili duishu biao* 比例对数表 (Table of the Proportional Logarithms), the *Bili sixian biao* 比例四線表 (Table of the Proportions of the Four Trigonometric Lines), the *Suan sanjiao fa* 算三角法 (Method of Calculating Triangles), the *Huofa* 火法 (Firearm Method), and the *Tianbu zhenyuan*.

and later, quoting Xue Fengzuo:

"I came to Nanjing in the Spring Day of the year *renchen* 壬辰 (11 Jan. 1652) and learnt trigonometry from Mr. Mu [Nige]. In addition, I also learnt from him logarithm and the table of logarithms of the four trigonometric lines. ... However, when I was engaged in the project of emendation and integration recently, I found that, whereas the Chinese method was too scattered and simple, the old [trigonometric] method was in sexagesimal system, both being incompatible with each other. Therefore, I integrated them by converting the sexagesimal system in both the eight trigonometric lines and the related books into centesimal system, so that the old and new, Chinese and Western can be unified into one system, which might become a ladder for this study [i.e. astronomy]."

and

(...) Xue Fengzuo really revised these works in accordance with the centesimal system, and the *Bili sixian xinbiao* is actually an independent work by himself, rather than by Smogulecki and him together as was believed before. Mei Wending also observed that "the *Bili sixian* 比例四 (Proportions of the Four Trigonometric Lines) was taught by Mr. Mu [Nige]", while "Yifu [i.e. Xue Fengzuo] compiled again the *Sixian xinbili* 四新比例 (New Proportions of the Four Trigonometric Lines), which gives the values of the four trigonometric lines. He uses 'degree' in the same way [as Mu Nige does], but divides one degree into hundred minutes."

3.85 Wang Ai-ling (2009)

Wang Ai-ling has recently published a communication on the method of making tables of logarithms developed by Dai Xu [戴煦, 1805–1860] ("Research on the Thinking of Logarithm and Binomial Express and Euler's Numbers in the Late Qing Dynasty," *2009 International Conference on Information and Multimedia Technology*, Jeju Island, South Korea, December 16–December 18) and we redirect to this article for more information on the subject.[78]

In this context, Han Qi's 1992 article on Briggs should also be consulted.

[78] `http://doi.ieeecomputersociety.org/10.1109/ICIMT.2009.100`