Introduction to Edward Sang's table of logarithms to 15 places

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1 Introduction

Edward Sang (1805–1890) was probably the greatest calculator of logarithms of the 19th century [4, 12, 13, 14, 26, 34]. Sang spent 40 years computing tables of logarithms and trigonometric functions, with the assistance from his daughters Flora (1838-1925) and Jane (1834-1878). The result fills about 50 manuscript volumes, plus a number of transfer duplicates,[1] which are kept at Edinburgh University Library and at the National Library of Scotland, Edinburgh. I have reconstructed a number of these tables and an overview of the tables and reconstructions can be found in a separate guide [41].

Sang's purpose was in particular to provide fundamental tables, including for the decimal division of the quadrant. In 1890 [4, p. 189], he wrote that

> In addition to the results being accurate to a degree far beyond what can ever be needed in practical matters, [the collection of computations] contains what no work of the kind has contained before, a complete and clear record of all the steps by which those results were reached. Thus we are enabled at once to verify, or if necessary, to correct the record, so making it a standard for all time.
>
> For these reasons it is proposed that the entire collection be acquired by, and preserved in, some official library, so as to be accessible to all interested in such matters; so that future computers may be enabled to extend the work without the need of recomputing what has been already done; and also so that those extracts which are judged to be expedient may be published.

2 The computation of logarithms

The details of the computation of the logarithms of integers are given in a separate document [43]. Here I am only outlining Sang's methods. Sang started with the computation of the logarithms of all primes up to 10037 to 28 places (with a view of being exact at 25 places) and the logarithms of all integers from 100000 to 370000 to 15 places,[2] going beyond the corresponding work in the French *Tables du cadastre* [13, p. 55]. Sang started to work on logarithms in the 1840s, when he was involved in editing Shortrede's tables [76], [15, pp. 904–907]. The logarithms of numbers were then computed between 1848 and the mid 1870s. Sang used interpolation (like Briggs and Vlacq) to compute a number of logarithms [13, p. 77]. According to Craik [13, p. 68], Sang's tables are more accurate than those of Prony [39].[3]

Sang's method to compute the logarithm of a prime number involves finding several equations relating this prime to already computed primes and to a number differing by

[1] For details on the process of transfer duplicates and photocopying techniques, see [35].

[2] In July 1865, Sang had reached all primes up to 2000 [13, p. 74]. In 1872, he had computed the primes up to 2600 and the numbers up to 260000 [64]. In 1874, he was at 3600 and 320000 [65]. And in 1875, all the primes to 10000 had been computed [66].

[3] For the accuracy of Sang's tables, see (presumably) Knott's article [4]. Sang found that the error on the 15th place in his tables could reach five units in a few cases, and therefore that the error never reached one unit in the 14th place [4, pp. 186–187].

one from a number ending with several 0s [13]. For instance, in order to compute log 8447, Sang may have considered the two equations

$$2 \cdot 3769 \cdot 10^8 + 1 = 3 \cdot 37 \cdot 251 \cdot 3203 \cdot 8447$$
$$643 \cdot 10^7 - 1 = 3 \cdot 89 \cdot 2851 \cdot 8447$$

where the logarithms of 2, 3, 37, 89, 251, 643, 2851, 3203, and 3769 were assumed to be already known.

Then, Sang would write

$$\begin{aligned}\log(2 \cdot 3769 \cdot 10^8 + 1) &= \log\left(\frac{2 \cdot 3769 \cdot 10^8 + 1}{2 \cdot 3769 \cdot 10^8}\right) + \log(2 \cdot 3769 \cdot 10^8) \\ &= \log\left(1 + \frac{1}{2 \cdot 3769 \cdot 10^8}\right) + \log 2 + \log 3769 + 8 \log 10\end{aligned}$$

and

$$\log 8447 = \log(2 \cdot 3769 \cdot 10^8 + 1) - \log 3 - \log 37 - \log 251 - \log 3203$$

$\log\left(1 + \frac{1}{2 \cdot 3769 \cdot 10^8}\right)$ can easily be computed, using the familiar development of $\ln(1+x)$ and a value of ln 10. It is in order to ease the computation of $\ln(1+x)$ that Sang chose $\frac{1}{x}$ to be an integer ending with as many 0s as possible. The divisions are then much easier to perform. Moreover, Sang made use of Burckhardt's factor tables [9] in order to factor the numbers $n \times 10^m$ appearing in the above method.

Sang chose to use two equations for every new prime, so that he could perform independent calculations and avoid errors. Such an approach is of course much less systematic than the one used by Prony [39]. Incidentally, Sang's method of finding adequate relations between primes had already been employed by Isaac Wolfram at the end of the 18th century [40].

In order to compute ln 10, Sang considered the two series

$$\ln 8 = 2\left[1 + \frac{1}{3} \cdot \frac{1}{9} + \frac{1}{5} \cdot \frac{1}{9^2} + \frac{1}{7} \cdot \frac{1}{9^3} + \cdots\right]$$
$$\ln \frac{10}{8} = 2\left[\frac{1}{9} + \frac{1}{3} \cdot \frac{1}{9^3} + \frac{1}{5} \cdot \frac{1}{9^5} + \frac{1}{7} \cdot \frac{1}{9^7} + \cdots\right]$$

obtained from $\ln\left(\frac{1+a}{1-a}\right) = 2a\left[1 + \frac{a^2}{3} + \frac{a^4}{5} + \frac{a^6}{7} + \cdots\right]$ with $a = \frac{1}{3}$ and $\frac{1}{9}$, and by adding these two series, he obtained a series for ln 10 [13, p. 75].

Apart from Shortrede's tables, the only tables of logarithms published by Sang were a short table giving the logarithms to five places published in 1859 [60], and a larger one giving the logarithms to seven places of all numbers from 20000 to 200000, published in 1871 [61, 18].[4] According to Archibald, it was the first (complete and published) logarithmic table going beyond 108000 [6]. For these tables, Sang made use of electrotype

[4]Sang's table is available at `http://www.archive.org/stream/anewtablesevenp01sanggoog`. Sang has also computed trigonometric tables and he was an eager defender of the decimal subdivision of angles [4, 5, 6, 13, 22, 31, 32, 68, 71, 72, 73].

plates which might still exist [13, p. 61]. The 1871 tables also included a notice for the projected “Million table of nine-place logarithms.” Sang's real purpose was to compute a nine-place table of logarithms of all integers from 100000 to one million [64], and in order to be sure of the digits of the planned final table, he needed a number of other logarithms to a higher accuracy [13, p. 74]. This fundamental table was the 15-place table of logarithms, reconstructed here. The nine-place table was never completed by Sang, but I have also reconstructed it [36].

One of Sang's incentives for computing his logarithms were the *Tables du cadastre*. In his presentation of the specimen pages of the “Million table” [64], Sang mentions the involvement of the British Government in 1819 in a joint printing of the French tables, but that the negotiation was without result [39], and that there was a want of more extensive tables.[5]

In 1914, Knott suggested the photographic reproduction of some of Sang's manuscript tables, but this never occured. This initiative was doomed by the outbreak of war [13, p. 68]. Eventually, projects such as Thompson's led to the abandonment of the publication of Sang's tables [79].

3 Scope and faithfulness of the reconstruction

Sang's table only covers the range from 100 000 to 370 000 (10800 pages), and spans 27 volumes at the NLS and 14 volumes at Edinburgh University Library. For the sake of completeness, and because it didn't cost much effort, I have extended the table to 1 million (36000 pages) split in 90 volumes of 400 pages each, each volume covering a range of 10000 integers.

The layout of Sang's table is straightforward and mostly regular. Each page usually contains 26 logarithms, although volumes K7 to K11 usually contain only 25 logarithms. In the general case, the last value of a page is also the first value of the next page. This is the scheme I have adopted in the reconstruction.

Sang separated the (usually repeating) prefixes of the logarithms. There are some cases where these prefixes are dropped, and it was not always done consistently in the original tables. Here, I have systematically given the prefixes of the first and second differences on the first line and only when they differ from previous prefixes. In the original tables, the first prefix of the first differences is sometimes given in the heading. Moreover, I have adopted suffixes of constant lengths, whereas the second differences are not always split the same way by Sang across the table. Finally, the first digit of the second differences is sometimes also given in the heading of the table, but I have not

[5]When Edward Sang's project of a nine-place table became known and when Sang's article on his discovery of errors in Vlacq's table was published with this project [65], an article in *Nature* [2] appeared very critical of Sang's claims and asserted that, contrary to his writings, the *Tables du cadastre* had been used to check Vlacq's table, and that the errors found by Vlacq had mostly already been found by Lefort [27]. The article in *Nature* led Sang to publish a more detailed article on the *Tables du cadastre*, and on the need for new tables [66], and an exchange with Lefort followed [28, 67], since Sang had actually not seen the *Tables du cadastre*, and only seen one of Lefort's articles, not his analysis published in the *Annales de l'Observatoire*. In 1875, the *Comptes rendus hebdomadaires des séances de l'Académie des sciences* also had a short note echoing Govi's article [21] on Sang's project [3].

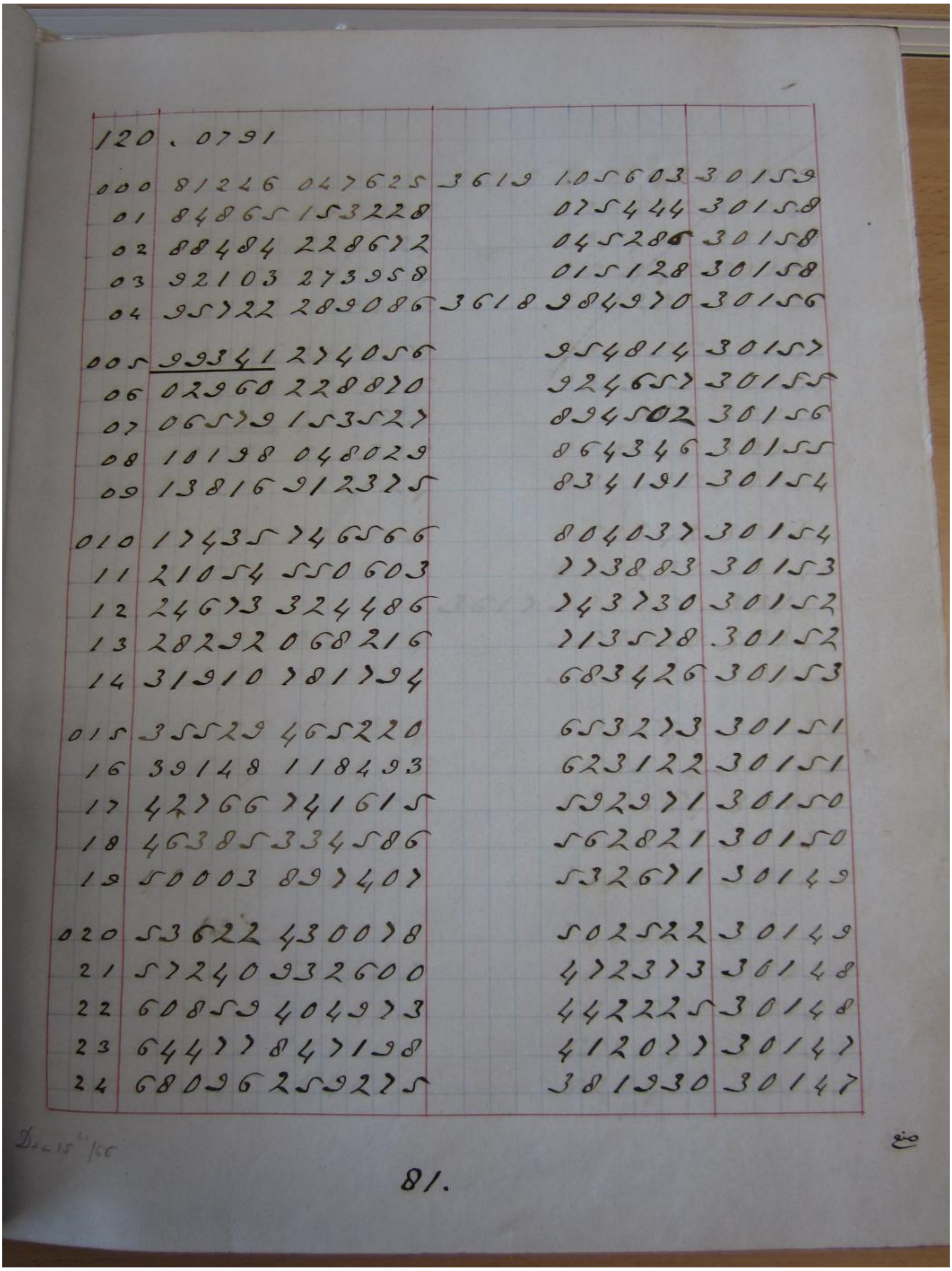
adopted this feature. These departures from the original tables are mere details and should not hinder a comparison with the original tables.

Sang numbered the pages by groups of 8, starting with 500 for the first page covering the range 100000 to 100025. This may mean that Sang considered filling the interval 0–100000, which would have covered $500 \times 8 = 4000$ pages. However, the first computation of the interval 100000 to 150000 (volumes K7 to K11) starts with page 1, and the groups cover 10 pages and not 8.

The exact values of the logarithms were computed with the GNU `mpfr` library [16] and the 15th place of each logarithm is correctly rounded. I have checked that this is the case on all tangent cases over the whole range of the table. The first and second differences were computed from the rounded logarithms, and are therefore the tabulated differences, not the rounded exact differences. This is what is desired, since Sang also computed the differences from the logarithms to 15 places, in order to check them.

4 Accuracy

Sang's table appears quite accurate, and apart from possible transcription errors and judging from a number of samples, the errors seem to be no greater than three units of the last decimal place of the logarithms. A more comprehensive examination of the original tables is however required to assess the real accuracy of the tables, and also in order to understand how each logarithm has been computed.



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000	81226 047625	3619	1.05603	30159
01	84865 153228		075444	30158
02	88484 228672		045286	30158
03	92103 273958		015128	30158
04	95722 289086	3618	984970	30156
005	99341 274056		954814	30157
06	02960 228870		924657	30155
07	06579 153527		894502	30156
08	10198 048029		864346	30155
09	13816 912375		834191	30154
010	17435 746566		804037	30154
11	21054 550603		773883	30153
12	24673 324486		743730	30152
13	28292 068216		713578	30152
14	31910 781794		683426	30153
015	35529 465220		653273	30151
16	39148 118493		623122	30151
17	42766 741615		592971	30150
18	46385 334586		562821	30150
19	50003 897407		532671	30149
020	53622 430078		502522	30149
21	57240 932600		472373	30148
22	60859 404973		442225	30148
23	64477 847198		412077	30147
24	68096 259275		381930	30147

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81.

Figure 1: Excerpt from Sang's table of logarithms. (Courtesy Edinburgh University Library)

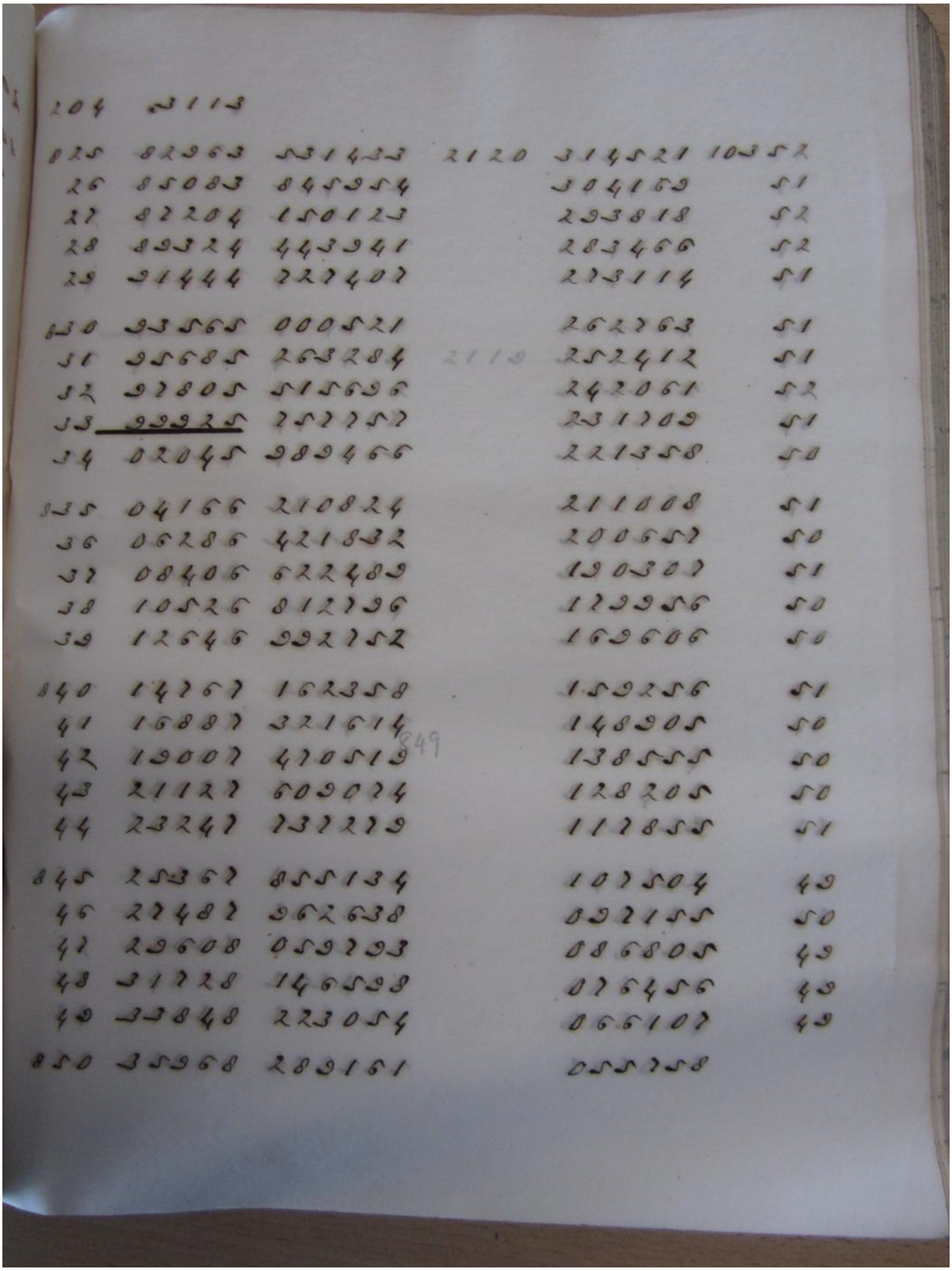

Figure 2: Excerpt from Sang's table of logarithms (transfer duplicate). (Courtesy Edinburgh University Library)

References

The following list covers the most important references[6] related to Sang's tables. Not all items of this list are mentioned in the text, and the sources which have not been seen are marked so. I have added notes about the contents of the articles in certain cases.

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[6]**Note on the titles of the works:** Original titles come with many idiosyncrasies and features (line splitting, size, fonts, etc.) which can often not be reproduced in a list of references. It has therefore seemed pointless to capitalize works according to conventions which not only have no relation with the original work, but also do not restore the title entirely. In the following list of references, most title words (except in German) will therefore be left uncapitalized. The names of the authors have also been homogenized and initials expanded, as much as possible.

The reader should keep in mind that this list is not meant as a facsimile of the original works. The original style information could no doubt have been added as a note, but I have not done it here.

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