A reconstruction of the tables of the Shuli Jingyun (1713–1723)

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Introduction

The Shuli Jingyun (Collected Essential Principles of Mathematics) is an encyclopedia of mathematics, commissioned by the order of the Emperor Kangxi 康熙帝 (1654–1722) and covering almost all mathematical knowledge known in China at that time. It was part of a larger collection, the 律曆淵源 (lǘlì-yuānyuán, Sources of musical harmonics and mathematical astronomy), which was composed of three parts: the Lixiang kaocheng 曆象考成 (Compendium of observational and computational astronomy), the Shuli Jingyun, and the Lülü zhengyi 律呂正義 (Exact meaning of the pitch-pipes). The compilation of the Shuli Jingyun started in 1713 and one of the main editor of the work was the mathematician Mei Juecheng (1681–1763) [31, p. 163]. However, as observed by Jami and Han, "the Shuli jingyun was not merely the result of ten years of work of the scholars appointed to the Office of Mathematics. Instead, it was the final outcome of the lectures in mathematics that Kangxi received from several Jesuits since 1688, that is, 25 years before the work was commissioned, and 35 years before it was printed." [27, p. 3]

1 General structure of the tables in the Shuli Jingyun (數理精蘊)

The Shuli Jingyun is divided in three parts totalling 53 chapters (卷, juǎn). The first part is made of five chapters covering theoretical notions, the second part of made of forty chapters describing a number of mathematical techniques, and the third part is made of eight chapters of tables. This is the part with which we are concerned here.[1]

It is not known when exactly the tables were completed, but the whole encyclopedia was completed in 1723. The main part of the encyclopedia was set using movable copper type [23, p. 76], but the tables were certainly printed with xylography.[2] The Shuli Jingyun was imported in other countries, such as

[1]We have consulted the original tables at the *Institut des Hautes Études Chinoises* in Paris, and at the Lyon municipal library which only holds volumes 5–8. Catherine Jami and Han Qi are working towards an identification of the sources of the various parts of the Shuli Jingyun [27, p. 10]. We hope that our work will also add a small contribution to this effort. For some sources of the Shuli Jingyun, see Jami [20] and Peng [39, pp. 371–367].

[2]On the history of xylography in China, see [8].

Korea in 1729 [17, p. 483]. And sixty years later, the whole Shuli Jingyun was included in the Siku Quanshu [22, p. 194].

In the original Shuli Jingyun the tables were occupying eight volumes (table 1). According to Jami, these volumes were chapters 41 to 48 of the Shuli Jingyun [19, p. 406], but in fact the volumes only number the tables from 1 to 8.

The first two volumes gave tables of the six trigonometric functions, computed every 10 seconds of the quadrant (540 pages). The next two volumes gave tables of factors and prime numbers from 1 to 100000 (702 pages). The next two volumes gave the logarithms of numbers from 1 to 100000 (1000 pages). The final two volumes gave the logarithms of the trigonometric functions, every 10 seconds of the quadrant (540 pages).

Volume	Content
1	trigonometric functions (0° to 22°)
2	(22° to 45°)
3	primes and factors (1 to 50000)
4	(50001 to 100000)
5	logarithms of numbers (1 to 50000)
6	(50001 to 100000)
7	logarithms of trigonometric functions (0° to 22°)
8	(22° to 45°)

Table 1: Structure of the tables in the original Shuli Jingyun.

2 The Siku Quanshu (四庫全書)

The Shuli Jingyun came to be included in the Siku Quanshu (四庫全書, Complete Library of the Four Treasuries) collection. This collection was commissioned by the Qianlong emperor 乾隆帝 (1711–1799), emperor Kangxi's grandson, and was compiled between 1773 and 1782. It contained 3461 titles, bound in 36381 volumes and containing more than 79000 chapters. This collection is divided into four "treasuries," one of them being 子 (zǐ, masters), to which the Shuli Jingyun belongs.

The Siku Quanshu collection was copied by hand by 3826 copists, contains 2.3 million pages, and seven copies were made [58]. Three of these copies were partly or totally destroyed, and the four remaining copies are kept in the National Library of China in Beijing, the National Palace Museum in Taipei, the Gansu Library in Lanzhou, and the Zhejiang Library in Hangzhou. The Siku Quanshu, as well as a selection of the Siku Quanshu, namely the Siku Quanshu Huiyao (四庫全書薈要) [6, p. 152], are now available online on the Internet Archive. Part or all of these books seem to originate from the Hangzhou library.[3]

[3]The tables of the Shuli Jingyun are available at `http://www.archive.org/details/`x,

The original tables of the Shuli Jingyun were divided into eight volumes, and in the Siku Quanshu each volume was now divided in two or five new chapters. The Siku Quanshu version of the tables of the Shuli Jingyun therefore contains 28 chapters (table 2). These chapters are numbered 10864 to 10891, but they are bound in only 20 volumes in the main Hangzhou version. The first volume contains a list of these 28 chapters.

In addition, there has been a slight change of layout between the original Shuli Jingyun and the Siku Quanshu copy, in that the logarithms of numbers appear to be formatted differently.

We have reconstructed the eight volumes of tables of the Shuli Jingyun, as well as the 28 parts of the Siku Quanshu version of the tables of the Shuli Jingyun.

3 The influence of Briggs' and Vlacq's work

The tables of logarithms of the Shuli Jingyun were directly borrowed from Vlacq's *Arithmetica logarithmica* and *Trigonometria artificialis*, with only minor changes. Vlacq's tables were the fundamental tables on which most Western tables of logarithms were based until the beginning of the 20th century. The influence of Vlacq is also quite noticeable in the main part of the Shuli Jingyun, whose 38th chapter[4] appears to be inspired by Vlacq's *Arithmetica logarithmica*. As an illustration, we can compare the table of square root extractions in Briggs' table (figure 2), in Vlacq's *Arithmetica logarithmica* (figure 3) and in the Shuli Jingyun (figures 4 and 5).

Logarithms were actually first introduced in China in 1653 by Nikolaus Smogulecki (1610–1656), a Jesuit missionary and his pupil Xue Fengzuo who adapted them. Smogulecki's work was probably also based on Vlacq's.[5] There may have been other smaller tables of logarithms, but at this point, our knowledge on these tables is still very scarce.

At the end of the 17th century, Mei Wending (1633–1721) adapted many parts of Western mathematics and also rehabilitated ancient Chinese techniques [31, p. 25]. A major part of his work was included in the Shuli Jingyun.

where x ranges from `06076316.cn` to `06076335.cn` (as part of the 四庫全書薈要). In addition, there is also a partial version of the tables of a second version of the Shuli Jingyun, apparently also from Hangzhou, and found with x ranging from `06055001.cn` to `06055004.cn` (trigonometric functions, in four volumes), and from `06055005.cn` to `06055013.cn` (factors and prime numbers, in 9 volumes, hence slightly differently bound than the first series). The latter carries the title 四庫全書, and not 四庫全書薈要. These two versions are not the same, as a comparison of the handwriting shows (compare for instance `06076318.cn` and `06055003.cn`). It should also be noted that the Taipei copy of the Siku Quanshu was printed in 1500 volumes in 1983–1986 and in a reduced size in 1987 [58, p. 275].

[4] `http://www.archive.org/details/06076314.cn` (Siku Quanshu version).

[5] See our reconstructions of Smogulecki and Xue's tables [49, 50].

Vol.	Chapter			Content
1	10864	1.1	(1)	trigonometric functions (0° to 10°)
2	10865	1.2	(2)	(10° to 22°)
3	10866	2.1	(3)	(22° to 30°)
4	10867	2.2	(4)	(30° to 45°)
5	10868	3.1	(5)	primes and factors (1 to 10000)
	10869	3.2	(6)	(10001 to 20000)
6	10870	3.3	(7)	(20001 to 30000)
7	10871	3.4	(8)	(30001 to 40000)
	10872	3.5	(9)	(40001 to 50000)
8	10873	4.1	(10)	(50001 to 60000)
	10874	4.2	(11)	(60001 to 70000)
9	10875	4.3	(12)	(70001 to 80000)
	10876	4.4	(13)	(80001 to 90000)
10	10877	4.5	(14)	(90001 to 100000)
11	10878	5.1	(15)	logarithms of numbers (1 to 10000)
	10879	5.2	(16)	(10001 to 20000)
12	10880	5.3	(17)	(20001 to 30000)
13	10881	5.4	(18)	(30001 to 40000)
	10882	5.5	(19)	(40001 to 50000)
14	10883	6.1	(20)	(50001 to 60000)
	10884	6.2	(21)	(60001 to 70000)
15	10885	6.3	(22)	(70001 to 80000)
	10886	6.4	(23)	(80001 to 90000)
16	10887	6.5	(24)	(90001 to 100000)
17	10888	7.1	(25)	logarithms of trigonometric functions (0° to 10°)
18	10889	7.2	(26)	(10° to 22°)
19	10890	8.1	(27)	(22° to 30°)
20	10891	8.2	(28)	(30° to 45°)

Table 2: Structure of the tables in the Siku Quanshu version of the Shuli Jingyun. The first column gives the volume in the Siku Quanshu, the second column gives the number of the chapter in the Siku Siku Quanshu, and the third column gives the chapter from the Shuli Jingyun. We have confined ourselves to the structure observed in the complete set of tables from the 四庫全書薈要 on the Internet Archive, and other sets may differ. Our reconstruction is made of one separate document for every chapter.

4 Detailed structure of the tables in the Shuli Jungyun

As we mentioned above, the tables of the Shuli Jingyun are divided in eight parts.[6] Several tables contain introductions with examples of usage and some tables contain additional appended material.[7]

In the original Shuli Jingyun, every page bears the central title 御製數理精蘊, as well as a title of the table and its number.

In the Siku Quanshu version, every page of the tables contains the name of the collection (欽定四庫全書, Imperial Siku Quanshu), one side indicates the name of the book (御製數理精蘊表, Imperial Shuli Jingyun tables), and the other side gives the table part, for instance 表卷一上 (table volume 1 beginning). Page numbers are also given, but they are not identical with those of the original Shuli Jingyun.

4.1 Trigonometric functions

These tables give seven-place values of the six trigonometric functions for every 10 seconds of the quadrant.[8] The headings are as follows: 弦正 (sine), 線切正 (tangent), 線割正 (secant), 弦餘 (cosine), 線切餘 (cotangent), and 線割餘 (cosecant), 線 meaning "line." These headings are all read from right to left, these functions being now usually written 正弦, 正切, 正割, 餘弦, 餘切, and 餘割. The angles are given in degrees (度), minutes (分) and seconds (秒). Figure 8 shows an excerpt of these tables as reproduced in the Siku Quanshu.

An important matter is to identify the source of the trigonometric values. In order to try to clarify this matter, we can examine the values of the cotangents and cosecants for small angles. For 40″, for instance, the Shuli Jingyun gives $\csc 40'' = 515.72971573$ and $\cot 40'' = 515.72970603$, the exact values being $\csc 40'' = 515.66201885$ and $\cot 40'' = 515.66200915$. Now, it appears that Rheticus' *Opus palatinum* (1596) [43] has $\csc 40'' = 515.66194234383$ and $\cot 40'' = 515.66193264939$ which are much more accurate values than those of

[6]As mentioned above, the original tables are found at the *Institut des Hautes Études Chinoises* in Paris and at the Lyon municipal library. The latter only holds volumes 5 to 8. Both sets are contained in folded boxes, and the Lyon set, albeit incomplete, must have been sent from China that way. The boxes of both libraries were obviously made at the same place in China. The box in Lyon bears the handwritten note "Logarithmes chiffres pour supputer les saisons, imprimerie impériale de la Chine, 4 vol. dans ce tao." It is somewhat curious that these tables were viewed as a tool for calendar computing, when in fact they are more general than that.

[7]We have not reproduced these introductions, but we might do so in a future version of our reconstructions.

[8]It should however be remarked that the headings of the tables refer to *eight* trigonometric lines. These eight lines were the six lines given in the tables, and the 正矢 (versine) and 餘矢 (coversine) functions, which can be obtained indirectly from the values in the table, as explained in the introduction to the tables. We must acknowledge 钱赛 for his help in analyzing this introduction.

the Shuli Jingyun. Pitiscus' 1612 canon [40] also has more accurate values than the Shuli Jingyun.

It is however easy to find at least a partial explanation for these errors. A first hypothesis is that the values of the cosecants were computed from the sines. Indeed, we have

$$1/515.72971573 = 0.00193900015\ldots$$

and the tables of the Shuli Jingyun have $\sin 40'' = \tan 40'' = 0.001939$. This first suggests that the cosecants and cotangents were not computed with the same procedure from the sines and tangents, as their values should then have been identical for $40''$. Moreover, this also suggests that the source of the sines were either inaccurate sines to 10 places or sines to 7 places, where $\sin 40'' = 0.001939$, and that the computation of the cosecant was slightly incorrect. The same observations apply for other values of the cosecants. In any case, the sines used for the computation of the cosecants cannot have been the full sines from the *Opus palatinum*, as it gives quite different values. The *Opus palatinum* has $\sin 40'' = \tan 40'' = 0.001939255$, the correct values being $0.0019392535\ldots$ and $0.0019392571\ldots$. Given that the *Opus palatinum* was the only table giving values every 10 seconds, we are led to think that its sines were taken, then rounded, then incorrectly used for the computation of the cosecants.

The cotangents, however, were not computed as reciprocals of the tangents, but probably merely by multiplying the cosecants by the cosines. However, if we take the value of $\cos 40'' = 0.9999999$ given by the Shuli Jingyun, we obtain

$$\cot 40'' = \csc 40'' \times \cos 40'' = 515.72966\ldots$$

which is not the value given by the Shuli Jingyun.

But if we take instead a more accurate value of the cosine, such as $\cos 40'' = 0.9999999812$ found in the *Opus palatinum*, we obtain

$$\cot 40'' = \csc 40'' \times \cos 40'' = 515.72970603\ldots$$

which is exactly the value in the Shuli Jingyun.

It seems therefore likely that many values were taken from the *Opus palatinum*, but that the cosecants were computed as reciprocals of the sines, with some errors, and that the cotangents were computed using the cosecants and the more accurate cosines from the *Opus palatinum*. In addition, the layout of these tables was clearly influenced by Vlacq's *Trigonometria artificialis*.

In any case, the makers of the table could have obtained a more accurate result by copying the *Opus palatinum* without any recomputation. The results would not have been totally correct, as the *Opus palatinum* is known to contain errors in the cosecants and cotangents, but the errors would have been smaller than they are in the Shuli Jingyun now.

4.2 Logarithms of trigonometric functions

The tables of logarithms of trigonometric functions have exactly the same layout as those of the trigonometric functions. The headings are the same, and there is no mention of logarithms in the tables themselves. However, the values are those of the logarithms of the six trigonometric functions, to 10 places, and every 10 seconds of the quadrant.[9]

Four of the six functions are given by Vlacq in 1633 with the same step and number of places [57]. It seems that the values given by the Shuli Jingyun are identical with those of Vlacq, as it appears by comparing the logarithms of sines between $30°10'$ and $30°20'$, for instance. There is not a single difference, although Vlacq's table contains errors in this interval. It also seems that the values of the four functions are identical with those of the 3-volume set (see § 5).

Vlacq did not give the logarithms of the secants, nor of the cosecants, but they can of course be obtained easily, since we have

$$\log\sec\alpha = -\log\cos\alpha$$
$$\log\csc\alpha = -\log\sin\alpha$$

For instance, for $\alpha = 30°10'40''$, the Shuli Jingyun has $10 + \log\sin\alpha = 9.7012956660$ and this would lead to

$$10 + \log\csc\alpha = 10 - \log\sin\alpha = 10.2987043340$$

which is the value given by the Shuli Jingyun (instead of the more correct 10.2987043339).

For $\alpha = 30°12'20''$, the Shuli Jingyun has $10 + \log\cos\alpha = 9.9366273399$ (instead of the correct 9.9366273397) which leads to $10+\log\sec\alpha = 10.0633726601$ which is the value given by the Shuli Jingyun (instead of the more correct 10.0633726603).

So, it appears that the tables of logarithms of trigonometric functions were based on Vlacq's tables, although the differences between values were dropped. The layout of the tables is also based on the layout of Vlacq's *Trigonometria artificialis*.

The tables in the Shuli Jingyun are possibly the first tables giving the logarithms of all six trigonometric functions to 10 places, every 10 seconds of the quadrant.

4.3 Logarithms of numbers

4.3.1 Main table

The tables of logarithms of numbers give the 10-place decimals logarithms of all numbers from 1 to 100000. Each page covers an interval of 150 numbers (three columns of 50) in the Shuli Jingyin and 100 numbers (two columns of 50) in the

[9] The headings of these tables also refer to eight trigonometric lines, see our note above.

Siku Quanshu. The characteristics are given, but are not separated from the fractional parts. Figure 7 shows an excerpt of these tables as reproduced in the Siku Quanshu.

These logarithms were most certainly copied from Vlacq's *Arithmetica logarithmica* (1628) [56]. The values seem to be identical with those printed in the 3-volume set, although the latter are laid out differently (see § 5).

4.3.2 Appendix to the table

At the end of the last volume of the logarithms of numbers, there is an annex on the relationships between certain parameters in polygons and polyhedra. We did not reconstruct this appendix, but we give here an overview of it.

A number of "theorems" or "laws" are given, such as the "theorem of the circumference", which states that if the diameter (徑) is 1, then the circumference (周) is 三一四一五九二六五 (3[.]14159265). If the circumference is 1, then the diameter is 三一八三〇九八八 ([0.]31830988), etc. The logarithm of every value which appears in this appendix is also given beneath it. Other laws are given for the area of the circle, etc. Then, the regular polygons are considered. The first case is that of the polygons of side 1 and the areas of all such polygons are given up to the decagon (十邊). Several other "laws" are considered, for instance that giving the side of a polygon whose surface is 1. Polygons inscribed in a circle are also considered, etc., and eventually come the regular polyhedra. The first law gives the volumes of the polyhedra whose edge is 1. The tetrahedron (四面), cube (六方), octahedron (八面), dodecahedron (十二面) and icosahedron (二十面) are all given. The volume of the sphere (球) is also given, but the "edge" or "side" of the sphere is taken as its diameter. Among the other laws enumerated is that giving the sides of the regular polyhedra inscribed in a sphere of diameter 1. No formula is given, only the numerical relationships.

This annex was probably influenced by the results of Mei Wending (梅文鼎) (1633-1721) on polyhedra. His researches were first collected in 1723 in the 曆算全書 (Lisuan quanshu), published by his grandson Mei Juecheng (1681–1761), who was also in charge of the Shuli Jingyun. There have been several editions of Mei Wending's work [30, p. 34] and it was taken to Japan as soon as 1726 [18, pp. 259–260]. In the 1874 edition of the 梅氏叢書輯要 [33], Mei Wending's work on polyhedra is contained in the Jihe bubian, forming chapters 25 to 28 [30, p. 40].

Martzloff suggested [30, p. 40] that Mei Wending may have seen the figures in Clavius' untranslated commentary of books 15 and 16 of Euclid's elements [7]. Clavius' commentary was in fact the basis of the first Chinese adaptation of the first six books of Euclid's elements in 1607 [29, p. 27]. Martzloff gave an overview of Mei Wending's work on the geometry of polyhedra [30, pp. 265–267] and Mei Wending apparently discovered the exact relations between the inscriptions of various polyhedra around 1691 [29, pp. 34–35], but according to Martzloff's analysis, Mei Wending does not seem to have used logarithms in his work, and of course Clavius did not make use of logarithms.

On the other hand, Briggs' *Arithmetica logarithmica* (1624) [3] and Vlacq's *Arithmetica logarithmica* (1628) [56] contain a chapter where the various parameters of each polyhedron are related to those of the circumscribed sphere, and it is very easy to derive the values in the annex of the Shuli Jingyun from those given by Briggs and Vlacq.[10] Briggs' and Vlacq's *Arithmetica logarithmica* also contain a chapter on the relationships in polygons.

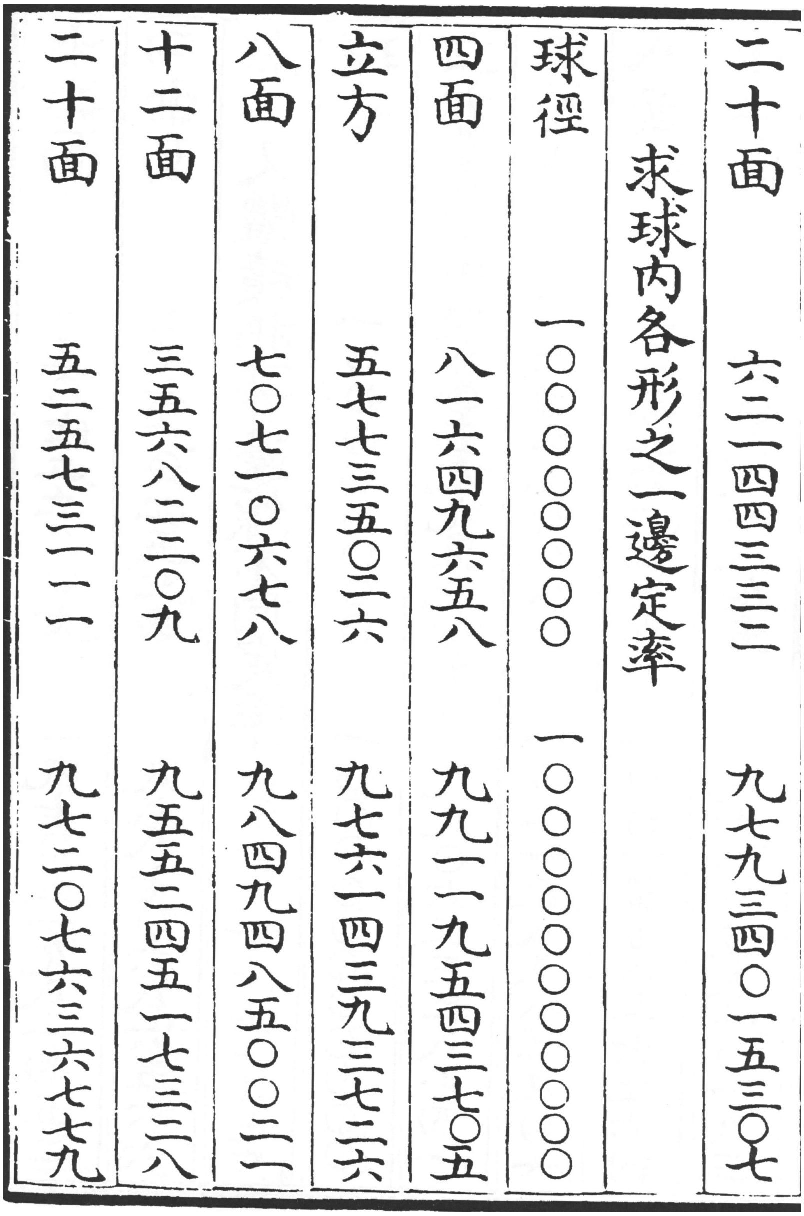
The appendix is concluded by a list of densities of various materials, such as gold, silver, mercury, copper, copper-nickel alloy, brass, lead, and many others. The value given for gold is 1680, that for mercury 1228 and that for lead 993. These values seem to be in about the same ratios as the densities of gold (19.3kg/l), mercury (13.5 kg/l), and lead (11.34kg/l). Differences with the real values are normal, given that the materials were certainly not completely pure, and the measurement techniques not as sophisticated. The list ends with the densities of several types of wood, of oil (83) and of water (93).

4.4 Factors and prime numbers

The tables of logarithms are supplemented by tables of factors, which can be used for checking the values of the logarithms, or for simplifying computations before using logarithms. These tables are divided in ten parts, each corresponding to an interval of 10000. Each page contains 160 numbers, in eight columns of 20 numbers, and the main table of each part therefore covers 62 pages and a half. A decomposition into two factors is given for every non-prime in this list. In addition, a list of the primes only is appended at the end of each part. Figure 6 shows an excerpt of these tables as reproduced in the Siku Quanshu.

The decompositions do not always obey the same rules throughout the tables. In some parts, the two factors given are such that their ratio is closest to 1. In other words, $n = a \times b$ is chosen such that a/b is as close as possible to 1, and in addition $a \geq b$. Finding these factors is in some cases time consuming and this alone makes it likely that the optimal cases were not always given. This table also gives the prime numbers. We do not know the source of these prime numbers, or whether they have been computed independently by the Chinese. It is however likely that they were copied from a European source,

[10] For instance, figure 1 gives the edges of the polyhedra inscribed in a sphere of diameter 1. For the tetrahedron, we have 八一六四九六五八 ($= 0.81649658 = \sqrt{2/3}$), for the cube we have $0.57735026 = 1/\sqrt{3}$, for the octahedron we have $0.70710678 = 1/\sqrt{2}$, for the dodecahedron we have $0.35682209 = \frac{1}{2}\sqrt{2-\sqrt{20/9}}$, and for the icosahedron we have $0.5257312 = \frac{1}{2}\sqrt{2-\sqrt{4/5}}$. Beneath these values, we have their decimal logarithms, shifted by 10. But Briggs' *Arithmetica logarithmica* [3, p. 87], as well as Vlacq's *Arithmetica logarithmica* [56, p. 78] (Latin edition) give the values $1\frac{6329931618}{10000000000} = \sqrt{8/3}$, $1\frac{1547005384}{10000000000} = \sqrt{4/3}$, etc., which, when truncated to 8 places and divided by 2, give the values found in the Shuli Jingyun. This is so merely because Briggs took a sphere of radius 1, instead of a diameter of 1. Briggs and Vlacq also give the logarithms of these values and the corresponding values in the Shuli Jingyun could be derived from them merely by subtracting $\log 2$ and adding 10.



二十面 六二一四四三三二 九七九三四〇一五三〇七

求球內各形之一邊定率

球徑 一〇〇〇〇〇〇〇 一〇〇〇〇〇〇〇〇〇〇

四面 八一六四九六五八 九九一一九五四三七〇五

立方 五七七三五〇二六 九七六一四三九三七二六

八面 七〇七一〇六七八 九八四九四八五〇〇二一

十二面 三五六八二二〇九 九五五二四五一七三二八

二十面 五二五七三一一一 九七二〇七六三六七七九

Figure 1: An excerpt of the table appended to the table of logarithms of numbers. The columns are read from top to bottom, and from right to left. The first column to the right is the end of a previous table. The last five columns to the left give the lengths of the edges of the regular polyhedra inscribed in a sphere of diameter 1. The logarithms of the lengths are given below them. The third column to the right corresponds to the sphere. The second column gives the title of the 'recipe,' that is 'obtain one side of each shape inside the sphere.' We thank 钱赛 for his help in analyzing this appendix.

perhaps Brancker's table published in 1668 and which covers the same interval to 100000 [42].[11]

We have found several errors in the lists of primes:[12] 27099, 29691 and 45107 are for instance given as primes, but are not; 59629 and 99989 are not given in the list of primes, although they are. These errors seem to occur in both manuscript copies of the table of factors. Such errors are easy to spot, as they alter the number of primes, and a comparison with a reconstruction makes them stand out. There may however be other errors, which cancel each other out, and which are therefore more difficult to locate, especially given the low quality of the scanned versions of the tables.

Some of the above errors are oversights, given that the complete list shows that $27099 = 9033 \times 3$ and $29691 = 9897 \times 3$. Incidentally, we did not give the same decompositions in our reconstruction, but the more balanced decompositions $27099 = 3011 \times 9$ and $29691 = 3299 \times 9$. The list of factors gives 45107 without factors, although it is divisible by 43. This error does not appear in Brancker's table, at least in that printed by Maseres in 1795 [32]. (We have not seen Brancker's original table.) And the main list gives 59629 and 99898 without factors, but these two numbers were forgotten in the final list.

Consequently, it turns out that four of these five errors were oversights, and that only one of them corresponds to a forgotten factor.

It now remains to be determined how often the decomposition rules depart from ours, and what is the general accuracy of these tables. Given the bad image quality of the digitizations, this is currently a difficult task, but it might be undertaken by someone with access to the original volumes, and with the aid of our reconstructions.

5 Relationship with the set in three volumes[13]

The tables of the Shuli Jingyun were not the only Chinese tables of logarithms completed at the end of Kangxi's reign. In about 1720, another set of tables was printed containing only the logarithms of numbers and of trigonometric functions. These tables were bound in three volumes. They are close, but not identical to the tables of the Shuli Jingyun, although they are clearly related to them. These two versions are sometimes confused.

The layouts of the two sets of tables are clearly different. On the one hand, the 3-volume table is very faithful to Vlacq's tables, except that differences between logarithms are not given. On the other hand, the tables of the Shuli

[11] For a recent survey of early tables of factors, see Bullynck [4].

[12] These errors have only been identified in the Siku Quanshu version and we have not yet compared the corresponding values in the original Shuli Jingyun.

[13] At the time of the reconstruction of the 3-volume set in 2010, we had not yet seen the Shuli Jingyun, neither in its original version, nor in the Siku Quanshu, and it was not clear to us that there are actually two sets of tables, with different layouts, and not merely identical tables bound differently.

Jingyun use a different layout, although they too are clearly influenced by Vlacq's tables. The Shuli Jingyun tables also contain values which were given neither by Vlacq nor by Briggs. One possible explanation is therefore that the 3-volume set was produced first, apparently under supervision of the Jesuits, and perhaps as a draft for the Shuli Jingyun, and that it was later extended to the final form of the Shuli Jingyun. Another possibility — but which seems less likely — is that the three volume set was produced afterwards, as a special version for the West, or as a summary of the essential tables of the Shuli Jingyun, and printed in color. The versions sent by the Jesuits seem to have been these colored tables. In any case, a close analysis reveals that the two sets of tables were not obtained from the same wooden blocks, although the reuse of wooden blocks was a common practice.[14]

In any case, the 3-volume set was also reconstructed in 2010, so that it is now easy to compare the two sets of tables. The reconstruction was described in our analysis of Vlacq's tables in Chinese [51].

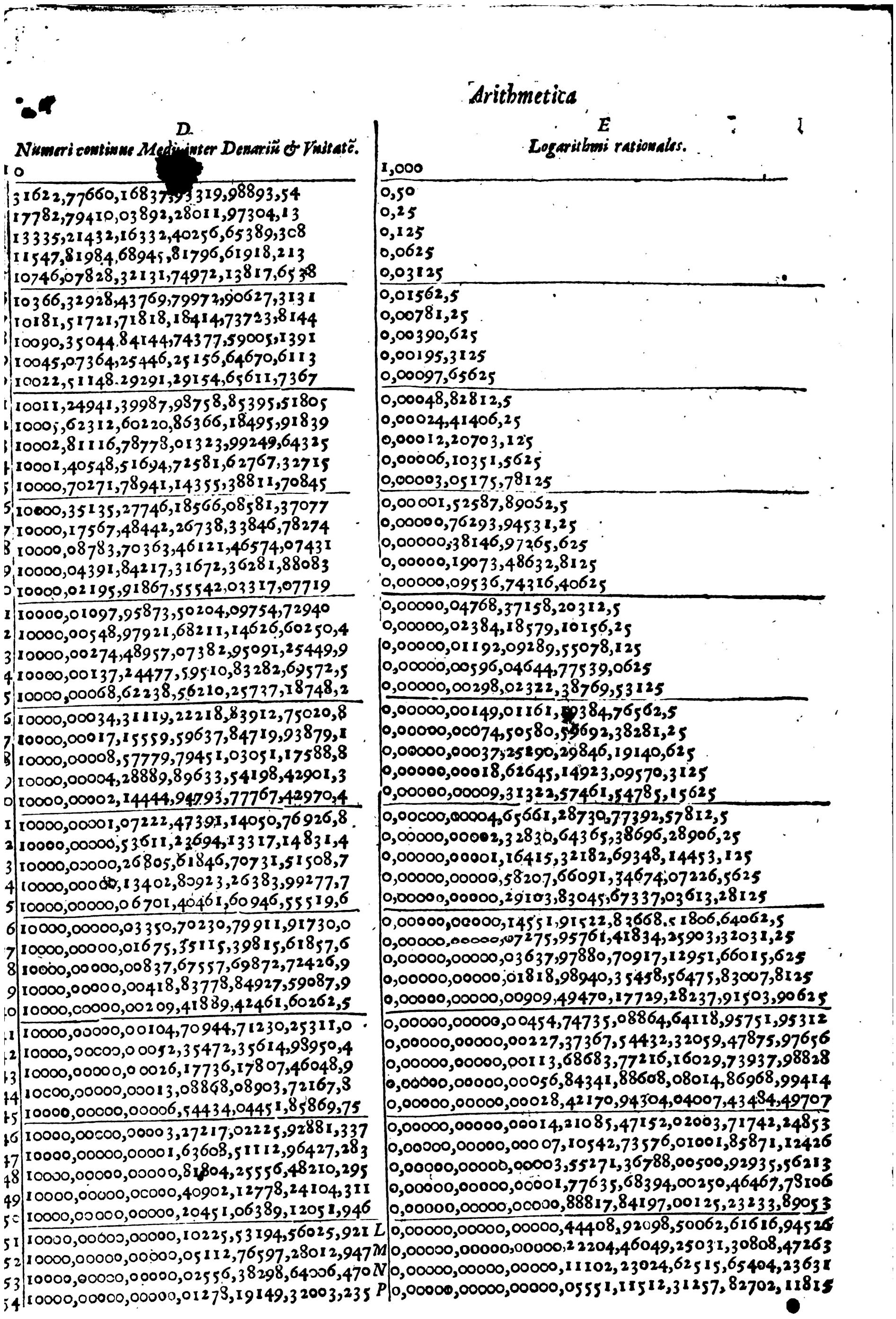
6 The reconstructions

Our reconstructions give the ideal tables, and can be used to assess the accuracy of the original tables, as well as conveying their contents faster. Since our tables give the exact values, they are not exact copies of the Chinese tables, which included errors borrowed from Vlacq's tables. The tables were recomputed using the `mpfr` library [11] and the computation was straightforward.

Regarding the order of the pages, the beginning of the tables is at the (Western) end of the volume, as is customary in Chinese. If these tables are printed, they should be bound on the *left* (as usual), and then read from the (Western) end.

The headings of the original tables use traditional Chinese characters, and they were retained here. We have also kept the original page numbers, and the ends (beginnings) of the volumes do not always start with 1, as there is usually introductory material which was not reproduced here.

[14]On the tradition of reuse of printing blocks, see Shi [53].



Arithmetica

	D. Numeri continue Medii inter Denariū & Vnitatē.	E Logarithmi rationales.
	10	1,000
	31622,77660,16837,93319,98893,54	0,50
	17782,79410,03892,28011,97304,13	0,25
	13335,21432,16332,40256,65389,308	0,125
	11547,81984,68945,81796,61918,213	0,0625
	10746,07828,32131,74972,13817,6538	0,03125
	10366,32928,43769,79972,90627,3131	0,01562,5
	10181,51721,71818,18414,73723,8144	0,00781,25
	10090,35044,84144,74377,59005,1391	0,00390,625
	10045,07364,25446,25156,64670,6113	0,00195,3125
	10022,51148.29291,29154,65611,7367	0,00097,65625
	10011,24941,39987,98758,85395,51805	0,00048,82812,5
	10005,62312,60220,86366,18495,91839	0,00024,41406,25
	10002,81116,78778,01323,99249,64325	0,00012,20703,125
	10001,40548,51694,72581,62767,32715	0,00006,10351,5625
	10000,70271,78941,14355,38811,70845	0,00003,05175,78125
5	10000,35135,27746,18566,08581,37077	0,00001,52587,89062,5
7	10000,17567,48442,26738,33846,78274	0,00000,76293,94531,25
8	10000,08783,70363,46121,46574,07431	0,00000,38146,97265,625
9	10000,04391,84217,31672,36281,88083	0,00000,19073,48632,8125
0	10000,02195,91867,55542,03317,07719	0,00000,09536,74316,40625
1	10000,01097,95873,50204,09754,72940	0,00000,04768,37158,20312,5
2	10000,00548,97921,68211,14626,60250,4	0,00000,02384,18579,10156,25
3	10000,00274,48957,07382,95091,25449,9	0,00000,01192,09289,55078,125
4	10000,00137,24477,59510,83282,69572,5	0,00000,00596,04644,77539,0625
5	10000,00068,62238,56210,25737,18748,2	0,00000,00298,02322,38769,53125
6	10000,00034,31119,22218,83912,75020,8	0,00000,00149,01161,19384,76562,5
7	10000,00017,15559,59637,84719,93879,1	0,00000,00074,50580,59692,38281,25
8	10000,00008,57779,79451,03051,17588,8	0,00000,00037,25290,29846,19140,625
9	10000,00004,28889,89633,54198,42901,3	0,00000,00018,62645,14923,09570,3125
0	10000,00002,14444,94793,77767,42970,4	0,00000,00009,31322,57461,54785,15625
1	10000,00001,07222,47391,14050,76926,8	0,00000,00004,65661,28730,77392,57812,5
2	10000,00000,53611,23694,13317,14831,4	0,00000,00002,32830,64365,38696,28906,25
3	10000,00000,26805,61846,70731,51508,7	0,00000,00001,16415,32182,69348,14453,125
4	10000,00000,13402,80923,26383,99277,7	0,00000,00000,58207,66091,34674,07226,5625
5	10000,00000,06701,40461,60946,55519,6	0,00000,00000,29103,83045,67337,03613,28125
6	10000,00000,03350,70230,79911,91730,0	0,00000,00000,14551,91522,83668,51806,64062,5
7	10000,00000,01675,35115,39815,61857,6	0,00000,00000,07275,95761,41834,25903,32031,25
8	10000,00000,00837,67557,69872,72426,9	0,00000,00000,03637,97880,70917,12951,66015,625
9	10000,00000,00418,83778,84927,59087,9	0,00000,00000,01818,98940,35458,56475,83007,8125
40	10000,00000,00209,41889,42461,60262,5	0,00000,00000,00909,49470,17729,28237,91503,90625
41	10000,00000,00104,70944,71230,25311,0	0,00000,00000,00454,74735,08864,64118,95751,95312
42	10000,00000,00052,35472,35614,98950,4	0,00000,00000,00227,37367,54432,32059,47875,97656
43	10000,00000,00026,17736,17807,46048,9	0,00000,00000,00113,68683,77216,16029,73937,98828
44	10000,00000,00013,08868,08903,72167,8	0,00000,00000,00056,84341,88608,08014,86968,99414
45	10000,00000,00006,54434,04451,85869,75	0,00000,00000,00028,42170,94304,04007,43484,49707
46	10000,00000,00003,27217,02225,92881,337	0,00000,00000,00014,21085,47152,02003,71742,24853
47	10000,00000,00001,63608,51112,96427,283	0,00000,00000,00007,10542,73576,01001,85871,12426
48	10000,00000,00000,81804,25556,48210,295	0,00000,00000,00003,55271,36788,00500,92935,56213
49	10000,00000,00000,40902,12778,24104,311	0,00000,00000,00001,77635,68394,00250,46467,78106
50	10000,00000,00000,20451,06389,12051,946	0,00000,00000,00000,88817,84197,00125,23233,89053
51	10000,00000,00000,10225,53194,56025,921 L	0,00000,00000,00000,44408,92098,50062,61616,94526
52	10000,00000,00000,05112,76597,28012,947 M	0,00000,00000,00000,22204,46049,25031,30808,47263
53	10000,00000,00000,02556,38298,64006,470 N	0,00000,00000,00000,11102,23024,62515,65404,23631
54	10000,00000,00000,01278,19149,32003,235 P	0,00000,00000,00000,05551,11512,31257,82702,11815

Figure 2: Briggs' table (1624).

10 D *ARITHMETICA* E

Numeri continue Medij inter Denarium & Unitatem.	10	*Logarithmi Rationales.* 1,000
1	31622,77660,16837,93319,98893,54	0,50
2	17782,79410,03892,28011,97304,13	0,25
3	13335,21432,16332,40256,65389,308	0,125
4	11547,81984,68945,81796,61918,213	0,0625
5	10746,07828,32131,74972,13817,6538	0,03125
6	10366,32928,43769,79972,90627,3131	0,01562,5
7	10181,51721,71818,18414,73723,8144	0,00781,25
8	10090,35044,84144,74377,59005,1391	0,00390,625
9	10045,07364,25446,25156,64670,6113	0,00195,3125
10	10022,51148,29291,29154,65611,7367	0,00097,65625
11	10011,24941,39987,98758,85395,51805	0,00048,82812,5
12	10005,62312,60220,86366,18495,91839	0,00024,41406,25
13	10002,81116,78778,01323,99249,64325	0,00012,20703,125
14	10001,40548,51694,72581,62767,32715	0,00006,10351,5625
15	10006,70271,78941,14355,38811,70845	0,00003,05175,78125
16	10000,35135,27746,18566,08581,37077	0,00001,52587,89062,5
17	10000,17567,48442,26738,33846,78274	0,00000,76293,94531,25
18	10000,08783,70363,46121,46574,07431	0,00000,38146,97265,625
19	10000,04391,84217,31672,36281,88083	0,00000,19073,48632,8125
20	10000,02195,91867,55542,03317,07719	0,00000,09536,74316,40625
21	10000,01097,95873,50204,09754,72940	0,00000,04768,37158,20312,5
22	10000,00548,97921,68211,14626,60250,4	0,00000,02384,18579,10156,25
23	10000,00274,48957,07382,95091,25449,9	0,00000,01192,09289,55078,125
24	10000,00137,24477,59510,83282,69572,5	0,00000,00596,04644,77539,0625
25	10000,00068,62238,56210,25737,18748,2	0,00000,00298,02322,38769,53125
26	10000,00034,31119,22218,83912,75020,8	0,00000,00149,01161,19384,76562,5
27	10000,00017,15559,59637,84719,93879,1	0,00000,00074,50580,59692,38281,25
28	10000,00008,57779,79451,03051,17588,8	0,00000,00037,25290,29846,19140,625
29	10000,00004,28889,89633,54198,42901,3	0,00000,00018,62645,14923,09570,3125
30	10000,00002,14444,94793,77767,42970,4	0,00000,00009,31322,57461,54785,15625
31	10000,00001,07222,47391,14050,76926,8	0,00000,00004,65661,28730,77392,57812,5
32	10000,00000,53611,23694,13317,14831,4	0,00000,00002,32830,64365,38696,28906,25
33	10000,00000,26805,61846,70731,51508,7	0,00000,00001,16415,32182,69348,14453,125
34	10000,00000,13402,80923,26383,99277,7	0,00000,00000,58207,66091,34674,07226,5625
35	10000,00000,06701,40461,60946,55519,6	0,00000,00000,29103,83045,67337,03613,28125
36	10000,00000,03350,70230,79911,91730,0	0,00000,00000,14551,91522,83668,51806,64062,5
37	10000,00000,01675,35115,39815,61857,6	0,00000,00000,07275,95761,41834,25903,32031,25
38	10000,00000,00837,67557,69872,72426,9	0,00000,00000,03637,97880,70917,12951,66015,625
39	10000,00000,00418,83778,84927,59087,9	0,00000,00000,01818,98940,35458,56475,83007,8125
40	10000,00000,00209,41889,42461,60262,5	0,00000,00000,00909,49470,17729,28237,91503,90625
41	10000,00000,00104,70944,71230,25311,0	0,00000,00000,00454,74735,08864,64118,95751,95312
42	10000,00000,00052,35472,35614,98950,4	0,00000,00000,00227,37367,54432,32059,47875,97656
43	10000,00000,00026,17736,17807,46048,9	0,00000,00000,00113,68683,77216,16029,73937,98828
44	10000,00000,00013,08868,08903,72167,8	0,00000,00000,00056,84341,88608,08014,86968,99414
45	10000,00000,00006,54434,04451,85869,75	0,00000,00000,00028,42170,94304,04007,43484,49707
46	10000,00000,00003,27217,02225,92881,337	0,00000,00000,00014,21085,47152,02003,71742,24853
47	10000,00000,00001,63608,51112,96427,283	0,00000,00000,00007,10542,73576,01001,85871,12426
48	10000,00000,00000,81804,25556,48210,295	0,00000,00000,00003,55271,36788,00500,92935,56213
49	10000,00000,00000,40902,12778,24104,311	0,00000,00000,00001,77635,68394,00250,46467,78106
50	10000,00000,00000,20451,06389,12051,946	0,00000,00000,00000,88817,84197,00125,23233,89053
51	10000,00000,00000,10225,53194,56025,921 *L*	0,00000,00000,00000,44408,92098,50062,61616,94526
52	10000,00000,00000,05112,76597,28012,947 *M*	0,00000,00000,00000,22204,46049,25031,30808,47263
53	10000,00000,00000,02556,38298,64006,470 *N*	0,00000,00000,00000,11102,23024,62515,65404,23631
54	10000,00000,00000,01278,19149,32003,235 *P*	0,00000,00000,00000,05551,11512,31257,82702,11815

Figure 3: Vlacq's table (1628), adapted from Briggs.

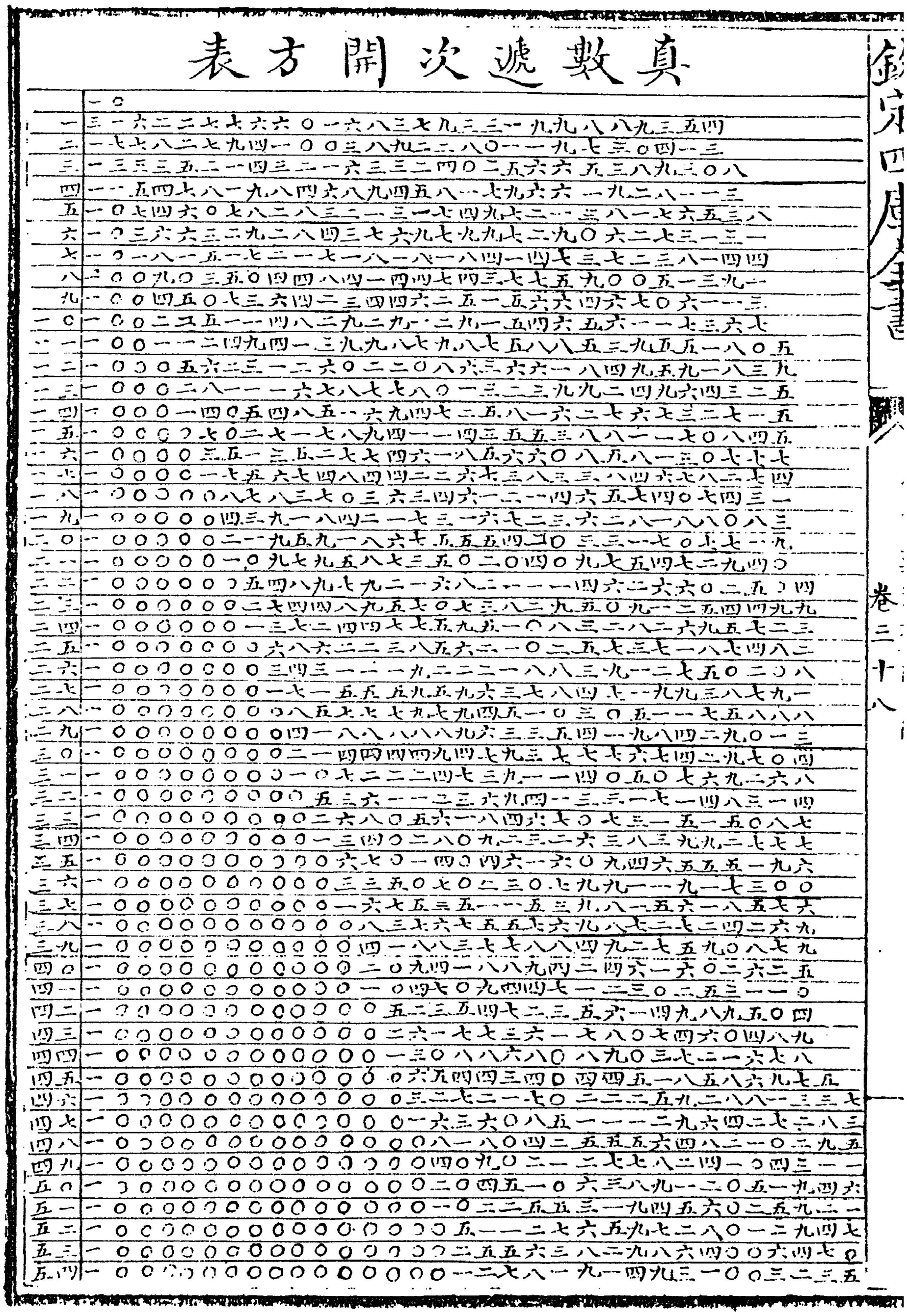


Figure 4: Excerpt of volume 38 of the Shuli Jingyun.

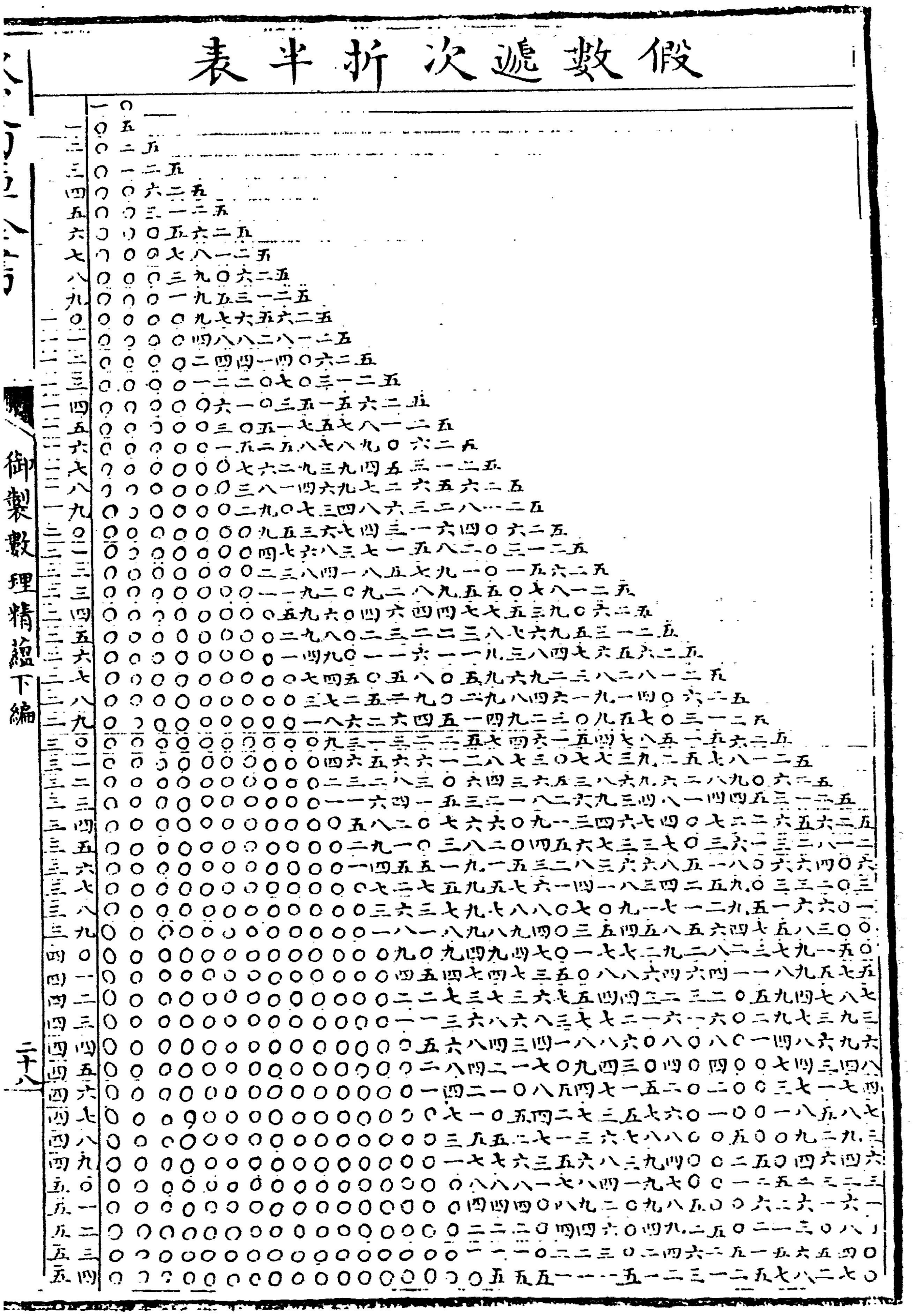


Figure 5: Excerpt of volume 38 of the Shuli Jingyun.

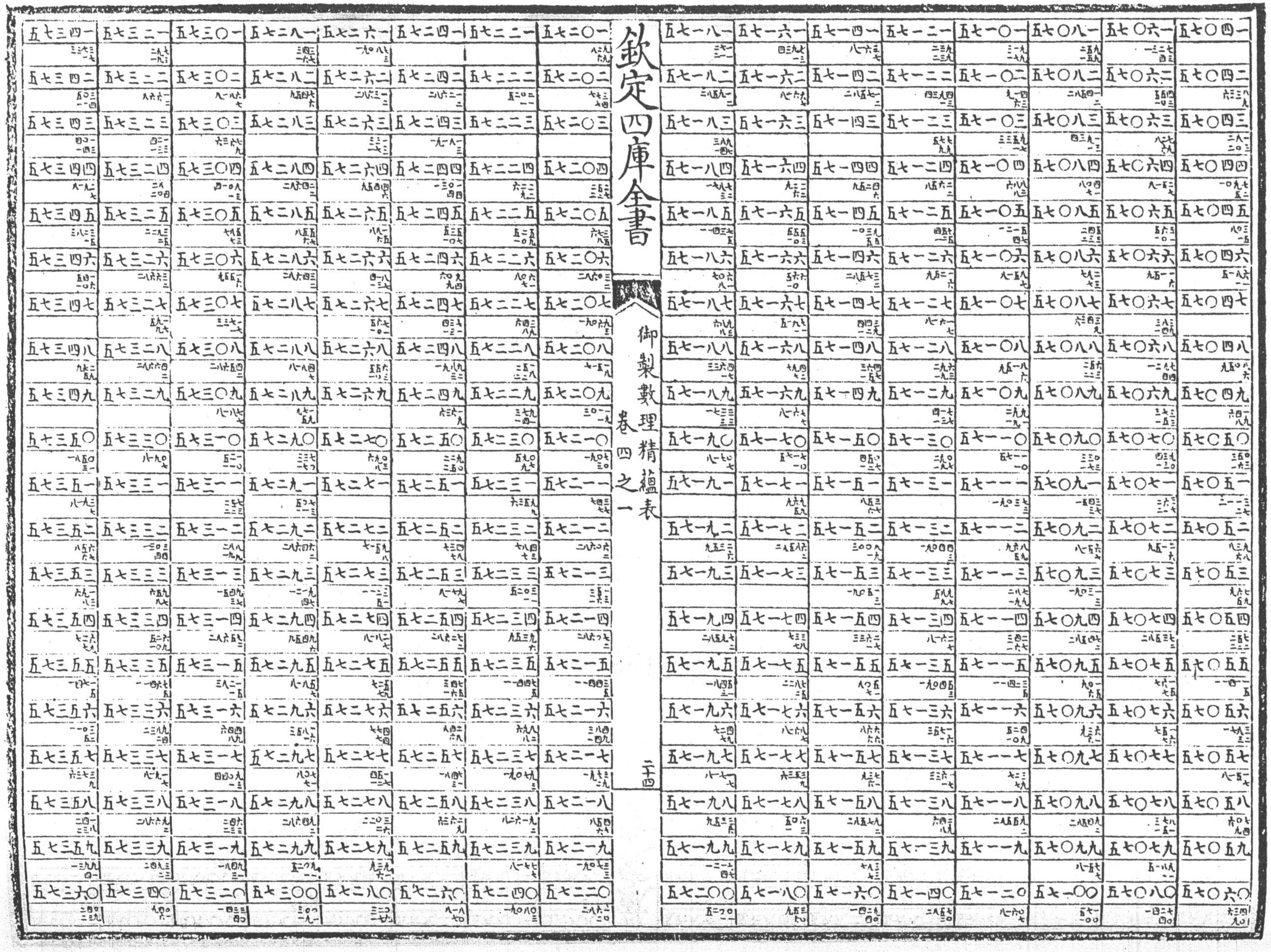


Figure 6: Excerpt of the Siku Quanshu version of the table of factors (reduced printing from [35]).

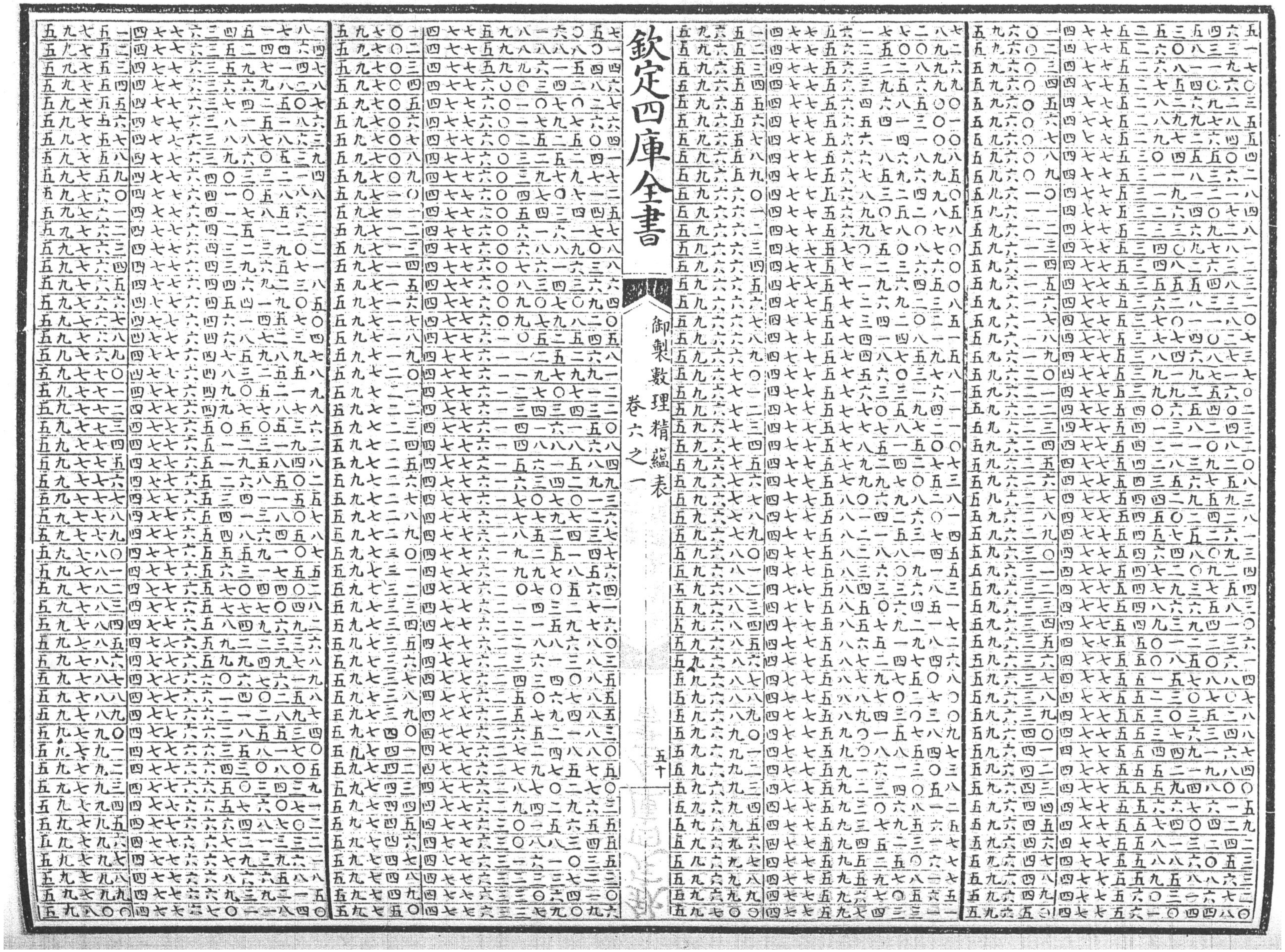


Figure 7: Excerpt of the Siku Quanshu version of the table of logarithms of numbers (reduced printing from [35]).

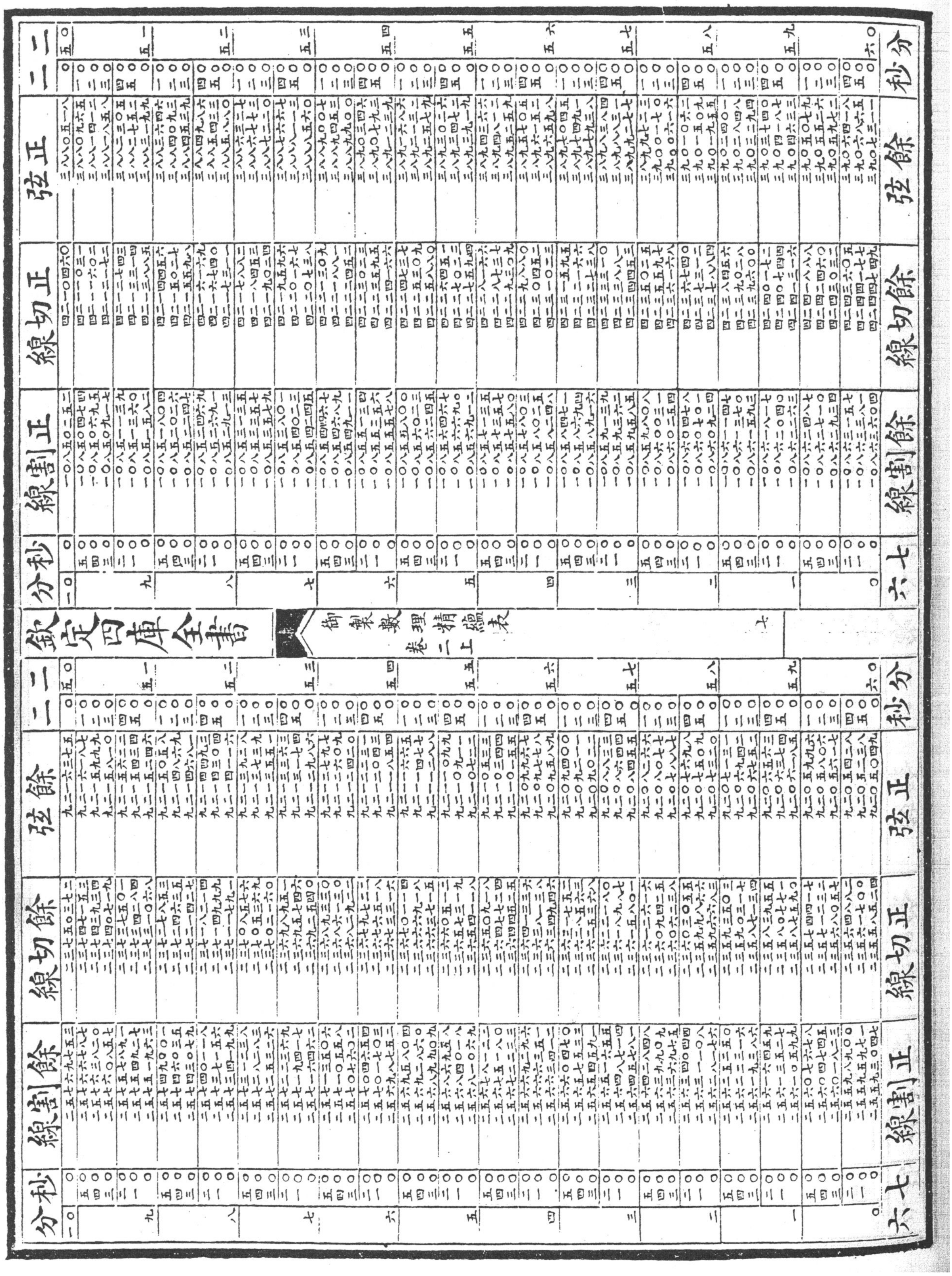


Figure 8: Excerpt of the Siku Quanshu version of the table of trigonometrical functions (reduced printing from [35]).

References

The following list covers the most important Western references[15] related to the tables in the Shuli Jingyun. Not all items of this list are mentioned in the text, and the sources which have not been seen are marked so. We have added notes about the contents of the articles in certain cases.

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[15]**Note on the titles of the works:** Original titles come with many idiosyncrasies and features (line splitting, size, fonts, etc.) which can often not be reproduced in a list of references. It has therefore seemed pointless to capitalize works according to conventions which not only have no relation with the original work, but also do not restore the title entirely. In the following list of references, most title words (except in German) will therefore be left uncapitalized. The names of the authors have also been homogenized and initials expanded, as much as possible.

The reader should keep in mind that this list is not meant as a facsimile of the original works. The original style information could no doubt have been added as a note, but we have not done it here.

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