A survey of the main fundamental European trigonometric tables printed in the 15th and 16th centuries

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Abstract

This document is a survey of the main European fundamental trigonometric tables printed in the 15th and 16th centuries. After a review of the work done before the 15th century in Greece, India and the Arab world, the starting points in Europe are examined. The seminal work of Regiomontanus is carefully studied and the lineage of all later works is established.

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1 Introduction

The purpose of this survey is to sort out the many fundamental European purely (*i.e.*, non astronomical) trigonometric tables published in the 15th and 16th centuries, and specifically to clarify their relationships.[1] I am concerned here almost exclusively with tables of sines, tangents and secants, and not with more specialized trigonometric tables that might be used as auxiliary tables.

Although a study with a similar scope has been published by Glowatzki and Göttsche in 1990,[2] I feel that it is necessary to review the tables in the light of their ready access, and to see whether their understanding can be improved. I believe that my study brings new information and corrects some earlier mistakes.

This new examination is also made in the context of the LOCOMAT project,[3] where a number of historical tables have been reconstructed (computationally and typographically) and analyzed, enabling a better assessment of their accuracy and lineage. However, it must be stressed that the absolute accuracy of the historical tables under consideration here is less important than their relationships and the process that led to their computation or organization.

In the following sections, I first give a short review of the history of purely trigonometric tables before the 15th century, and follow their development through Greece, India, the Arab world, and finally Western Europe. I am then considering the work of four great innovators, Johannes von Gmunden, Giovanni Bianchini, Georg von Peuerbach and Johannes Regiomontanus. The latter was the one who greatly expanded the world of trigonometric tables, and set the background for almost all future work until the end of the 16th century. I am therefore examining what are Regiomontanus's seminal tables, and then journey through a century of tables, from Regiomontanus's *Tabulæ directionum profectionumque* of 1490 to Rheti-

[1]There is a vast literature on numerical tables, and I am directing the reader to a number of general surveys, such as [Hutton (1785)], [De Morgan (1842)], [De Morgan (1851)], [Glaisher (1873)], [Davis (1933), pp. 1-40], [Campbell-Kelly et al. (2003)], etc. This document also mentions many people, and I am not always directing to specific biographical information for each of them. Valuable informations can in particular be found in the notices of [Hockey (2014)], in particular on Al-Battāni, Abū al-Wafāʾ, Al-Khwārizmī, Al-Zarqālī, Apian, Bürgi, Clavius, Copernicus, Engel, Fine, Gemma Frisius, Lansberge, Magini, Maurolico, Peucer, Peuerbach, Regiomontanus, Reinhold, Rheticus, and many others.

[2][Glowatzki and Göttsche (1990)]

[3]`https://locomat.loria.fr`, see [Roegel (2012)].

cus's *Opus palatinum* of 1596, which was itself the start of a new era, but the end of this survey. In this journey, I am in particular examining the genealogy of the tables. In other words, I am trying to find out who copied on whom, and I am also trying to shed a new light on the computations that were made, whenever possible.

Finally, this survey is also a companion document to a number of modern reconstructions, that is, reconstructions usually giving the exact values, but also trying to reproduce the original layout of the tables, so as to make their comparison straightforward. These reconstructions are those of Regiomontanus's table of tangents (1490),[4] Engel's table of sines (1490, but here reproduced from the 1504 edition),[5] Peuerbach's arctangent table (1516),[6] the tables of sines of Fine (1530),[7] Apian (1533),[8] Regiomontanus (1541),[9] Rheticus (1542)[10] and again Fine (1550),[11] and eventually the trigonometric tables of Rheticus (1551),[12] Reinhold (1554),[13] Maurolico (1558),[14] Viète (1579),[15] Fincke (1583),[16] Lansberge (1591),[17] Rheticus & Otho (1596)[18] and Pitiscus (1613).[19]

2 Before the 15th century

I give here a quick and rough sketch of the history of trigonometric tables before the 15th century, so as to serve as a background for the development of trigonometry in the 15th and 16th centuries. More detailed (although sometimes incorrect or dated) surveys can be found in the works of Braun-

[4] [Regiomontanus (1490)]
[5] [Regiomontanus (1490), Regiomontanus (1504)]
[6] [von Peuerbach (1516)]
[7] [Fine (1530)]
[8] [Apian (1533)]
[9] [von Peuerbach and Regiomontanus (1541)]
[10] [Copernicus (1542)]
[11] [Fine (1550)]
[12] [Rheticus (1551)]
[13] [Reinhold (1554)]
[14] [Maurolico (1558)]
[15] [Viète (1579)]
[16] [Fincke (1583)]
[17] [van Lansberge (1591)]
[18] [Rheticus and Otho (1596)]
[19] [Pitiscus (1613)]

mühl,[20] Tropfke,[21] Bond,[22] Zeller[23] and more recently of Brummelen.[24] More general works on the history of mathematics may also sometimes be of interest, for instance those of Montucla,[25] Kästner,[26] Zeuthen,[27] Katz[28], Boyer/Merzbach[29] or Scriba/Schreiber,[30] but they may at times be inaccurate.

2.1 Greek chord tables

Trigonometry started with triangles inscribed in circles of some radius R. This radius was typically taken to be 60, but other values were also used. Within such a circle, some quantities can then be defined. In particular chords are segments subtended by an arc (figure 1) and there is a simple relationship between chords in a circle of radius R (which I denote Chd_R or often merely Chd) and sines (which I assume to be defined in a unit circle). We have $\mathrm{Chd}_R\,\alpha = 2R\sin(\alpha/2)$. In the case of sine tables, R was later called the *sinus totus*. This radius was not made equal to unity before Abū al-Wafāʾ in the 10th century (see § 2.3).[31]

We know that Hipparchus (c.190-c.120 BC) and Menelaus of Alexandria (c.70-c.140) wrote treatises on chords, but these works are unfortunately lost.[32] It is not known if they contained tables of chords. But we know that the use of tables in Greek mathematics apparently takes its roots in Babylonian sources.[33]

In 1974, Toomer suggested that Hipparchus may have had a table of

[20] [von Braunmühl (1900, 1903)]

[21] [Tropfke (1902-1903), v. 2, pp. 189-221, and 296-306]

[22] [Bond (1921)]

[23] [Zeller (1944)]

[24] See [van Brummelen (2009)] and [van Brummelen (2021)].

[25] [Montucla (1758)]

[26] [Kästner (1796)]

[27] [Zeuthen (1903)]

[28] [Katz et al. (2007)]

[29] [Merzbach and Boyer (2010)]

[30] [Scriba and Schreiber (2015)]

[31] In the sequel, sines will often be defined in non-unit circles, and I will use Sin for this purpose, often leaving the radius implicit. We have of course $\mathrm{Sin}_R\,\alpha = R\sin\alpha$. Some authors call this Sin the R-sine, but I will always use "sine" alone, as the context should be unambiguous. I will also use other variants such as Tan, Sec, etc., when needed.

[32] [Bond (1921), pp. 297-298]

[33] [Sidoli (2014), p. 13]

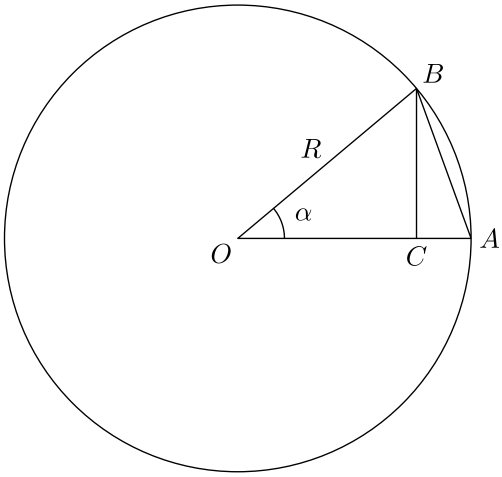

Figure 1: Chords and sines. AB is the chord of α and BC is its sine, for a radius R. We have $\mathrm{Chd}_R\,\alpha = 2R\sin(\alpha/2)$ and $BC = \mathrm{Sin}_R\,\alpha$.

chords with a circle of radius $R = 3438$ and at intervals of $7.5°$,[34] and that this radius was then copied by Indian mathematicians, but this is still debated, by Toomer himself,[35] as well as by Klintberg in 2005 who believes that Hipparchus may have had instead a chord table with $R = 3600$.[36] On the other hand, Duke, also in 2005, and using the analysis of two eclipse trios, concurs with Toomer's original suggestion.[37]

Later, Ptolemy (2nd century AD) gathered the earlier works and covered the computation and use of chords in the first book of the Almagest. His table gives the chords for every $30'$ of the quadrant, using a circle of diameter 120 (figure 2).[38]

The way Ptolemy computed his table of chords was to find first the sides of the inscribed regular triangle, quadrilateral, pentagon, hexagon and decagon in a circle divided in 60 parts, that is, of radius 60.[39] This gave

[34][Toomer (1974), p. 7] See [van Brummelen (2009), pp. 41-45] for a recent discussion on this topic.

[35][Ptolemaeus (1984), p. 215]

[36][Klintberg (2005)]

[37][Duke (2005)]

[38]See [Ptolemaeus (1813-1816), v. 1, pp. 38-45], [Ptolemaeus (1898-1903), v. 1, pp. 48-63], [Ptolemaeus (1984), pp. 57-60].

[39]Besides Toomer's edition of the Almagest [Ptolemaeus (1984), pp. 57-60], see [Neugebauer (1975), pp. 21-24], [Pedersen (2011), ch. 3], [Bond (1921), pp. 301-303], [Clagett (1957), pp. 200-205], [Kneale (1965)], [Glowatzki and Göttsche (1976)], [Thurston (1996), pp. 235-

him the chords of 36°, 60°, 72°, 90°, 108°, 120°, and 144°.

Using the theorem known as Ptolemy's theorem (a relation between the four sides and two diagonals of a cyclic quadrilateral), Ptolemy was able to compute the chord of the difference of two arcs, when the chords of these arcs are known, and also the chord of their sum. He also was able to compute the chord of the half arc from that of the arc. Eventually, Ptolemy computed the chords of 0.75° and of 1.5°.

Then Ptolemy used an interpolation to find the chord of 1°:

$$\mathrm{Chd}\,1^\circ = 1^{\mathrm{p}}2'50''$$

This means that the chord of 1° is a bit more than one part, given that the radius is equal to 60 parts. Of course, $\mathrm{Chd}\,180^\circ = 2R = 120$.

The above value for $\mathrm{Chd}\,1^\circ$ is correct, since we actually have $\mathrm{Chd}\,1^\circ = 2 \cdot 60 \cdot \sin 0.5^\circ = 1.047184\ldots \approx 1 + 2/60 + 50/60^2$. This value will also be written $1;2,50$, following a convention used by many authors.[40]

After the computation of $\mathrm{Chd}\,1^\circ$, Ptolemy obtained $\mathrm{Chd}\,0.5^\circ$ and eventually all the other values in his table of chords. Glowatzki and Göttsche recomputed Ptolemy's table using the procedure he described in the Almagest.[41]

The beginning of Ptolemy's table of chords as given by Halma is shown in figure 2.

In the following excerpt of Ptolemy's table

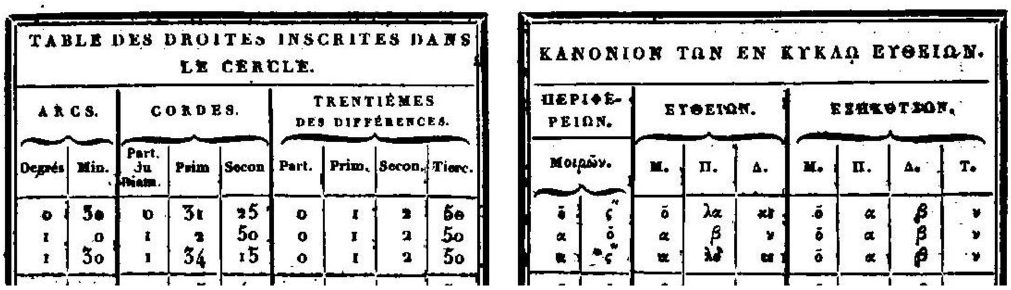

TABLE DES DROITES INSCRITES DANS LE CERCLE.

ARCS.		CORDES.			TRENTIÈMES DES DIFFÉRENCES.			
Degrés	Min.	Part. du Diam.	Prim	Secon	Part.	Prim.	Secon.	Tierc.
0	30	0	31	25	0	1	2	50
1	0	1	2	50	0	1	2	50
1	30	1	34	15	0	1	2	50

ΚΑΝΟΝΙΟΝ ΤΩΝ ΕΝ ΚΥΚΛΩ ΕΥΘΕΙΩΝ.

ΠΕΡΙΦΕΡΕΙΩΝ. Μοιρῶν.		ΕΥΘΕΙΩΝ.			ΕΞΗΚΟΣΤΩΝ.			
		Μ.	Π.	Δ.	Μ.	Π.	Δ.	Τ.
ō	ς″	ō	λα	κε	ō	α	β	ν
α	ō	α	β	ν	ō	α	β	ν
α	ς″	α	λδ	ιε	ō	α	β	ν

236], [van Brummelen (2009), pp. 70-77], [Buscherini and Panaino (2010)], [Otero (2020)] and especially [van Brummelen (1993), pp. 46-73] for an extensive analysis of Ptolemy's chord table and its underlying mathematics.

[40]Throughout this document, I will count decimal places beyond this radius, and not including it, so that the value of $\mathrm{Chd}\,1^\circ$ given here will be considered given to two (sexagesimal) places and not three.

[41]See [Glowatzki and Göttsche (1976)]. Glowatzki and Göttsche give the listings of the PL/I programs they used.

the values of $\mathrm{Chd}\,0^\circ 30'$, $\mathrm{Chd}\,1^\circ$ and $\mathrm{Chd}\,1^\circ 30'$ are given, together with differences. These differences are given in thirtieth of the actual differences, so that $31'25''$ becomes $\frac{31'25''}{30} = \frac{2\times 31'25''}{60} = \frac{62'50''}{60} = 62''50''' = 1'2''50'''$.

In Greek (right hand side), letters are used for numbers, in particular $\bar{o}$ for 0, α for 1, β for 2, ι for 10, $\varkappa$ for 20, λ for 30, ν for 50, $\iota\varepsilon$ for 15, $\varkappa\varepsilon$ for 25, $\lambda\alpha$ for 31, $\lambda\delta$ for 34, etc. Note however that Halma uses ς'' for $30'$, when actually Ptolemy used a symbol for the half degree.[42]

As the chords are twice the sines of the half angles, Ptolemy's table would make it very easy to obtain the sines at intervals of $15'$.

2.2 Indian tables

The history of mathematics in India is complex and a lot of details are shady or lost.[43] As far as trigonometry is concerned, some elements of Greek chord tables were probably taken to India, but they were then converted to sines.[44] It seems that it was for practical reasons that Indian astronomers replaced the chords (*jyā*) by the sines, that is by half-chords (*jyā-ardha*, eventually shortened to *jyā*), with various values of the radius R of the base circle.[45]

This move from chords to sines may seem to be a detail, but it had in fact far-reaching consequences, connecting trigonometric functions with right triangles and therefore to the Pythagorean theorem.

However, even though a transmission from Greece to India is compelling, there is no certainty about the origins of the calculations, and whether the values were borrowed from Greek sources or computed independently.[46]

In any case, once the sine (*jyā*) had been defined for radius R, we had

$$\text{jyā}(\theta) = R\sin\theta = \mathrm{Sin}\,\theta.$$

Among the oldest sine tables, Neugebauer and Pingree mention the *Paitāmahasiddhānta*, possibly of the 1st century AD, which had a table based

[42] [Ptolemaeus (1898-1903), v. 1, p. 48]

[43] For summaries of the history of mathematics and astronomy in India and extensive discussions on trigonometry or tables, see [Srinivasiengar (1967)], [Pingree (1978)], [Bag (1979)], [Katz et al. (2007)], [Plofker (2009)], [González-Velasco (2011), pp. 25-34], [van Brummelen (2009), pp. 94-134], [Puttaswamy (2012), pp. 108-116], [Divakaran (2018)], [Montelle and Plofker (2018)] (especially page 57) and [Ramasubramanian (2019)].

[44] [van Brummelen (2009), p. 99]

[45] [van Brummelen (2009), p. 96]

[46] [van Brummelen (2009), p. 99]

38 ΜΑΘΗΜΑΤΙΚΗΣ ΣΥΝΤΑΞΕΩΣ ΒΙΒΛΙΟΝ Α.

TABLE DES DROITES INSCRITES DANS LE CERCLE.

ARCS.		CORDES.			TRENTIÈMES DES DIFFÉRENCES.			
Degrés	Min.	Part. du Diam.	Prim	Secon	Part.	Prim.	Secon.	Tierc.
0	30	0	31	25	0	1	2	50
1	0	1	2	50	0	1	2	50
1	30	1	34	15	0	1	2	50
2	0	2	5	40	0	1	2	50
2	30	2	37	4	0	1	2	48
3	0	3	8	28	0	1	2	48
3	30	3	39	52	0	1	2	48
4	0	4	11	16	0	1	2	47
4	30	4	42	40	0	1	2	47
5	0	5	14	4	0	1	2	46
5	30	5	45	27	0	1	2	45
6	0	6	16	49	0	1	2	44
6	30	6	48	11	0	1	2	43
7	0	7	19	33	0	1	2	42
7	30	7	50	54	0	1	2	41
8	0	8	22	15	0	1	2	40
8	30	8	53	35	0	1	2	39
9	0	9	24	54	0	1	2	38
9	30	9	56	13	0	1	2	37
10	0	10	27	32	0	1	2	35
10	30	10	58	49	0	1	2	33
11	0	11	30	5	0	1	2	32
11	30	12	1	21	0	1	2	30
12	0	12	32	36	0	1	2	28
12	30	13	3	50	0	1	2	27
13	0	13	35	4	0	1	2	25
13	30	14	6	16	0	1	2	23
14	0	14	37	27	0	1	2	21
14	30	15	8	38	0	1	2	19
15	0	15	39	47	0	1	2	17
15	30	16	10	56	0	1	2	15
16	0	16	42	3	0	1	2	13
16	30	17	13	9	0	1	2	10
17	0	17	44	14	0	1	2	7
17	30	18	15	17	0	1	2	5
18	0	18	46	19	0	1	2	2
18	30	19	17	21	0	1	2	0
19	0	19	48	21	0	1	1	57
19	30	20	19	19	0	1	1	54
20	0	20	50	16	0	1	1	51
20	30	21	21	12	0	1	1	48
21	0	21	52	6	0	1	1	45
21	30	22	22	58	0	1	1	42
22	0	22	53	49	0	1	1	39
22	30	23	24	39	0	1	1	36

ΚΑΝΟΝΙΟΝ ΤΩΝ ΕΝ ΚΥΚΛΩ ΕΥΘΕΙΩΝ.

ΠΕΡΙΦΕΡΕΙΩΝ. Μοιρῶν.		ΕΥΘΕΙΩΝ.			ΕΞΗΚΟΣΤΩΝ.			
		Μ.	Π.	Δ.	Μ.	Π.	Δ.	Τ.
ō	ς″	ō	λα	κε	ō	α	β	ν
α	ō	α	β	ν	ō	α	β	ν
α	ς″	α	λδ	κε	ō	α	β	ν
β	ō	β	ε	μ	ō	α	β	ν
β	ς″	β	λζ	δ	ō	α	β	μη
γ	ō	γ	η	κη	ō	α	β	μη
γ	ς″	γ	λθ	νβ	ō	α	β	μη
δ	ō	δ	ια	ις	ō	α	β	μζ
δ	ς″	δ	μβ	μ	ō	α	β	μζ
ε	ō	ε	ιδ	δ	ō	α	β	μς
ε	ς″	ε	με	κζ	ō	α	β	με
ς	ō	ς	ις	μθ	ō	α	β	μδ
ς	ς″	ς	μη	ια	ō	α	β	μγ
ζ	ō	ζ	ιθ	λγ	ō	α	β	μβ
ζ	ς″	ζ	ν	νδ	ō	α	β	μα
η	ō	η	κβ	ιε	ō	α	β	μ
η	ς″	η	νγ	λε	ō	α	β	λθ
θ	ō	θ	κδ	νδ	ō	α	β	λη
θ	ς″	θ	νς	ιγ	ō	α	β	λζ
ι	ō	ι	κζ	λβ	ō	α	β	λε
ι	ς″	ι	νη	μθ	ō	α	β	λγ
ια	ō	ια	λ	ε	ō	α	β	λβ
ια	ς″	ιβ	α	κα	ō	α	β	λ
ιβ	ō	ιβ	λβ	λς	ō	α	β	κη
ιβ	ς″	ιγ	γ	ν	ō	α	β	κζ
ιγ	ō	ιγ	λε	δ	ō	α	β	κε
ιγ	ς″	ιδ	ς	ις	ō	α	β	κγ
ιδ	ō	ιδ	λζ	κζ	ō	α	β	κα
ιδ	ς″	ιε	η	λη	ō	α	β	ιθ
ιε	ō	ιε	λθ	μζ	ō	α	β	ιζ
ιε	ς″	ις	ι	νς	ō	α	β	ιε
ις	ō	ις	μβ	γ	ō	α	β	ιγ
ις	ς″	ιζ	ιγ	θ	ō	α	β	ι
ιζ	ō	ιζ	μδ	ιδ	ō	α	β	ζ
ιζ	ς″	ιη	ιε	ιζ	ō	α	β	ε
ιη	ō	ιη	μς	ιθ	ō	α	β	β
ιη	ς″	ιθ	ιζ	κα	ō	α	β	ō
ιθ	ō	ιθ	μη	κα	ō	α	α	νζ
ιθ	ς″	κ	ιθ	ιθ	ō	α	α	νδ
κ	ō	κ	ν	ις	ō	α	α	να
κ	ς″	κα	κα	ιβ	ō	α	α	μη
κα	ō	κα	νβ	ς	ō	α	α	με
κα	ς″	κβ	κβ	νη	ō	α	α	μβ
κβ	ō	κβ	νγ	μθ	ō	α	α	λθ
κβ	ς″	κγ	κδ	λθ	ō	α	α	λς

Figure 2: The beginning of Ptolemy's table of chords as retranscribed by Halma in 1813 [Ptolemaeus (1813-1816)]. The part on the right shows the numerical values as represented by Greek letters. The left part is the modern translation. One should take note that the layout of the table in the Greek manuscripts differs from that displayed here and half-degrees are marked with a special symbol [Ptolemaeus (1898-1903), v. 1, p. 48].

on $R = 3438$.[47]

As mentioned above, Toomer has suggested that this radius 3438, which is $60 \cdot 360$ divided by an approximation of 2π, was actually borrowed from Hipparchus,[48] but this claim may now be questioned. In any case, the simultaneous choice in India of a radius of 3438 and a measure of the circumference of $360 \cdot 60'$ means that the radius was actually measured in the same units as the circumference, thus anticipating the concept of radians.[49]

The *Sūrya Siddhānta*, a Sanskrit treatise on Indian astronomy, which in its original version goes back to the 4th century, may also have been one of the earliest texts giving a table of sine. The version now known of this work which had been heavily amended gives a table of sines with the same $R = 3438$ and for every multiple of $3°45'$.[50] This interval of $3°45'$ may go back to an interval of $7°30'$ for chords.[51]

Around 499 AD, Āryabhaṭa's *Āryabhaṭīya* also used $R = 3438$ and had tables of sines ($\mathrm{Sin}\, x$) and versines (or versed sines, *utkramajyā*, $R - \mathrm{Cos}\, x$) for every x multiple of $3°45'$.[52]

Then in the sixth century AD, Varāhamihira (c505-c587) gave a table of sines with $R = 120$ again for every multiple of $3°45'$.[53] Neugebauer and Pingree write that this table uses a terminology derived from that of the *Paitāmahasiddhānta* mentioned above. In any case, Varāhamihira's table may be much older than the 6th century as his work, the *Pañca-siddhāntikā* is a summary of five earlier *siddhāntas*. And since, as observed by Bag,[54] we have the chord of $7°30'$ in a circle of radius $R = 60$ which is equal to the sine of $3°45'$ in a circle of radius $R = 120$, and that therefore a table of chords can right away become a table of sines in a circle twice as large, it may be that Varāhamihira's table goes back to a table of chords with $R = 60$.

In his *Brāhma-sphuṭa-siddhānta*, Brahmagupta (c598-668)[55] computed a

[47] [Neugebauer and Pingree (1970-1971), part 2, p. 37]

[48] See [Toomer (1974), p. 6] and [Divakaran (2018), p. 198].

[49] [Neugebauer (1956)]

[50] [Burgess (1860), pp. 58-60]

[51] [van Brummelen (2009), p. 97]

[52] See [Clark (1930), p. 19], [Srinivasiengar (1967), pp. 40-54], [Filliozat (1988)], and [Mazars (1974)].

[53] See [Neugebauer and Pingree (1970-1971), part 2, pp. 37-38] and [Plofker (2009), p. 51]

[54] [Bag (1969), p. 84]

[55] [Bhattacharyya (2011)]

table of sines but with the radius 3270 for every multiple of $3°45'$.[56] Gupta suggested that this peculiar value of R is rounded from $21600/6.6$, where $6.6/2$ is an approximation of $\sqrt{10}$.[57]

Bag[58] gave a comparative view of the main early Indian tables of sines and examined how Varāhamihira and others may have computed their values.

Brahmagupta has also used the value $R = 150$ in his *Khaṇḍakhādyaka* (665).[59] This value was then used again by Al-Khwārizmī.

2.3 Arabic tables

I merely sketch here the main milestones in the developement of trigonometric tables between the 8th and 13th centuries, before their wider transmission to Western Europe.[60]

At the end of the 8th century, during the first years of the Abbasid Caliphate (750-1258), men of learning were gathered in Baghdad and they translated into Arabic the works of the Hindus and the Greeks.[61] In particular, excerpts of Brahmagupta's *Brāhma-sphuṭa-siddhānta* were brought to the calif Al-Mansur (714-775) by a scholar named Kaṅka[62] and a translation was made.

The Persian mathematician and astronomer Al-Khwārizmī (c780-850) wrote a revised edition of this translation, the *Zīj al-Sindhind*.[63] The word *zīj* is a generic name used for tabular astronomical works in Arabic and Persian, and it is derived from a Persian word meaning "cord" or "string", the tables with their columns and lines bearing some similarity with strings.[64]

The Indian word *jyā* for the chord was translated to *jib* and was later probably incorrectly translated in the Latin *sinus*, based on the similar

[56] [Plofker (2009), p. 81, 157]

[57] [Gupta (1978)]

[58] [Bag (1969)]

[59] See [Chatterjee (1970), p. 206], [Gupta (1978)], [Pingree (1996), p. 43], and [Pingree (2003)].

[60] For more extensive descriptions of Arabic mathematics and astronomy, see in particular the surveys of [von Braunmühl (1900, 1903), v. 1, pp. 42-86] and [Rashed and Morelon (1996)]. Heydari-Malayeri's short survey may also be of interest [Heydari-Malayeri (2007)]. On trigonometric tables in the Islamic world, see [Berggren (1986), p. 144] and [van Brummelen (2009), pp. 135-222].

[61] [Bond (1921), p. 307]

[62] [Bag (1969), p. 84]

[63] See [Kennedy (1956), pp. 148-154], [Pingree (1996), p. 41] and [van Dalen (1996)].

[64] [Heydari-Malayeri (2007)]

unvocalized Arabic word *jaib* meaning "cavity".[65]

For the *Zīj al-Sindhind* Al-Khwārizmī computed c820 a table of sines. In fact, according to McCarthy and Byrne, Al-Khwārizmī's treatise contained two sine tables.[66] The main table used the radius $R = 60$ and a step of 1°,[67] and was likely based on Ptolemy's table of chords,[68] but Al-Khwārizmī also used another simpler table with $R = 150$,[69] that is Brahmagupta's radius from the *Khaṇḍakhādyaka*.[70] This simpler table only contained the sines at intervals of 15° and was especially known from a commentary by Al-Biruni.[71] In particular, McCarthy and Byrne convincingly discard Hogendijk's suggestion[72] of a possible candidate for a full $R = 150$ sine table that could be attributed to Al-Khwārizmī.[73]

Al-Khwārizmī's main table survives in Adelard of Bath (c1080-1152)'s Latin translation (1126) of Maslama al-Majriti (c950-c1007)'s late 10th century Cordova edition of the original table. It was reproduced by Suter in 1914.[74] The table with $R = 150$ is not found in Adelard of Bath's translation, but its radius $R = 150$ made it to the tables of Toledo.

Al-Khwārizmī also had a table of shadows with a gnomon of 12,[75] following the Hindu custom.[76] The shadows seem to have been viewed as apart from the cosines and they were gathered in the same category only by the Europeans in the 15th century.[77]

Al-Khwārizmī's *Zīj al-Sindhind* was brought to Al-Andalus, the Muslim-ruled area of the Iberian Peninsula, sometime between 821 and 852, that is only a short time after its conception. The Umayyad dynasty, after their replacement by the Abbasid dynasty in 750, had reestablished itself there, first as an emirate, then as a caliphate.

During the 9th century, Ptolemy's Almagest was also translated in

[65] See [Folkerts (2006), pp. 75-76], [Goldstein (2019), p. 132] and [Filliozat (1988), p. 261]

[66] [McCarthy and Byrne (2003), p. 247]

[67] [Neugebauer (1962), p. 104]

[68] [McCarthy and Byrne (2003), pp. 265-266]

[69] See [Neugebauer (1962), p. 104] and [Chabás Bergón and Goldstein (2012), p. 19].

[70] [Pingree (1996), p. 43]

[71] [McCarthy and Byrne (2003), p. 246]

[72] [Hogendijk (1991)]

[73] [McCarthy and Byrne (2003), pp. 264-265]

[74] [Suter (1914), tab. 58 and 58a]

[75] [Suter (1914), tab. 60]

[76] [Bond (1921), p. 307]

[77] [Bond (1921), p. 308]. See also [Moussa (2010)] who considers the process by which the tangent and cotangent functions became more abstract, especially with Abū al-Wafāʾ.

Arabic, so that his table of chords was then also known in Arabic.[78]

Around the year 860 the Iranian astronomer al-Marwazi (al-Hasib) (766-after 869) borrowed Ptolemy's table of chords and gave the sines for every 15 minutes.[79] He also constructed the first systematic table of tangents/cotangents[80] (from $0°30'$ to $89°$ at intervals of $30'$ and to three places[81]), although the tangent function had been tabulated before without being identified as such.[82] From then on, the tangent could have a place comparable to that of the sine. But in the West, tangents were only rediscovered in the 15th century by Bianchini and Regiomontanus.

It is interesting to note that a table equivalent to a table of tangents has appeared elsewhere before al-Hasib's table, namely in China. Indian mathematics had actually been exported to China and such a table was constructed there in the 8th century in the form of a shadow-list, but this table was a false start for Chinese trigonometry.[83]

In the *Ṣābi Zīj*,[84] the Syrian astronomer Al-Battāni (Albategnius) (c858-929) put forward the advantages of sines. His table gave the sines for $R = 60$ and for every half-degree and to two sexagesimal places.[85] Al-Battāni also computed a table of cotangents (table of shadows) for every degree.[86]

But the first original arabic constructions of sine tables were the works of Abū al-Wafāʾ and Ibn Yūnus.[87]

The Persian mathematician and astronomer Abū al-Wafāʾ (940-998) gave a better method for the computation of trigonometric tables and his sine table for $R = 60$ has a step of $15'$ and the values were computed on four sexagesimal places.[88] It should be observed, however, as already mentioned, that Ptolemy's table itself already gave the means to construct a

[78] See [Glowatzki and Göttsche (1976), pp. 12-13] and [Folkerts (2006), p. 76].

[79] [Debarnot (1996), p. 524]

[80] [Joseph (2011), p. 497]

[81] [Debarnot (1996), p. 512]. These "three places" probably include the radius.

[82] [Debarnot (1996), p. 509]

[83] See [Cullen (1982)], [Gupta (1987), p. 241], [Qu Anjing (2002)] and [Divakaran (2018), p. 209]. A recent summary of Indian and Islamic trigonometry in China is given in [van Brummelen (2021), pp. 185-191].

[84] See [Al-Battāni (1899-1907)] and [Kennedy (1956), pp. 154-156].

[85] [Al-Battāni (1899-1907), vol 2, pp. 55-56]

[86] [Al-Battāni (1899-1907), vol 2, p. 60]

[87] [Debarnot (1996), p. 524]

[88] This is what Folkerts writes [Folkerts (2006), p. 76], but it may mean four places including the integer part, which would then mean three sexagesimal places with our conventions.

table of sines at intervals of $15'$, since the sine of $15'$ is half the chord of $30'$.

As an indication of Abū al-Wafāʾ's work, let us mention that he found[89]

$$\mathrm{Sin}\,30' = 0;31,24,55,54,55.$$

The correct value is

$$\mathrm{Sin}\,30' = 0;31,24,55,54,0,12,\ldots,$$

that is

$$\mathrm{Sin}\,30' = 0 + 31/60 + 24/60^2 + 55/60^3 + 54/60^4 + 0/60^5 + 12/60^6 + \cdots$$

Abū al-Wafāʾ was also the first to take the radius as unity. He constructed a table of tangents and cotangents with the radius 1, but it was still subdivided sexagesimally.[90] He seems also to have been the one who introduced the secant and cosecant.[91]

Ibn Yūnus (950-1009), astronomer of Cairo, wrote the Hakemite tables. He also recomputed a table of sines. According to Debarnot, Ibn Yūnus's table of sines is more directly based on the Almagest[92] than that of Abū al-Wafāʾ. Ibn Yūnus gave the sines for every minute, for $R = 60$ and to four sexagesimal places. An excerpt of that table is shown by Berggren and King.[93]

Ibn Yūnus obtained $\mathrm{Sin}\,1^\circ = 1;2,49,43,4$,[94] while the correct value is

$$\mathrm{Sin}\,1^\circ = 1;2,49,43,11,14,44,\ldots$$

As observed by Glowatzki and Göttsche, the values of Ibn Yūnus's table were obtained by interpolation.[95] Ibn Yūnus very likely computed the sines at intervals of $10'$ and filled the intermediate values by interpolation.[96] For instance, he gives[97] $\mathrm{Sin}\,28^\circ 10' = 28;19,20,11,0$ which is a rather good

[89] [Debarnot (1996), p. 527]

[90] [Bond (1921), p. 311]

[91] [Joseph (2011), p. 497]

[92] [Debarnot (1996), p. 524]

[93] See [Berggren (1986), p. 150], [Berggren (2016), p. 181] and [King (1975), p. 43].

[94] See [Debarnot (1996), p. 525] and [Schoy (1923), pp. 382-383].

[95] [Glowatzki and Göttsche (1990), p. 9]

[96] Note however that [Debarnot (1996), p. 524] misleadingly states that Ibn Yūnus gave his sines only every $10'$.

[97] [Schoy (1923), p. 394]

approximation (the correct value is $\mathrm{Sin}\, 28°10' = 28; 19, 20, 12, 0$), but the values for $\mathrm{Sin}\, 28°1'$, $\mathrm{Sin}\, 28°2'$, etc., $\mathrm{Sin}\, 28°9'$, are all less accurate, with the least accurate being that for $\mathrm{Sin}\, 28°5'$.

A table with such a small interval would not be available in Western Europe before the work of Regiomontanus in 1462 (see § 3.4). Incidentally, Regiomontanus's table was also obtained by interpolation, albeit certainly using a more elaborate scheme.

At about the same time as Ibn Yūnus, the Iranian scholar Al-Biruni (973-c1050) had obtained the very accurate value $\mathrm{Sin}\, 1° = 1; 2, 49, 43, 11, 14$.[98] His table[99] gives the sines at intervals of $15'$ and uses $R = 1$.

In 1031, the Córdoban caliphate came to an end, the state was divided in a number of smaller kingdoms, and it is during the period 1031 to 1085 that Andalusian science flourished.[100] In particular, around 1070 or 1080 a group of astronomers in Toledo, including Al-Zarqālī (c1028-1087) and perhaps also Ṣā^cid al-Andalusī (1029-1070),[101] put together the "Toledan tables."[102] The tables of Toledo were closely based on those of Al-Khwārizmī[103] and Al-Battāni[104] which had been available in Al-Andalus since the 10th century.[105] Al-Zarqālī is often credited as the author of these tables, but this is not sure and he may also not be the author of their canons.[106]

The original Arabic Toledan tables are no longer extant, but they are known through many Latin editions from the 12th century onward and they had an important influence on Western European astronomy.[107] The tables may have been organized (rather than translated) in Latin by Gerard of Cremona who died in 1187.[108]

The Toledan tables contained a sine table with $R = 150$ (from $1°$ to $180°$ for every degree and with two sexagesimal places)[109] (see figures 3 and 4

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98 [Schoy (1923), p. 386]

99 [Schoy (1923), p. 396]

100 [Samsó Moya (2020)]

101 [Richter-Bernburg (1987)]

102 See [Pingree (1996), p. 46], [Chabás Bergón (2019), pp. 47-75], and [Samsó Moya (2020)].

103 [Suter (1914)]

104 [Toomer (1968)]

105 [van Brummelen (2018), p. 547]

106 See [Busard (1971a), p. 74]. The canons of the tables were published by Curtze in 1900 [Curtze (1900), p. 337].

107 [Zinner (1936), Toomer (1968), Pedersen (2002)]

108 [Zinner (1936), p. 747]

109 See [Zinner (1936), table 25, p. 749], [Toomer (1968), table 12, pp. 27-28], [Pedersen (2002), pp. 946-952], [Millás Vallicrosa (1950), pp. 62-63] and [Kennedy (1956), p. 128].

for Latin editions) and another sine table with $R = 60$ (for every half degree of the quadrant and with two sexagesimal places)[110] (see figures 5 and 6 for Latin editions).

As mentioned above, the radius 150 of this table[111] (but not the values) possibly goes back to Al-Khwārizmī's table, and consequently to Brahmagupta's *Khaṇḍakhādyaka*. However, as observed by van Dalen,[112] the idiosyncrasies of the table indicate that it was likely derived from a table with $R = 60$ (probably by Al-Battāni[113]) by multiplying the values by 2.5 and McCarthy and Byrne[114] believe that Al-Zarqālī was the one who made this transformation, perhaps in the hope of restoring a table which he thought to be that of Al-Khwārizmī.[115]

The second sine table in the Toledan tables, with $R = 60$, originates neither in Al-Khwārizmī's treatise (because Al-Khwārizmī's table only gives the sines at intervals of one degree), nor in Al-Battāni's treatise (because of distinctive discrepancies).[116] It may possibly be based on Ptolemy's table of chords.

The table of shadows of the Toledan tables (see figures 7 and 8) is the same as that in Al-Khwārizmī and Al-Battāni's tables.[117]

In the 12th century, the Christians, assisted by Jewish scholars, translated many Arabic works. In particular, Gerard of Cremona (c1114-1187) translated in Latin the canons of the tables of Toledo and, as mentioned above, Adelard of Bath made Al-Khwārizmī's astronomical tables accessible to the Latins.

Around 1272, the *Alfonsine tables* were constructed in Toledo under the guidance of King Alfonso X of Castile (1221-1284).[118] They were the last major astronomical work by Spanish astronomers before the Renaissance.[119]

The canons of these Castilian Alfonsine tables are still extant in a unique

[110] See [Zinner (1936), table 135, p. 757], [Toomer (1968), table 13, p. 29] and [Pedersen (2002), pp. 954-959].

[111] [Toomer (1968), table 12, pp. 27-28]

[112] [van Dalen (1996), p. 206]

[113] [McCarthy and Byrne (2003), p. 266]

[114] [McCarthy and Byrne (2003), pp. 252-253]

[115] [McCarthy and Byrne (2003), p. 264]

[116] [McCarthy and Byrne (2003), p. 265] Pedersen, however, attributes this table to Al-Battāni [Pedersen (2002), p. 954].

[117] [Toomer (1968), table 15, p. 32]

[118] See [Dreyer (1920)], [Poulle (1988)], [Chabás Bergón (2002)], [Chabás Bergón and Goldstein (2003)], [Swerdlow (2004)] and [Chabás Bergón (2019), pp. 125-132].

[119] [Heydari-Malayeri (2007), p. 10]

manuscript, but the original tables are not. These Alfonsine tables arrived in Paris in the early 14th century and they spread in a modified form in Latin,[120] becoming the Parisian Alfonsine tables. These tables were only superseded in the 16th century by the Prutenic tables based on Copernicus's theory.

More accurate trigonometric tables were constructed in the Arabic world after those of Al-Khwārizmī and Al-Battāni. Chabás and Goldstein mention for instance a 14th century manuscript giving a table of sines for 2700 arguments, at $1'$ intervals,[121] so presumably up to 45° and giving sines and cosines.

And during the next century in Samarkand (now in Uzbekistan), Ulugh Beg (1394-1449) also computed a table of sines for intervals of one minute.[122]

And finally, let's mention that at the beginning of the 15th century, the Persian mathematician Al-Kāshī (c1380-1429) was able to obtain

$$\mathrm{Sin}\,1^\circ = 1;2,49,43,11,14,44,16,19,16$$

(correct value: $\mathrm{Sin}\,1^\circ = 1;2,49,43,11,14,44,16,26,18,\ldots$) by solving numerically the equation $\sin 3x = 3\sin x - 4\sin^3 x$ for $x = 1^\circ$.[123]

[120] See [Goldstein and Chabás Bergón (2004), p. 455] and [Chabás Bergón (2019), pp. 237-276].

[121] [Chabás Bergón and Goldstein (2012), p. 20]

[122] See [Bond (1921), p. 304], [Schoy (1923), pp. 398-399], [Archibald (1949), p. 31], and [Gloden (1950), p. 10]. For the development of table literature in Indian and Arabic mathematics, especially after Ulugh Beg's tables, see for instance the surveys of [Ghori (1985)] and [Plofker (2009)]. Gloden's text just cited, as well as a number of others, should be taken cautiously, as they contain many approximations.

[123] [Aaboe (1954)]

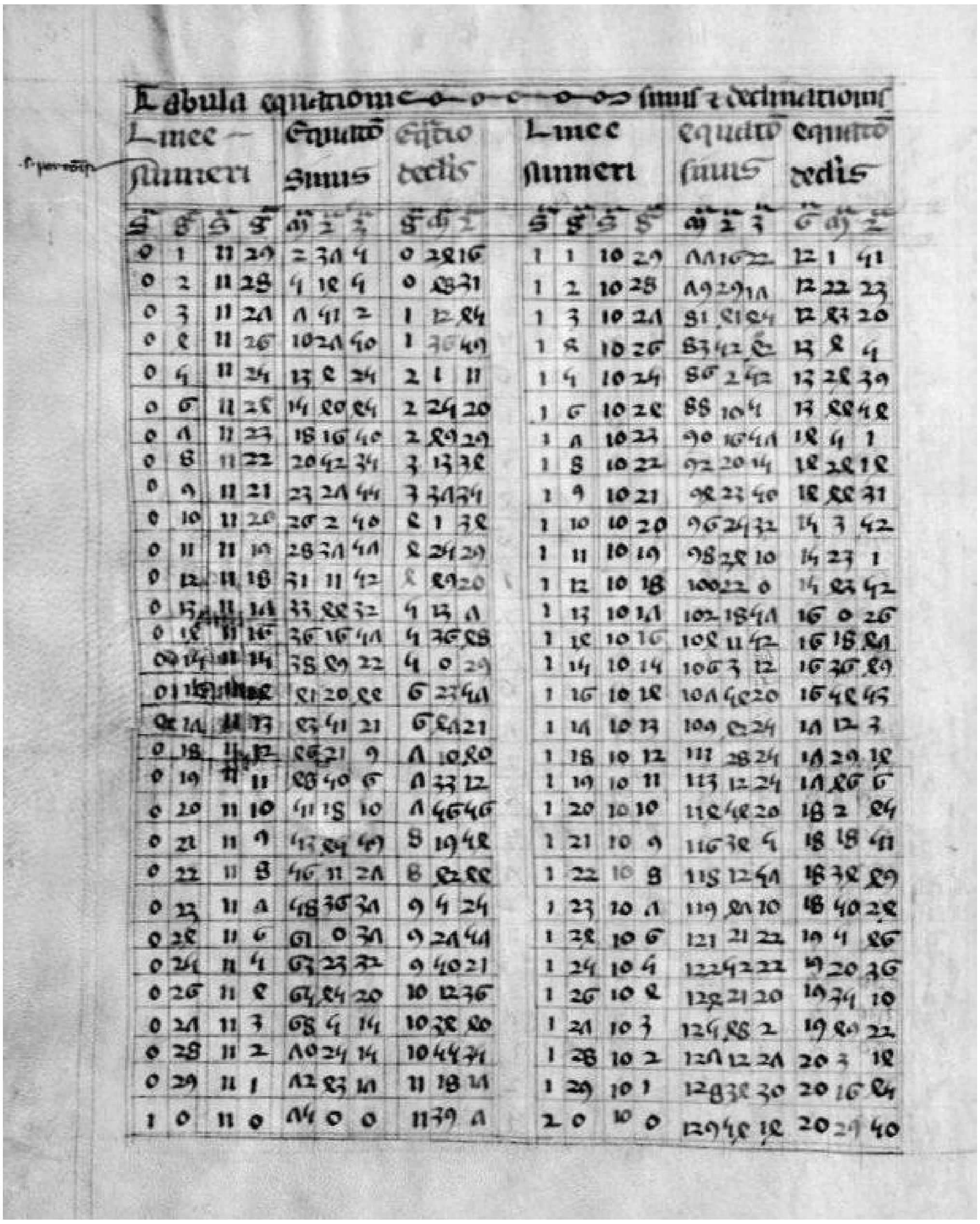

Figure 3: The beginning of the table of sines to $R = 150$ in a Latin edition of the tables of Toledo (BNF Ms. Latin 16211, f°26v).

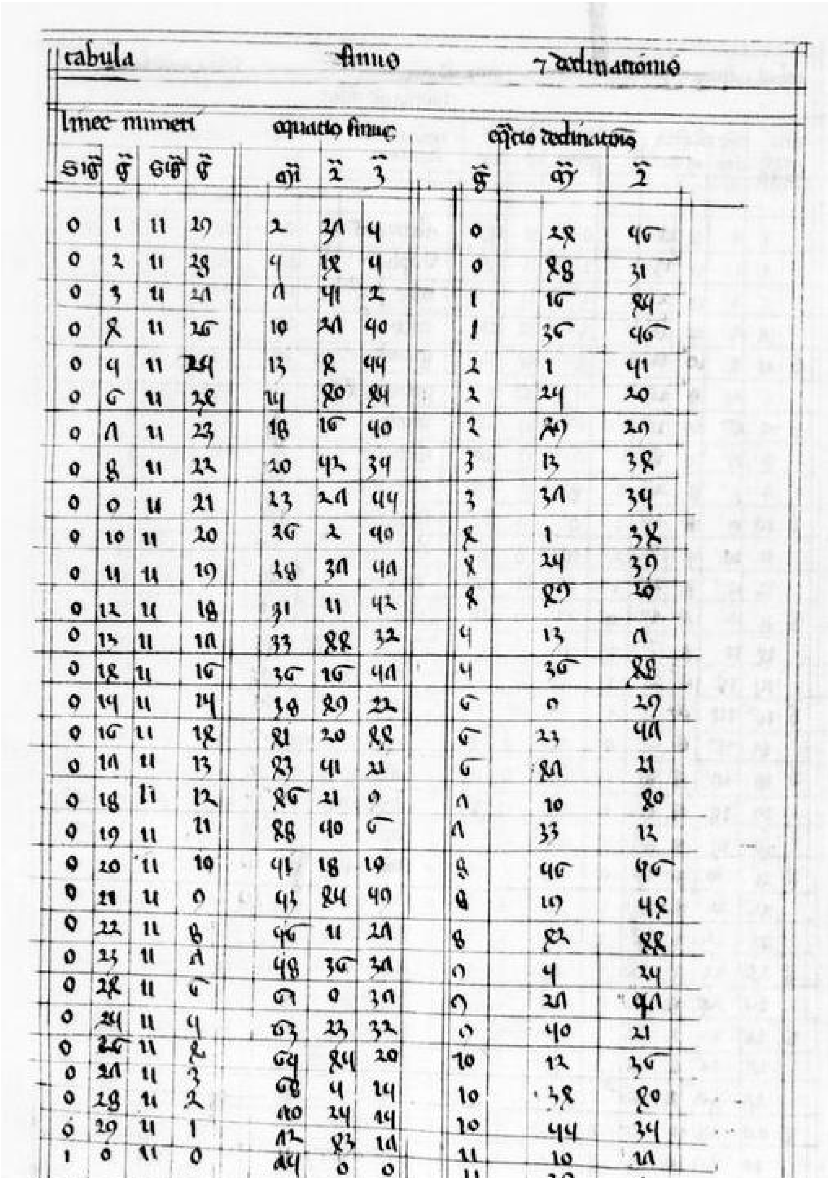

Figure 4: The beginning of the table of sines to $R = 150$ in a Latin edition of the tables of Toledo (BNF Ms. Latin 16655, f°24v).

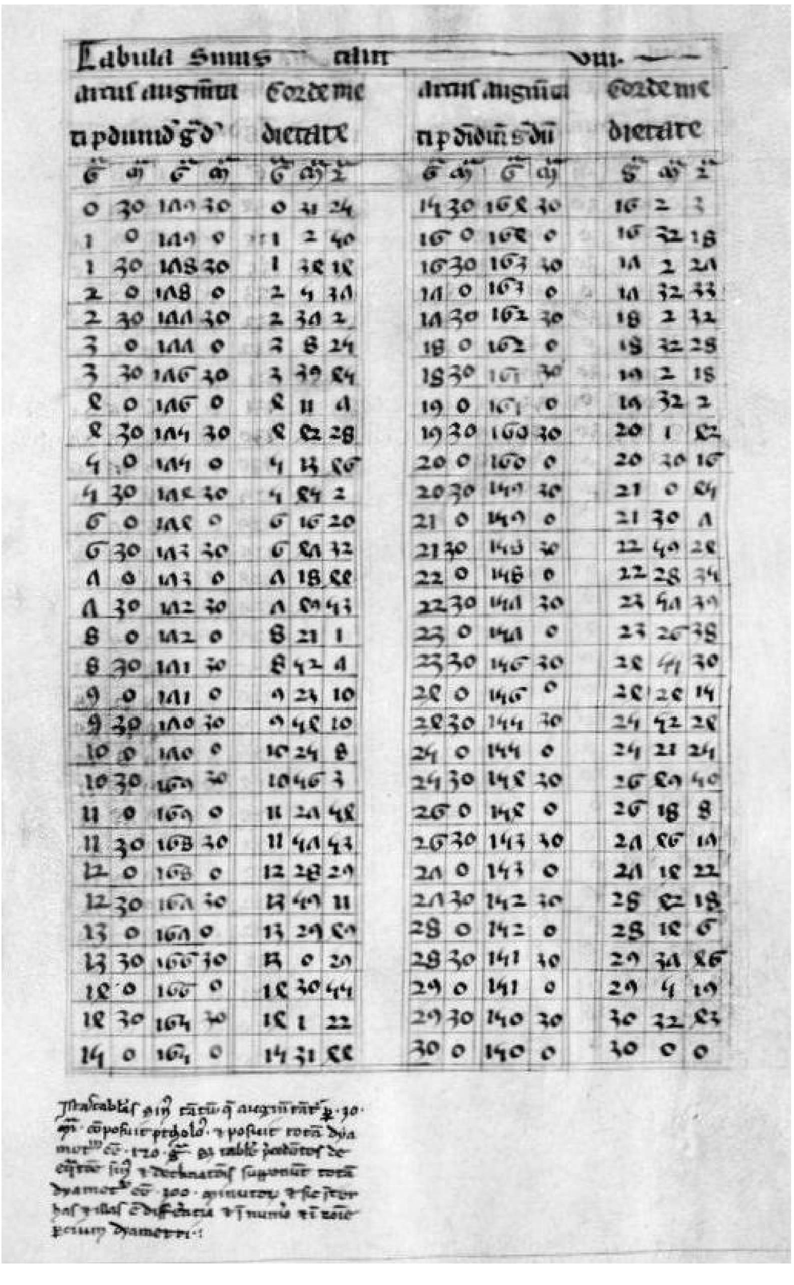

Figure 5: The beginning of the table of sines to $R = 60$ in a Latin edition of the tables of Toledo (BNF Ms. Latin 16211, f°28r).

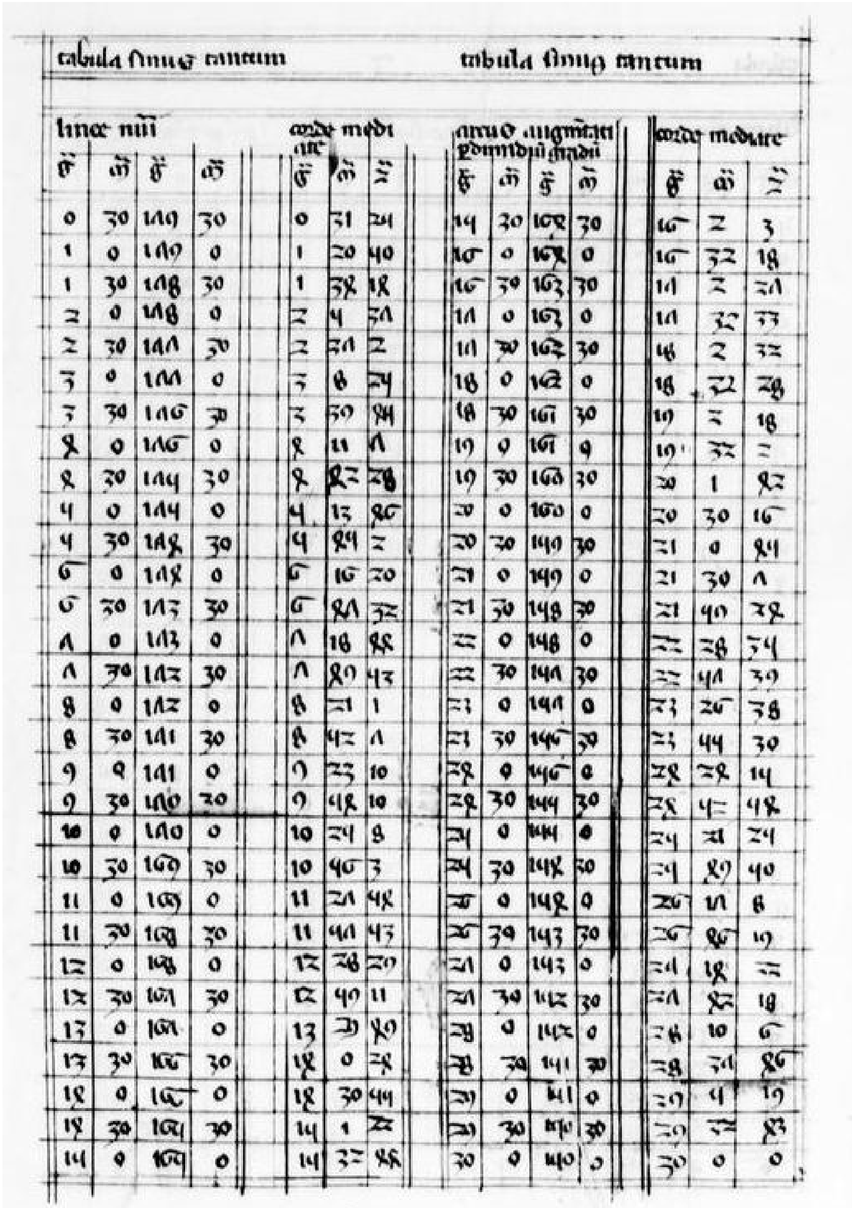

Figure 6: The beginning of the table of sines to $R = 60$ in a Latin edition of the tables of Toledo (BNF Ms. Latin 16655, f°27v).

Figure 7: The table of shadows in a Latin edition of the tables of Toledo (BNF Ms. Latin 16211, f°24v).

tabula			umbre			altitudinis		
grad. altitud.	umbra		grad. altitud.	umbra		grad. altitud.	umbra	
numerus	puncta	minuta	numerus	puncta	minuta	numerus	puncta	minuta
1	687	26	31	19	58	61	6	39
2	343	39	32	19	12	62	6	39
3	293	28	33	18	29	63	6	7
4	171	42	34	17	47	64	5	51
5	137	10	35	17	8	65	5	36
6	114	10	36	16	30	66	5	21
7	97	44	37	15	55	67	5	6
8	85	28	38	15	21	68	4	51
9	75	46	39	14	49	69	4	36
10	68	3	40	14	18	70	4	22
11	61	44	41	13	48	71	4	8
12	56	27	42	13	20	72	3	54
13	51	59	43	12	52	73	3	40
14	48	8	44	12	26	74	3	27
15	44	46	45	12	0	75	3	13
16	41	51	46	11	35	76	3	0
17	39	15	47	11	2	77	2	46
18	36	54	48	10	48	78	2	33
19	34	51	49	10	26	79	2	20
20	32	58	50	10	4	80	2	7
21	31	16	51	9	23	81	1	54
22	29	42	52	9	22	82	1	41
23	28	16	53	9	3	83	1	28
24	26	57	54	8	43	84	1	16
25	25	44	55	8	24	85	1	3
26	24	36	56	8	6	86	0	50
27	23	33	57	7	48	87	0	38
28	22	34	58	7	31	88	0	25
29	21	40	59	7	13	89	0	13
30	20	47	60	6	56	90	0	0

Figure 8: The table of shadows in a Latin edition of the tables of Toledo (BNF Ms. Latin 16655, f°24v).

3 The starting point in Western Europe: from von Gmunden to Regiomontanus

In the 13th and 14th centuries, many writings appeared based on the canons of the Toledan tables, in particular the canons of John of Lignères (1322). These canons borrowed the details of the computation of the sines from the canons of the Toledan tables, but they also gave a sine table for $R = 60$ and for every half degree, as well as a table of shadows.[124]

Moreover, in 1175 Gerard of Cremona (c1114-87) translated Ptolemy's Almagest from the Arabic in Latin.[125] Hence, tables of sines and of chords were available to those willing to pick them up.

Several relatively independent works appeared in the following centuries, of which a few can be mentioned. For instance, in 1220, Leonardo of Pisa (c1170-c1250), known as Fibonacci, published his *Practica Geometriæ* where he gave a table of chords with a radius of 21 *perticæ* and a circumference of 132 *perticæ* (figure 9).[126] The *pertica* is a Roman length unit equal to 10 Roman feet or about 2.96 m. The ratio 132/21 corresponds to the approximation 22/7 for π. In Leonardo of Pisa's table, the arcs are measured with the circumference (first column), so that 90 degrees correspond to 33 *perticae*. In that case, the chord should have been $21\sqrt{2} = 29.69\ldots$ but it is given as 29. For 180 degrees (66 *perticae*), the chord is 42, corresponding to twice the radius.

In that same century, Campanus of Novara (c1220-1296) also supposedly constructed a table of tangents for each degree.[127]

In the 14th century, we should also note the work of Levi Ben Gershon (Gersonides) (1288-1344) who in 1342 independently constructed a table of sines for intervals of $15'$ with a radius $R = 60$ and two sexagesimal places.[128]

And in the fist quarter of the 15th century, Jean Fusoris (c1365-1436) has independently recomputed tables of sines and chords, also at intervals of $15'$, with a radius $R = 60$ and with three to six sexagesimal places.[129]

But the real starting point of new trigonometric computations in Europe were the investigations of Johannes von Gmunden and Giovanni Bianchini,

[124] See [Curtze (1900), pp. 411-412] and [Glowatzki and Göttsche (1990), pp. 73-79].

[125] See [Haskins (1924)] and [Glowatzki and Göttsche (1976), p. 15].

[126] See [Boncompagni (1862), p. 96] and [Hughes (2008), p. 355].

[127] [Bond (1921)]

[128] See [Goldstein (1974), pp. 153-155], [Goldstein (1985), pp. 134-140], and [Goldstein (2019), p. 133].

[129] See [Gassendi (1654), pp. 340-342] and [Poulle (1963), pp. 75-80].

Arcus pertice	Arcus pertice	Corde pertice	Ar pedes	Cuu vncie	M puncta	Arcus pertice	Arcus pertice	Corde pertice	Ar pedes	C V vncie	VM puncta
1	131	0	5	17	17	34	98	30	2	6	17
2	130	1	5	17	13	35	97	31	0	8	5
3	129	2	5	17	4+	36	96	31	4	8	7
4	128	3	5	17	2	37	95	32	2	5	15
5	127	4	4	12	10	38	94	33	0	1	9
6	126	5	5	16	7+	39	93	34	3	13	0
7	125	6	5	14	5	40	92	35	1	4	15
8	124	7	5	12	9	41	91	35	4	12	10
9	123	8	5	8	16	42	90	36	2	0	0
10	122	9	5	7	8	43	89	36	5	3	5
11	121	10	5	4	2	44	88	37	2	4	6
12	120	11	4	17	18	45	87	37	5	3	2
13	119	12	4	13	6	46	86	38	1	17	15
14	118	13	4	7	16	47	85	38	4	12	13
15	117	14	4	1	0	48	84	38	1	4	0
16	116	15	3	11	18	49	83	39	3	11	15
17	115	16	3	3	12	50	82	39	5	17	2
18	114	17	2	12	8	51	81	40	2	2	1
19	113	18	2	0	15	52	80	40	4	2	10
20	112	19	1	8	12	53	79	40	0	0	11
21	111	20	0	13	18	54	78	40	1	14	5
22	110	21	0	0	0	55	77	41	3	7	8
23	109	21	5	2	16	56	76	41	4	16	2
24	108	22	4	4	5	57	75	41	0	4	12
25	107	23	3	4	8	58	74	41	1	8	1
26	106	24	2	3	2	59	73	41	2	9	0
27	105	25	1	0	6	60	72	41	3	7	14
28	104	25	5	16	2	61	71	41	4	9+	2
29	103	26	4	8	0	62	70	41	4	15	10
30	102	27	3	0	3	63	69	41	5	6	9
31	101	28	1	9	7	64	68	41	5	12	17
32	100	28	5	16	4	65	67	41	5	6	14
33	99	29	4	3	9	66	66	42	0	0	0

Figure 9: Leonardo of Pisa's table of chords (1220) [Boncompagni (1862), p. 96].

which seem to have taken place independently at about the same time.

3.1 Johannes von Gmunden (c1384-1442)

Johannes von Gmunden (c1384-1442) founded the study of astronomy and trigonometry in Vienna in the early 1400s.[130] He had obtained his Master degree at the University in 1406. Johannes von Gmunden gave lectures on the construction of astronomical instruments and computed astronomical tables.[131] A few years before his death, he bequeathed his books to the University and thereby founded its first library.[132] Because very few of Johannes von Gmunden's works have been printed, he has been overshadowed by Georg Peuerbach and Regiomontanus.[133]

In 1437, he wrote a treatise *De sinibus, chordis et arcubus*.[134] He described the computation of sines using the Arabic (in fact Indian) methods with the sines of multiples of 15 degrees, as well as the computation of chords using the methods given by Ptolemy in the Almagest. In particular, he described the computation of the sine of the half-angle $\alpha/2$, as well as of the complementary angle $90^\circ - \alpha$, from the sine of α. The formulas are given without proof, like in the canons of the Toledan tables and in John of Lignères's canons.[135] This enabled von Gmunden to compute the sines for every multiple of $3^\circ 45'$ for $R = 150$ and $R = 60$.

Johannes von Gmunden's treatise is accompanied by several tables which, according to Glowatzki and Göttsche, were only computed in 1437 or later.[136]

In the first part, a table of sines with $R = 150$ is given for each degree, and for minutes and seconds of the unit of the *sinus totus*. It is attributed

[130] For summaries of Johannes von Gmunden's life and works, see [von Khauz (1755), pp. 27-32], [Aschbach (1865), pp. 455-467], [Klug (1943)], [Vogel (1973a)], [Grössing (1983), pp. 73-78], [Firneis (1988)], [Kaiser (1988)], [Shank (1997)], [Grössing (2002)], [Folkerts (2006)], [van Brummelen (2009), pp. 248-249], and [Simek and Klein (2012)]. For a survey of his tables, see [Porres de Mateo (2003)] and [Chabás Bergón (2019), pp. 321-336]. See also [Durand (1952), pp. 54-56], and [Duhem (1959), pp. 349-367], especially for the scientific context in Vienna. Gessner mentions von Gmunden very briefly [Gessner and Simmler (1574), p. 375].

[131] [Schmeidler (1977), p. 315]

[132] [von Khauz (1755), p. 29]

[133] [Sperl (1971a)]

[134] This treatise was published in [Busard (1971a)]. See also [Kaiser (1988), pp. 91-96] and [Folkerts (2006), p. 71].

[135] See [Busard (1971a), p. 78] and [Folkerts (2006), p. 81].

[136] [Glowatzki and Göttsche (1990), pp. 79-92]

to Al-Zarqālī and must come from the Toledan tables. I assume it does not originate in John of Lignères's canons as these canons only give the sines every 15 degrees for that *sinus totus*.[137]

Another table of sines, attributed to Ptolemy, with a *sinus totus* of 60, is also given for each degree. This table may be the Toledan table restricted to degrees, or it may be the table borrowed from Ptolemy's Almagest, but restricted to degrees. The two original tables are reproduced by Glowatzki and Göttsche.[138]

In the second part, Johannes von Gmunden gives tables of chords and sines for every half-degree from 0° to 180°, with a radius (*sinus totus*) of 60. The two original tables are also reproduced by Glowatzki and Göttsche.[139] Incidentally, Klug writes incorrectly[140] that Johannes von Gmunden was the first to compute a table of sines at intervals of $30'$.

The first of the tables in the second part is the table given by Ptolemy.[141] The second table may be the result of Johannes von Gmunden's computation, as it goes slightly beyond the table found in the Toledan tables. As a matter of fact, the sines are given to three sexagesimal places,[142] but the last place is always 0 (actually not shown at all) or 30. It may be derived from another table.

Glowatzki and Göttsche[143] drew the attention of a number of incorrect statements on Johannes von Gmunden's tables, in particular by von Braunmühl.[144] The latter, and later Bond[145] and Zeller[146] for instance incorrectly stated that Gmunden had a table with radius 600000.[147] Cantor and Eneström also made the mistake.[148] Some typos of Busard's transcription[149] are also corrected by Glowatzki and Göttsche.[150]

[137] See [Bond (1920), p. 319] and [Curtze (1900)].

[138] [Glowatzki and Göttsche (1990), p. 81]

[139] [Glowatzki and Göttsche (1990), pp. 85 and 89]

[140] [Klug (1943), p.57]

[141] [Ptolemaeus (1984), pp. 57-60]

[142] In figure 13, the number of places of sexagesimal tables is shown as $60; 60^n$, the first 60 being the value of R, and n being the number of additional sexagesimal places.

[143] [Glowatzki and Göttsche (1990), p. 92]

[144] [von Braunmühl (1900, 1903), v. 1, pp. 110-111]

[145] [Bond (1921), p. 320]

[146] [Zeller (1944), p. 16]

[147] [Glowatzki and Göttsche (1990), p. 72]

[148] [Eneström (1913-1914)].

[149] [Busard (1971a)]

[150] [Glowatzki and Göttsche (1990), p. 92]

Johannes von Gmunden's treatise was heavily used and Peuerbach borrowed much from it. Eventually, Peuerbach's own treatise made its way into Regiomontanus's works and was printed in 1541.[151]

Although Johannes von Gmunden's treatise did not contain any significant novelty, it brought the impetus for a new computation of sine tables,[152] which would find its completion in Pitiscus' *Thesaurus mathematicus* in 1613.[153]

3.2 Giovanni Bianchini (c1410-c1469)

Giovanni Bianchini was a merchant and businessman, probably born in Bologna or Florence around 1410. He later went to Ferrara, but also visited other cities. He became interested in astronomical calculations at an early age.[154] His scientific works were written between 1440 and 1460 and he is in particular the author of one of the few treatises of algebra written in the fifteenth century in Latin.[155] He corresponded with Regiomontanus during the latter's stay in Italy.[156]

Rosińska was the first to describe Bianchini's purely trigonometric tables, which consist in two decimal and two sexagesimal tables.[157] Earlier writers such as Boffito[158] and Birkenmajer[159] mentioned some of Bianchini's trigonometric tables, but did not describe them in detail.

Bianchini's table of sines for $R = 60 \cdot 10^3$ appears in his *Tabulae primi mobilis* and was reproduced and transcribed by Glowatzki and Göttsche.[160]

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[151] [Folkerts (2006), p. 87]

[152] [Busard (1971a), p. 76]

[153] [Pitiscus (1613)]

[154] See [Barotti (1792), vol. 1, p. 119-132], [Birkenmajer (1911)], [Federici Vescovini (1968)], [Goldstein and Chabás Bergón (2004)] and [Chabás Bergón and Goldstein (2009), p. 13]. For a survey of Bianchini's tables, see [van Brummelen (2018)] and [Chabás Bergón (2019), pp. 337-364]. See also [Gruyer (1897), v. 2, pp. 428-430] for some background on his astronomical tables. Gessner mentions Bianchini very briefly [Gessner and Simmler (1574), p. 346].

[155] [Rosińska (1994a), Rosińska (1997-1998)]

[156] See [von Murr (1786), vol. 1, p. 74-205], [Curtze (1902)] and [Gerl (1989)].

[157] See [Rosińska (1981a)], [Rosińska (1981b)], [Rosińska (1987)], and [Rosińska (2006)]. A more complete summary of Bianchini's trigonometric tables was recently given by Chabás [Chabás Bergón (2016)].

[158] [Boffito (1908)]

[159] [Birkenmajer (1911), p. 273]

[160] See [Chabás Bergón and Goldstein (2012), p. 20] and [Chabás Bergón (2019), p. 361].

They also reproduced his table of cotangents.[161] For both tables, Bianchini appears to have been the first to use a step of $10'$ and also the first to use a partly decimal radius $R = 60 \cdot 10^3$.[162] However, as mentioned earlier, in Cairo and in the 10th century, Ibn Yūnus had computed a table of sines at intervals of $1'$.

Bianchini's table of cotangents uses $R = 12000$[163] and this table may have been adapted from, or inspired by, a *tabula umbræ* found in the tables of Toledo, but with $R = 12 \cdot 60$.[164] The table of shadows of the Toledan tables is itself the same as that in Al-Khwārizmī and Al-Battāni.[165]

Bianchini must have computed his tables of sines *ab novo*, at least in part, perhaps interpolating from the Toledan tables. Glowatzki and Göttsche[166] observed that Bianchini computed a number of values in his table of shadows by interpolation, and not from his table of sines, thereby resulting in some inaccuracies.

Bianchini also computed decimal tables, that is tables not involving 60 and only based on powers of 10. These tables are found in the set of eight trigonometric tables named *Tabulae magistrales*.[167] Some of these tables give the values of trigonometric functions multiplied by certain astronomical factors (for instance the cosine of the obliquity of the ecliptic), but two of the tables are decimal tables ($R = 10000$) for the tangent and cosecant.[168]

Among this set of tables, the *Tabula magistralis quarta*[169] gives the tangents at $10'$ intervals and with $R = 10^4$.

This table may have been the incentive for Regiomontanus to construct his own table of tangents in his *Tabulæ directionum profectionumque*,[170] for every degree and with $R = 10^5$ (figure 15). He did not, however, use Bianchini's values, but computed his tangents using his large sexagesimal

[161] [Glowatzki and Göttsche (1990), pp. 95-114]

[162] [Glowatzki and Göttsche (1990), p. 94]

[163] [Chabás Bergón (2019), p. 361]

[164] See for instance BNF, Manuscrit Latin 16655, f°31r, here reproduced in figure 8.

[165] [Toomer (1968), table 15, p. 32]

[166] [Glowatzki and Göttsche (1990), p. 105]

[167] See [Rosińska (1984), pp. 476-477] and [Chabás Bergón (2019), p. 349].

[168] In 1981, [Rosińska (1981a)] wrote mistakenly that the tangents are given with $R = 10^3$. This error was repeated by Rosińska in 1987 [Rosińska (1987)] and 2002 [Rosińska (2002), p. 12], by Chabás and Goldstein [Chabás Bergón and Goldstein (2009), p. 20] and Brummelen in 2009 [van Brummelen (2009), p. 262], but it was corrected by Chabás in 2016 [Chabás Bergón (2016)].

[169] [Chabás Bergón (2019), p. 351]

[170] [Regiomontanus (1490)]

table of sines.

Another of Bianchini's tables, the *Tabula magistralis quinta*[171] gives the cosecants at $10'$ intervals and with $R = 10^4$.

Rosińska assumed that Bianchini's extant decimal tables were derived from a decimal table of sines for $R = 10^4$, and this would make sense. Unfortunately, such a table is no longer extant.[172] This table for $R = 10^4$ may itself have been computed from Bianchini's sine table found in his *Tabulae primi mobilis*.

Bianchini's work did not stay confined in Italy but circulated until Krakow, as described by Walsh.[173]

3.3 Georg von Peuerbach (1423-1461)

Georg Aunpekh, known as Georg von Peuerbach (1423-1461), was an Austrian mathematician and astronomer.[174] He was born in Peuerbach, Austria. In 1446 he registered at the University of Vienna and between 1448 and 1451, he travelled to Italy. There he met Nicholas of Cusa (1401-1464) who had been papal legate in Germany since 1446 and cardinal since 1448. In Ferrara, Peuerbach may also have met Giovanni Bianchini.[175] The latter wanted to obtain positions for Peuerbach in Bologna or Padua, but Peuerbach did not accept them.[176] He then returned to Vienna.

Peuerbach was first influenced by Johannes von Gmunden who had

[171] [Chabás Bergón (2019), p. 351]

[172] As mentioned above, Rosińska actually wrote that the decimal table of tangents used $R = 10^3$ and therefore also posited a sine table with that radius.

[173] [Walsh (1996), pp. 289-291]

[174] For summaries of Peuerbach's life and works, see in particular [Gassendi (1654), pp. 335-373], [von Khauz (1755), pp. 33-57], [Montucla (1758), v. 1, pp. 443-445], [Martin (1764), pp. 157-158], [Aschbach (1865), pp. 479-493], [Gallois (1890a), pp. 1-11], [Thorndike (1929), ch. 8], [Sperl (1971b)], [Vogel (1973b)], [Rose (1975)], [Hellman and Swerdlow (1978)], [Grössing (1983), pp. 79-116], [Shank (1997)], [Samhaber (2000)], [Grössing (2002)], [Kaunzner (2006)], [van Brummelen (2009), pp. 249-252], [Malpangotto (2020), pp. 19-34], and [Horst (2019)]. Several other references not cited here are given in the *Geschichtsquellen des deutschen Mittelalters* (`https://www.geschichtsquellen.de/autor/749`). One of the first biographical notices on Peuerbach was that of Tannstetter, when he published Peuerbach's table of eclipses [von Peuerbach and Regiomontanus (1514)]. Gessner, on the other hand, only briefly mentions Peuerbach [Gessner and Simmler (1574), p. 231]. For the scientific context in Vienna, see [Durand (1952)] and [Duhem (1959), pp. 349-367].

[175] See [Hellman and Swerdlow (1978), p. 473], [Grössing (1983), p. 80] and [Malpangotto (2020), p. 24].

[176] See [von Khauz (1755), p. 38] and [Hellman and Swerdlow (1978), p. 473].

died in 1442. He can thus be considered as Gmunden's spiritual student. It seems unlikely that he knew him, but he certainly studied his works.[177]

In 1454, after his return from Italy, Peuerbach completed a *Theoricae Novae Planetarum* which actually started as lectures on the theory of planetary motion. This work was published in 1473[178] by Regiomontanus (1436-1476), Peuerbach's student and successor, who had certainly attended these lectures.[179] The *Theoricae Novae Planetarum* became a standard textbook of planetary theory for the next century.[180] It contains solid sphere representations of Ptolemaic planetary models, and this work was of great importance until the solid sphere hypothesis was disproved by Tycho Brahe at the end of the 16th century.[181]

Peuerbach was acquainted with Cardinal Bessarion (1403-1472) who was then papal legate in Germany. In 1460, Bessarion spent more than one year in Vienna[182] in order to gain imperial support for the war against the Turks and during this stay he became friends with Peuerbach. Bessarion, a Greek, wanted to produce a new translation of the Almagest, because he considered Trebizond's work to be flawed. George of Trebizond (c1395-c1472) was of Greek origin and has translated many works from Antiquity in Latin. In particular in 1451 he composed a Commentary on the Almagest, which has never been printed.[183] Bessarion had himself considered translating the Almagest from the Greek, but his duties didn't let him the time to.[184]

Bessarion asked Peuerbach (who did not know Greek) to write an *Epitome* (summary) of Ptolemy's Almagest.[185] He also wanted him to accompany him to Italy for further investigations on the Almagest. Peuerbach certainly wanted to take Regiomontanus with him to Italy, but Peuerbach died in

[177] [Vogel (1973a), p. 120]

[178] See [Malpangotto (2020), pp. 116-119 & 678-679]. Many sources give the date of publication as 1472, but I follow Malpangotto here. Note that Khauß wrote that the *Theoricae* were published in 1460 [von Khauz (1755), p. 46].

[179] [Shank (1996), p. 124]

[180] [Schmeidler (1977), p. 315]

[181] [Hellman and Swerdlow (1978), p. 475]

[182] [Malpangotto (2020), p. 33]

[183] [Glowatzki and Göttsche (1976), p. 16]

[184] Meskens writes that Bessarion had started the translation, but doesn't give any substantial proof of this statement [Meskens (2010), p. 136]. Meskens's statement may rest on [Glowatzki and Göttsche (1976), p. 17].

[185] See [Malpangotto (2020), p. 20] and [Shank (2002), p. 183].

1461 before the journey began.[186] By that time, Peuerbach had written six chapters of his *Epitome*, and not based on the Greek text.[187] Regiomontanus, who learned Greek, added the seven missing chapters to Peuerbach's work after Peuerbach's death. This *Epitome* was only printed in 1496 and was very influential, in particular on Copernicus.[188]

Together with Johannes von Gmunden, Peuerbach and Regiomontanus were in fact the most important members of the first Viennese mathematical school of the 15th century.[189] Peuerbach's work on the *Epitome* led him to work on reforming Ptolemy's astronomy. Gassendi later wrote that Peuerbach resurrected an almost dying astronomy and that without him, we would have neither Copernicus nor Brahe.[190] Or, as others have put it, Peuerbach and his pupil Regiomontanus[191] woke up the study of astronomy and built the necessary tables.[192] And Hellman and Swerdlow wrote that the "*Epitome* is the true discovery of ancient mathematical astronomy in the Renaissance because it gave astronomers an understanding of Ptolemy that they had not previously been able to achieve."[193]

But as Thorndike notes, "it very likely never occurred to Peurbach that his name would go down to posterity as the reviver of the mathematics of classical antiquity or as the reformer of the mathematics of his own time."[194]

For his work in trigonometry, Peuerbach was both influenced by Johannes von Gmunden and by Giovanni Bianchini. In particular, Peuerbach's treatise on sines and chords,[195] printed in 1541, contains literal excerpts of Johannes von Gmunden's treatise.[196] And Peuerbach copied Bianchini's sine table with $R = 60000$ and a step of $10'$.[197]

[186] [von Khauz (1755), p. 42]

[187] Peuerbach actually followed closely the *Almagestum minor*, a textbook from the late thirteenth century [Hellman and Swerdlow (1978), p. 477]. Bendefy incorrectly stated that Peuerbach's *Epitome* was translated from the Greek [Bendefy (1980), p. 244].

[188] [Rosen (1975a), p. 349]

[189] [Grössing (1983), p. 146]

[190] [Gassendi (1658), p. 518]

[191] [Folkerts (1977), Kaunzner (1980), Zinner (1968)]

[192] [Gerhardt (1877), p. 87]

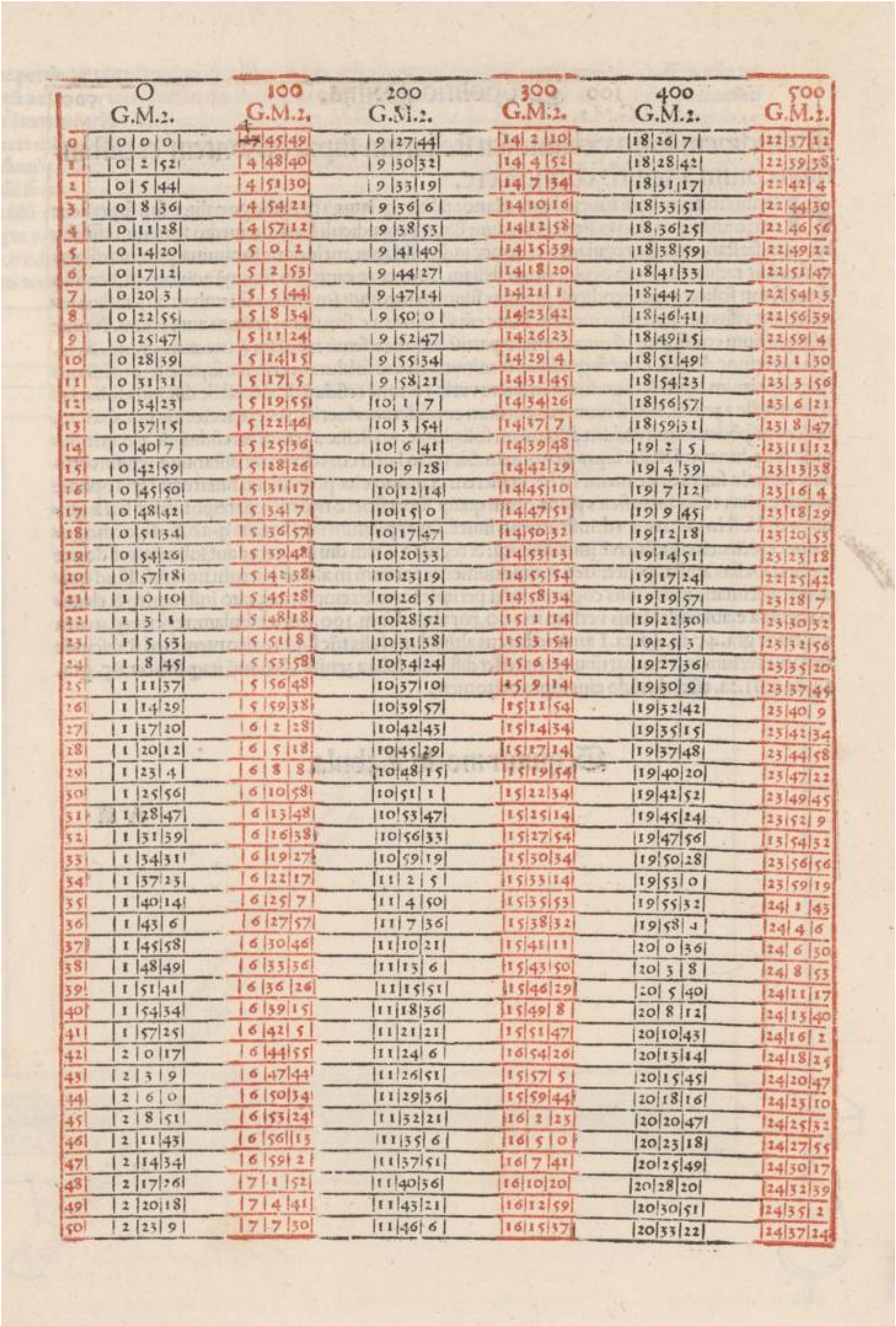
[193] [Hellman and Swerdlow (1978), p. 477]

[194] [Thorndike (1929), p. 143]

[195] [von Peuerbach and Regiomontanus (1541)]

[196] [Busard (1971a), p. 75]

[197] This table is reproduced by Glowatzki and Göttsche [Glowatzki and Göttsche (1990), pp. 116-123]. They draw the attention to incorrect statements by [Cantor (1900), p. 182]



	0 G.M.2.	100 G.M.2.	200 G.M.2.	300 G.M.2.	400 G.M.2.	500 G.M.2.
0	0 0 0	4 45 49	9 27 44	14 2 20	18 26 7	22 37 12
1	0 2 52	4 48 40	9 30 32	14 4 52	18 28 42	22 39 38
2	0 5 44	4 51 30	9 33 19	14 7 34	18 31 17	22 42 4
3	0 8 36	4 54 21	9 36 6	14 10 16	18 33 51	22 44 30
4	0 11 28	4 57 12	9 38 53	14 12 58	18 36 25	22 46 56
5	0 14 20	5 0 2	9 41 40	14 15 39	18 38 59	22 49 22
6	0 17 12	5 2 53	9 44 27	14 18 20	18 41 33	22 51 47
7	0 20 3	5 5 44	9 47 14	14 21 1	18 44 7	22 54 13
8	0 22 55	5 8 34	9 50 0	14 23 42	18 46 41	22 56 39
9	0 25 47	5 11 24	9 52 47	14 26 23	18 49 15	22 59 4
10	0 28 39	5 14 15	9 55 34	14 29 4	18 51 49	23 1 30
11	0 31 31	5 17 5	9 58 21	14 31 45	18 54 23	23 3 56
12	0 34 23	5 19 55	10 1 7	14 34 26	18 56 57	23 6 21
13	0 37 15	5 22 46	10 3 54	14 37 7	18 59 31	23 8 47
14	0 40 7	5 25 36	10 6 41	14 39 48	19 2 5	23 11 12
15	0 42 59	5 28 26	10 9 28	14 42 29	19 4 59	23 13 38
16	0 45 50	5 31 17	10 12 14	14 45 10	19 7 12	23 16 4
17	0 48 42	5 34 7	10 15 0	14 47 51	19 9 45	23 18 29
18	0 51 34	5 36 57	10 17 47	14 50 32	19 12 18	23 20 55
19	0 54 26	5 39 48	10 20 33	14 53 13	19 14 51	23 23 18
20	0 57 18	5 42 38	10 23 19	14 55 54	19 17 24	22 25 42
21	1 0 10	5 45 28	10 26 5	14 58 34	19 19 57	23 28 7
22	1 3 1	5 48 18	10 28 52	15 1 14	19 22 30	23 30 32
23	1 5 53	5 51 8	10 31 58	15 3 54	19 25 3	23 32 56
24	1 8 45	5 53 58	10 34 24	15 6 34	19 27 36	23 35 20
25	1 11 37	5 56 48	10 37 10	15 9 14	19 30 9	23 37 45
26	1 14 29	5 59 38	10 39 57	15 11 54	19 32 42	23 40 9
27	1 17 20	6 2 28	10 42 43	15 14 34	19 35 15	23 42 34
28	1 20 12	6 5 18	10 45 29	15 17 14	19 37 48	23 44 58
29	1 23 4	6 8 8	10 48 15	15 19 54	19 40 20	23 47 22
30	1 25 56	6 10 58	10 51 1	15 22 34	19 42 52	23 49 45
31	1 28 47	6 13 48	10 53 47	15 25 14	19 45 24	23 52 9
32	1 31 59	6 16 38	10 56 33	15 27 54	19 47 56	13 54 32
33	1 34 31	6 19 27	10 59 19	15 30 34	19 50 28	23 56 56
34	1 37 23	6 22 17	11 2 5	15 33 14	19 53 0	23 59 19
35	1 40 14	6 25 7	11 4 50	15 35 53	19 55 32	24 1 43
36	1 43 6	6 27 57	11 7 36	15 38 32	19 58 4	24 4 6
37	1 45 58	6 30 46	11 10 21	15 41 11	20 0 36	24 6 30
38	1 48 49	6 33 56	11 13 6	15 43 50	20 3 8	24 8 53
39	1 51 41	6 36 26	11 15 51	15 46 29	20 5 40	24 11 17
40	1 54 34	6 39 15	11 18 36	15 49 8	20 8 12	24 13 40
41	1 57 25	6 42 5	11 21 21	15 51 47	20 10 43	24 16 2
42	2 0 17	6 44 55	11 24 6	16 54 26	20 13 14	24 18 25
43	2 3 9	6 47 44	11 26 51	15 57 5	20 15 45	24 20 47
44	2 6 0	6 50 34	11 29 36	15 59 44	20 18 16	24 23 10
45	2 8 51	6 53 24	11 32 21	16 2 23	20 20 47	24 25 32
46	2 11 43	6 56 13	11 35 6	16 5 0	20 23 18	24 27 55
47	2 14 34	6 59 2	11 37 51	16 7 41	20 25 49	24 30 17
48	2 17 26	7 1 52	11 40 36	16 10 20	20 28 20	24 32 39
49	2 20 18	7 4 41	11 43 21	16 12 59	20 30 51	24 35 2
50	2 23 9	7 7 30	11 46 6	16 15 37	20 33 22	24 37 24

Figure 10: The first page of Peuerbach's table of arctangents (1516) [von Peuerbach (1516)] (e-rara).

A table with $R = 60000$ is again found in the 1490 edition of Regiomontanus's *Tabulæ directionum profectionumque*,[198] but it happens to be a table derived from Regiomontanus's large sexagesimal table, and not Peuerbach's table. Moreover, the 1490 table gives the sines at intervals of $1'$.

Around 1450, Peuerbach took $R = 600000$ and a step of $10'$ and went beyond what Johannes von Gmunden and Bianchini had done. But this table with $R = 600000$ is no longer extant.[199] We know of its existence because Peuerbach mentioned it in the *Propositio prima* of his *Quadratum geometricum* (or *Canones gnomonis*[200]) written in 1455 and published in 1516.[201] And another work of Peuerbach confirms that the step of the table was $10'$.

Gassendi[202] also mentions a table of sines by Peuerbach with $R = 6000000$ and a step of $10'$, and that this table had been extended to a step of $1'$ by Regiomontanus, but this is probably a typo, no such table with $R = 6000000$ being known of Peuerbach.[203]

Peuerbach was the one who provided the impetus for the replacement of Ptolemy's chords with the sines from Arabic mathematics, and Regiomontanus computed tables of sines for every minute of arc for radiuses of 6000000 and 10000000 units.

Among Peuerbach's other works is also his *Quadratum geometricum*[204] already mentioned, written in 1455 and published in 1516. This work describes the geometrical square, an instrument for measuring heights. A similar instrument was also described by Oronce Fine in his *De re et praxi geometrica* published in 1556.

Peuerbach's treatise contains what is basically a table of arctangents (figure 10). Peuerbach wrote that he used his now lost table with $R = 600000$ for the computation of the table. The possible values of the tangents range

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and [Zinner (1968), p. 36], [Zinner (1990), p. 23] about the radius of the table.

[198] [Regiomontanus (1490)]

[199] See [Glowatzki and Göttsche (1990), pp. iii and 115]. Earlier, Hellman and Swerdlow had mentioned a manuscript table with $R = 600000$, but this is in fact a table with $R = 60000$ [Hellman and Swerdlow (1978), p. 478]. Brummelen also seems to mention this no longer extant table [van Brummelen (2009), p. 249].

[200] [Hellman and Swerdlow (1978), p. 477]

[201] [von Peuerbach (1516)]

[202] See [Gassendi (1658), p. 520] and [von Khauz (1755), p. 54].

[203] This incorrect statement is also found in [Martin (1764), p. 158] (and in [Lublink and Meijer (1763), pp. 183-198] which must have the same source), and it was more recently repeated by [Bendefy (1980), p. 245].

[204] [von Peuerbach (1516)]. See [Roegel (2021a)] for a modern reconstruction.

from 0 to 1200 and, for an entry x, Peuerbach's table actually gives the value $\arctan(x/1200)$ in degrees. For instance, for $x = 1200$, Peuerbach's table gives 45°. For $x = 500$, Peuerbach's table gives $\arctan(5/12) = 22^\circ 37' 12''$. The value 1200 used in this table may have been influenced by the radius 12000 in Bianchini's table of cotangents.[205]

This table of arctangents was reprinted by Gemma Frisius[206] in 1545 and a similar table was given by Magini in 1592.[207]

3.4 Johannes Regiomontanus (1436-1476)

Regiomontanus, or rather Hans Müller, was probably born in 1436 in Königsberg, near Bamberg in Germany (figure 11).[208] He had latinized his name as Johannes de Monte Regio and it was only half a century after his death in 1476 that he became known as Regiomontanus.[209]

He established trigonometry as an independant field, separate from astronomy, in Western Europe, although the Persian al-Tūsī had already written a purely trigonometric treatise in the 13th century. Regiomontanus was the most famous Western mathematician of his time.[210]

[205] [Glowatzki and Göttsche (1990), pp. 124-125]

[206] [Gemma Frisius (1545)]

[207] [Magini (1592)]

[208] [Schmeidler (1977), p. 315]. Some authors, for instance recently [Meskens (2010)], have incorrectly confused this Königsberg with the modern Kaliningrad.

[209] According to some sources, the name Regiomontanus was coined by Philip Melanchthon. It does indeed appear in his *De capta Constantinopoli, Anno 1453* (1556). However, the earliest appearance I found of "Regiomontanus" (or rather Regiomontano) is that in Marcus Beneventanus's *Apologeticum opusculum* (1521). For summaries of Regiomontanus's life and works, see mainly [Zinner (1968)], which can be supplemented by [Gassendi (1654), pp. 335-373], [Doppelmayr (1730)], [Montucla (1758), v. 1, pp. 445-453], [Martin (1764), pp. 146-157], [Aschbach (1865), pp. 537-557], [Ziegler (1874)], [Günther (1885)], [Gallois (1890a), pp. 1-11], [Thorndike (1929), ch. 8], [Vogel (1973b)], [Rosen (1975a)], [Rose (1975)], [Hamann (1978)], [Hamann (1980)], [Grössing (1983), pp. 117-126], [Glowatzki and Göttsche (1990), p. 1-8], [Mett (1996)], [Grössing (2002)], [Malpangotto (2008)], [van Brummelen (2009), pp. 251-263], and [van Brummelen (2021), pp. 2-5]. One of the first biographical notices on Regiomontanus was that of Tannstetter, when he published Peuerbach's table of eclipses and Regiomontanus's table of the first mobile [von Peuerbach and Regiomontanus (1514)]. There are also many smaller articles of interest, some more specialized, some more introductory, such as [Shank (2017)], [Horst (2019)], [Götz (2003)], etc., but which are not all cited here. Gessner also mentions Regiomontanus [Gessner and Simmler (1574), p. 397]. And many sources on Peuerbach, not cited in the previous list, contain some information on Regiomontanus. For the scientific context in Vienna, see [Durand (1952)] and [Duhem (1959), pp. 349-367].

[210] [Glowatzki and Göttsche (1990), p. i]

Figure 11: Regiomontanus's probable birthplace in Königsberg, Bavaria. (photographs by the author)

After having studied in Leipzig, he came to Vienna around 1450 and became a friend and pupil of Georg von Peuerbach. In 1457, this is where he took his Master's degree and was appointed to the faculty, hence a colleague of Peuerbach.[211]

Peuerbach was supposed to go to Italy with Cardinal Bessarion who had asked him to write an *Epitome* (summary) of Ptolemy's Almagest. But after Peuerbach's death in 1461, it was Regiomontanus who accompanied him to Italy.[212] Regiomontanus completed the *Epitome*, probably in 1462.[213] He also studied Greek and it was during the time of the completion of the *Epitome* that Regiomontanus studied the copy he had made of Trebizond's translation of the Almagest.[214]

In Italy, Regiomontanus also became associated with Giovanni Bianchini. Part of their correspondence still survives.[215] Durand writes that

[211]See [Rosen (1975a), p. 348] and [Schmeidler (1977), p. 316].

[212]See [Rose (1975), pp. 90-117], [Schmeidler (1977), p. 316], [Grössing (1980)], [Mett (1989)] and [Moos (2020)]. On Regiomontanus's knowledge of Latin and Greek, see [Ben-Tov (2009), pp. 195-196] and [Jensen (1996), p. 65] who theorizes that Regiomontanus may not have mastered Latin as well as the Italian scholars.

[213]See [Zinner (1990), p. 52] and [Shank (1996), p. 125]. It was however only printed in 1496.

[214][Zinner (1990), p. 59]

[215]See [von Murr (1786), vol. 1, p. 74-205], [Curtze (1902)] and [Gerl (1989)]. See

"Regiomontanus envisaged an exchange of problems and answers to be based on friendly emulation, but the older Italian was speedily scared away by the precocity of the enthusiastic German."[216]

It was during this time that Regiomontanus constructed his *Tabula primi mobilis* which was only published in 1514.[217] This table gives the values of $\arcsin(\sin x \sin y)$ for $0 \leq x, y \leq 90^{\circ}$ and is useful for solving problems in spherical trigonometry. The table was computed using Regiomontanus's sine table with $R = 6 \cdot 10^6$.[218] Glowatzki and Göttsche gave a survey of similar tables or variants published until the 19th century.[219]

Regiomontanus returned from Italy around 1465,[220] he went to Pozsony (Pressburg, Bratislava) in 1467, at the invitation of Matthias Corvinus (1443-1490), King of Hungary,[221] of whom he became an astronomical adviser. Some time later, he was called to Buda.[222]

It was during this time in Hungary that Regiomontanus worked with the Polish astronomer Marcin Bylica (c1433-1493)[223] whom Regiomontanus met in Rome. Together they computed some tables, in particular Regiomontanus's *Tabulæ directionum profectionumque*.[224]

In 1471 Regiomontanus moved to Nuremberg. There he set up a printing press for the purpose of publishing the most important classical scientific works,[225] as well as some of his own works.[226] The first work to be published was Peuerbach's *Theoricae Novae Planetarum*. In 1475 Regiomontanus

also [Swerdlow (1990)].

[216] [Durand (1943), p. 13]

[217] See [Mett (1996), pp. 96-97], [Swerdlow (1999), p. 1], [van Brummelen (2009), p. 263], and [Chabás Bergón (2019), pp. 378-379].

[218] [Glowatzki and Göttsche (1990), p. 199]

[219] [Glowatzki and Göttsche (1990), pp. 197-207]

[220] [Hayton (2010), p. 33]

[221] [Schmeidler (1977), p. 317]

[222] [Orbán (2015), p. 118]

[223] [Domonkos (1968), Vargha and Both (1987), Hayton (2007), Hayton (2010), Orbán (2015)]

[224] There are several editions of the *Tabulæ directionum profectionumque*, in particular in 1490, 1504, 1550, 1552, 1559, 1584 and 1606. A French edition was published by Henrion in 1626 [Henrion (1626)]. For the collaboration between Bylica and Regiomontanus, see [Hayton (2007), p. 188] and [Chabás Bergón (2019), pp. 380-387]. On the relations between the astronomical schools of Vienna and Cracow, see [Markowski (1978), p. 268]. Bylica sent works from Peuerbach and Regiomontanus to the University of Cracow. See also [Walsh (1996)] and [Bendefy (1980)] on Regiomontanus's stay in Hungary.

[225] [Folkerts (1996), pp. 91-92]

[226] [Schmeidler (1977), p. 318]

returned to Rome at the invitation of Pope Sixtus IV in order to work on a reform of the Julian calendar, and this is where he died in 1476, probably from the plague. During all these years, Regiomontanus worked on a critique of Trebizond's translation of the Almagest, his *Theonis Alexandrini Defensio in sex voluminibus contra Georgium Trapezuntium*, a work which was probably only completed in the 1470s and still remains only in manuscript form.[227]

It seems that Regiomontanus started around 1460 to compute sines with a large radius in order to produce a table with $R = 6 \cdot 10^6$ for his *De triangulis* (1462?) (figure 22). This table was certainly inspired by Peuerbach's table with $R = 600000$, although Hallam claimed[228] that Regiomontanus was ignorant of that table. Glowatzki and Göttsche[229] give Regiomontanus's description of the computations, the *Compositio tabularum sinuum rectorum*, as well as a German translation. Regiomontanus's description is contained in the 1541 edition of Peuerbach's treatise on sines.[230] In section 4 below I analyze how Regiomontanus may have computed his table.

Around 1468, Regiomontanus composed another table with a radius of 10000000. Both the sexagesimal and the decimal tables were given at intervals of $1'$. These tables were first printed in 1541 (figures 23 and 24).[231] They were however not the first tables with such intervals, and they came after those of Ibn Yūnus and Ulugh Beg (see § 2.3).

Regiomontanus's table of sines with $R = 10^7$ was accessible in Cracow at the end of the 15th century[232] and was undoubtly one of the sources of Copernicus's trigonometric tables.

The move from a sexagesimal division to a decimal division, initiated by Bianchini, but greatly developped by Regiomontanus, made it much simpler to use the tables. With the new decimal radius, there is therefore no longer any need to mix the bases 10 and 60, as was the case in the older tables.

Regiomontanus's *Tabulæ directionum profectionumque* from 1467 and published in 1490 also contained a table of tangents (figure 15) which was probably inspired by Bianchini's table of tangents.[233] Cardano consid-

[227] See [Shank (2007)] for some excerpts.
[228] [Hallam (1837), p. 259]
[229] [Glowatzki and Göttsche (1990), pp. 11-24]
[230] [von Peuerbach and Regiomontanus (1541)]
[231] [von Peuerbach and Regiomontanus (1541), Roegel (2021b)]
[232] See [Rosińska (1984), pp. 503-504] and [Rosińska (1987), pp. 421-422].
[233] [van Brummelen (2018)]

ered that Regiomontanus's entire *Tabulæ directionum profectionumque* was largely drawn from Bianchini.[234] Folkerts,[235] however, considered that Regiomontanus's table of tangents was influenced by Al-Battāni. In fact, Regiomontanus's table of tangents was certainly computed using his large sexagesimal table as I shall show later. A modern reconstruction of this table of tangents is given separately.[236]

The *Tabulæ directionum profectionumque* also contains a table of sines with $R = 60000$ and at $1'$ intervals (figure 16). But contrary to what Bond,[237] Delambre,[238] or more recently Folkerts,[239] Zinner,[240] North,[241] Brummelen,[242] Husson,[243] and Chabás and Goldstein wrote,[244] this table is neither by Regiomontanus nor borrowed from Bianchini. It was appended to Regiomontanus's book, probably by Johannes Engel (or Johannes Angelus) (1453-1512),[245] and was derived from Regiomontanus's table with $R = 6000000$. Moreover, as observed by Glowatzki and Göttsche, the appended table was never used by Regiomontanus.[246] A modern reconstruction of Engel's table is given separately.[247]

In fact, most of Regiomontanus's writings were only published after his death. His main work on trigonometry, *De triangulis omnimodis*, was completed about 1464 but only printed in 1533, without any table.[248] It is the first systematic such treatise published in Europe and it was probably used by Copernicus. However, as observed by Stamm,[249] it is unlikely that Copernicus had access to Regiomontanus's treatise in manuscript form and he probably only saw the 1533 edition in the 1530s.

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[234] [Thorndike (1929), p. 148]
[235] [Folkerts (1977), p. 235]
[236] [Roegel (2021c)]
[237] [Bond (1921), p. 321]
[238] [Delambre (1819), p. 365]
[239] See [Folkerts (1977), p. 234], [Folkerts (1995), p. 224] and [Folkerts et al. (2016), p. 136].
[240] See [Zinner (1968), p. 345] and [Zinner (1990), p. 236].
[241] [North (2008), p. 275]
[242] [van Brummelen (2009), p. 262]
[243] [Husson (2014), p. 116]
[244] [Chabás Bergón and Goldstein (2012), p. 20]
[245] [Glowatzki and Göttsche (1990), p. 48] On Johannes Engel, see [Dobrzycki and Kremer (1996)].
[246] [Glowatzki and Göttsche (1990), p. iii]
[247] [Roegel (2021d)]
[248] [Regiomontanus (1533)], edited om [Regiomontanus (1967)].
[249] [Stamm (1933)]

Delambre was critical of Regiomontanus and wrote that except for his observations and trigonometrical work, Regiomontanus had hardly the time to do more than show his good intentions.[250] Delambre stresses that Regiomontanus was less advanced as a calculator than Ibn Yūnus and Abū al-Wafāʾ. However, this opinion may need to be revised in the light of my analysis of the construction of his tables.

Braunmühl[251] considered that Regiomontanus's work on triangles was influential, even if it didn't contain anything original.

And as observed by Glowatzki and Göttsche,[252] the tables computed by Regiomontanus are very modern and could still be used now, only the decimal point would have to be shifted.

Thorndike thought that Peuerbach and Regiomontanus's importance had perhaps been overestimated, among other things because Regiomontanus was more than a mathematician. He was a mathematical publisher, and he came at just the right time.[253]

[250] [Delambre (1819), p. 365]
[251] [von Braunmühl (1900, 1903), v. 1, pp. 124-133]
[252] [Glowatzki and Göttsche (1990), p. i]
[253] [Thorndike (1929), p. 150]

4 Regiomontanus's seminal tables

We can now pause and summarize the situation of Regiomontanus's tables at the end of the 15th century. There are four different trigonometric tables usually associated with Regiomontanus: a large table of sines with radius 6000000, another one with radius 10^7, a table of tangents with $R = 10^5$ and a smaller table of sines with $R = 60000$, but of which Regiomontanus is actually not the author. Most of the tables published during the 16th century are ultimately based on the table for $R = 10^7$.

I also include in this section some tables which are not directly from Regiomontanus, for instance the tables of secants, but which are nevertheless based on Regiomontanus's other tables.

The following tables by Regiomontanus have been reconstructed in separate documents:

- the table of tangents, as published in 1490 (figure 15)[254];
- the table of sines with $R = 6 \cdot 10^6$, as published in 1541 (figure 23)[255];
- the table of sines with $R = 10^7$, as published in 1541 (figure 24).[256]

4.1 Fundamental tables

When Regiomontanus set out to construct his new sine tables, he was certainly influenced by Peuerbach's work, and in particular by Peuerbach's sine table with $R = 600000$ and at intervals of $10'$.[257] This table is no longer extant, but it is likely that Regiomontanus used it as an inspiration for his further work.[258]

It seems that it was around 1460 that Regiomontanus first computed sines of values at $45'$ intervals with $R = 6 \cdot 10^8$ (figure 12), perhaps even before Peuerbach's death.[259] This was to be the fundamental table from

[254] [Regiomontanus (1490)]

[255] [von Peuerbach and Regiomontanus (1541)]

[256] [von Peuerbach and Regiomontanus (1541)]

[257] There have been some incorrect statements about the tables constructed by Regiomontanus and a table with $R = 600000$ is sometimes attributed to him, for instance by Günther in 1885 [Günther (1885), p. 573].

[258] In a long chapter, Glowatzki and Göttsche try to find the forerunners of Regiomontanus's large sexagesimal table and which may have influenced him [Glowatzki and Göttsche (1990), pp. 72-125].

[259] [Glowatzki and Göttsche (1990), pp. 10, 16, 22]

which a more complete table for $R = 6 \cdot 10^6$ could be computed.[260] This auxiliary table is only partially extant.

Once he had his pivots, Regiomontanus computed the sines at intervals of $15'$, dividing the sines at intervals of $45'$ obtained earlier in three parts in such a way that the sines vary smoothly.[261] Then Regiomontanus trisected each interval, again by ensuring that the differences vary smoothly. This gave him the sines at intervals of $5'$.[262] The same procedure was again applied to obtain the sines at intervals of $1'$.[263]

For the table with $R = 10^7$, Regiomontanus possibly also first computed a number of pivot values with $R = 10^9$, but these pivots have not been kept.

4.2 Sine table with $R = 6000000$

Regiomontanus's first large complete sine table was for a radius of 6000000 and was probably computed around 1462 in Rome.[264] It gives the sines for every minute. Figure 22 shows an excerpt of a manuscript of that table. This table is based on the computations made with $R = 6 \cdot 10^8$ as described in the previous sections.

After Regiomontanus's death, Regiomontanus's table was long kept in manuscript form. It was only published in 1541 with Peuerbach's *Tractatus super propositiones Ptolemæ etc.*,[265] and together with the table for $R = 10^7$ (figures 23 and 24). These two tables were then again published in 1561 in Regiomontanus's *De triangulis*.[266] Glowatzki and Göttsche gave a facsimile of the 1541 sexagesimal table and listed its errors.[267]

Regiomontanus's sine table appears rather accurate, although it is probably slightly less accurate than the table for $R = 10^7$. Sampling only the values for whole degrees, there are 25 last-place errors and one typo (for

[260] However, as observed by Glowatzki and Göttsche, an error in the computation of $\sin 45'$ caused other (small) errors, in particular in the interpolation leading to $\sin 1°$ [Glowatzki and Göttsche (1990), pp. 26-27].

[261] [Glowatzki and Göttsche (1990), p. 23]

[262] [Glowatzki and Göttsche (1990), p. 23]

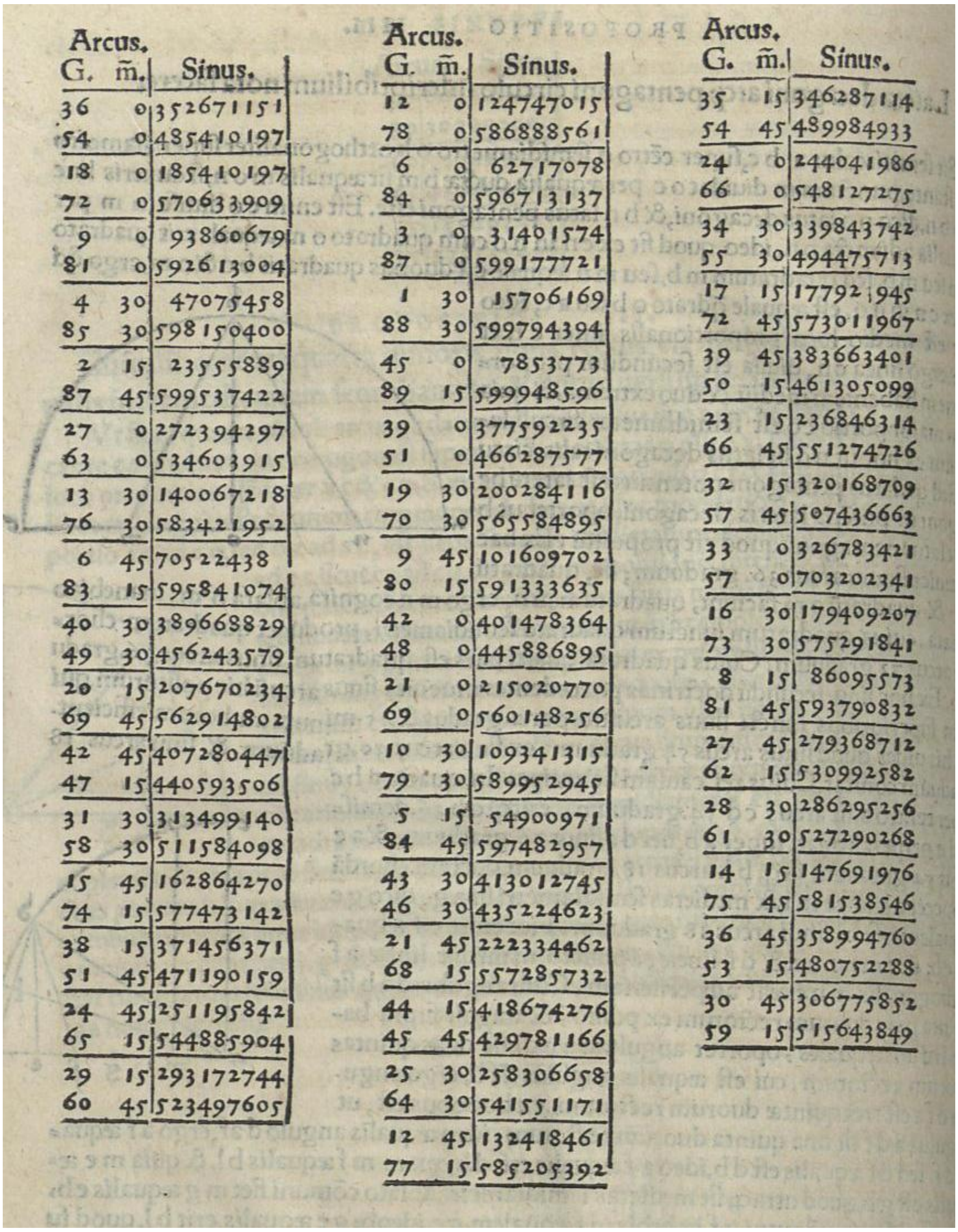
[263] [van Brummelen (2009), p. 263] gives Regiomontanus's implied value of $\mathrm{Sin}\, 1°$, but does not describe the actual interpolation process. See also [van Brummelen (2021), pp. 18-21], who hints at a procedure below $15'$ but without detailing it. Kästner gives also only a cursory description [Kästner (1796), pp. 540-560].

[264] See [Glowatzki and Göttsche (1990), p. 71] and [Mett (1996), p. 65].

[265] [von Peuerbach and Regiomontanus (1541)]

[266] [Regiomontanus (1561)]

[267] [Glowatzki and Göttsche (1990), pp. 28-47]



Arcus. G.	m̃.	Sinus.
36	0	352671151
54	0	485410197
18	0	185410197
72	0	570633909
9	0	93860679
81	0	592613004
4	30	47075458
85	30	598150400
2	15	23555889
87	45	599537422
27	0	272394297
63	0	534603915
13	30	140067218
76	30	583421952
6	45	70522438
83	15	595841074
40	30	389668829
49	30	456243579
20	15	207670234
69	45	562914802
42	45	407280447
47	15	440593506
31	30	313499140
58	30	511584098
15	45	162864270
74	15	577473142
38	15	371456371
51	45	471190159
24	45	251195842
65	15	544885904
29	15	293172744
60	45	523497605

Arcus. G.	m̃.	Sinus.
12	0	124747015
78	0	586888561
6	0	62717078
84	0	596713137
3	0	3140574
87	0	599177721
1	30	15706169
88	30	599794394
45	0	7853773
89	15	599948596
39	0	377592235
51	0	466287577
19	30	200284116
70	30	565584895
9	45	101609702
80	15	591333635
42	0	401478364
48	0	445886895
21	0	215020770
69	0	560148256
10	30	109341315
79	30	589952945
5	15	54900971
84	45	597482957
43	30	413012745
46	30	435224623
21	45	222334462
68	15	557285732
44	15	418674276
45	45	429781166
25	30	258306658
64	30	541551171
12	45	132418461
77	15	585205392

Arcus. G.	m̃.	Sinus.
35	15	346287114
54	45	489984933
24	0	244041986
66	0	548127275
34	30	339843742
55	30	494475713
17	15	17792 945
72	45	573011967
39	45	383663401
50	15	461305099
23	15	236846314
66	45	551274726
32	15	320168709
57	45	507436663
33	0	326783421
57	0	703202341
16	30	179409207
73	30	575291841
8	15	86095573
81	45	593790832
27	45	279368712
62	15	530992582
28	30	286295256
61	30	527290268
14	15	147691976
75	45	581538546
36	45	358994760
53	15	480752288
30	45	306775852
59	15	515643849

Figure 12: The list of pivots for Regiomontanus's large sexagesimal table [von Peuerbach and Regiomontanus (1541)] (source: Dresden).

40°, 3856796 which should be 3856726). Of the last-place errors, all are of one unit, and one (80°) is of two units. I have given separately a modern reconstruction of this table with the exact values which can be used for comparison with Regiomontanus's table.[268] And in § 5, I am giving a more detailed analysis of Regiomontanus's errors and computation procedure.

In Regiomontanus's table, the column of differences does not give the actual difference Δ, but the difference per second, in other words $\Delta/60$. These differences are given to one decimal place which is separated by a space.[269] For instance, the first difference is $\Delta = 1745$ and it is given as $29\ 1$, because $1745/60 = 29.08\ldots$. But this value can also be read 291, in which case it is the sixth of the actual difference.

These differences follow a layout similar to those in Bianchini's table with $R = 60 \cdot 10^3$, so that it is possible that Regiomontanus borrowed this layout.[270]

4.3 Sine table with $R = 10000000$

Regiomontanus's second large sine table was for a radius of 10^7 and was completed in 1468.[271] It came shortly after the smaller decimal table of tangents which was computed in 1467.

This large decimal table is probably not the first decimal table of sines, although Folkerts claimed so.[272] It has been assumed that Bianchini had a decimal table of sines, probably with a radius $R = 10^4$ (see § 3.2), but this table is no longer extant.

Regiomontanus's table is also not based on his large sexagesimal table.[273] Regiomontanus may have computed a number of pivot values, perhaps with $R = 10^9$, or he may have reused the sexagesimal pivots by multiplying them by $10/6$. In any case, these pivots have not been kept. Then, Regiomontanus must have proceeded by interpolation as in the sexagesimal table.

Like in the previous table, sines are given for every minute. This table was also published in 1541 and 1561 together with the sexagesimal table (figures 23 and 24). Glowatzki and Göttsche gave a facsimile of the entire

[268] [Roegel (2021b)]
[269] [Glowatzki and Göttsche (1990), p. 27]
[270] [Glowatzki and Göttsche (1990), p. 94]
[271] See [Folkerts (1977), p. 234] and [Mett (1996), p. 96].
[272] [Folkerts et al. (2016), p. 136]
[273] [Glowatzki and Göttsche (1990), p. 126]

1541 edition of the table, and listed its typos.[274]

The differences are expressed like in Regiomontanus's sexagesimal table and the first difference is for instance $\Delta = 2909$ and it is given as $48\ 5$, corresponding to $\Delta/60 = 48.48\ldots$

I have given separately a modern reconstruction of this table.[275]

It is interesting to note that Regiomontanus's table is slightly more accurate than the previous one with $R = 6 \cdot 10^6$.[276] Sampling only the sines for whole degrees, we can for instance see that there are only seven incorrect values, one of which (for 25°) being an obvious typo (4226583 which should be 4226183), and the other six values being only off by one unit of the last place. This suggests of course that the decimal table was not merely obtained from the sexagesimal table, but must have been obtained either from the pivots of the sexagesimal table, or from newly computed pivots, as described above.

4.4 Sine table with $R = 60000$

The *Tabulæ directionum profectionumque* published in 1490 contains a 30 pages long sine table with $R = 60000$ giving the sines for every minute (figure 16),[277] but this table was certainly computed by Johannes Engel for that edition (see § 6.1), and not by Regiomontanus.

4.5 Table of tangents

Regiomontanus's *Tabulæ directionum profectionumque*,[278] from 1467 and printed in 1490, also contained a short table of tangents, which he called *tabula fecunda* (figure 15).[279] The name "tangent" was as a matter of fact only introduced in 1583 by Fincke.[280]

Regiomontanus's table is only one page long and gives the tangents for every degree, and for a radius of 100000. The tangents were computed from the 1462 table of sines (with $R = 6000000$), by first dropping two digits and rounding the values, and then by mere division.[281] This procedure

274[Glowatzki and Göttsche (1990), pp. 127-147]
275[Roegel (2021b)]
276[Glowatzki and Göttsche (1990), p. 147]
277[Regiomontanus (1490)]
278[Regiomontanus (1490)]
279[Chabás Bergón (2019), p. 383]
280[Fincke (1583)]
281[Glowatzki and Göttsche (1990), p. 183]

actually gives exactly Regiomontanus's values, except for the angles $43°$, $73°$, $85°$ and $89°$. In these four cases, Regiomontanus very likely got the computations wrong, or these are typos. Incidentally, the same procedure fails miserably when using the decimal table of sines, and it is almost impossible to obtain the values of the table of tangents with this starting point.

Regiomontanus's table was not the first table of tangents, as tangents had already been used in eastern Islam, as mentioned above (see § 2.3).

Regiomontanus's table was reproduced in subsequent editions of his *Tabulæ directionum profectionumque*, by Gemma Frisius in 1545 (but as cotangents)[282] (also with Peuerbach's 1516 *quadratum* table[283]), by Gaurico in 1557,[284] by Maurolico in 1558 (at least partially, and he called it *umbra versa*),[285] by Schreckenfuchs in 1569,[286] and in subsequent editions of these works.[287]

Gaurico's table (1557)[288] only goes up to $50°$, and is attributed to Campanus. But neither Glowatzki and Göttsche,[289] nor von Braunmühl,[290] nor Zinner[291] were able to understand this attribution.

Curiously, Gaurico also gives a sine table with the heading *tabula fecunda* and also only up to $50°$.

And finally, mention should be made of Bendefy who mistakenly wrote in 1980 that Regiomontanus had constructed a table of tangents for a radius $R = 10^7$ and for every minute, and that it was only Reinhold who published it in 1554.[292]

[282]See [Gemma Frisius (1545)]. It was also reprinted in 1557.

[283][von Peuerbach (1516)]

[284][Gaurico (1557)]

[285][Maurolico (1558)]

[286]See [Glowatzki and Göttsche (1990), p. 180] and [Schreckenfuchs (1569), p. 153].

[287][Glowatzki and Göttsche (1990), pp. 180-181]

[288][Gaurico (1557)]

[289][Glowatzki and Göttsche (1990), pp. 180-181]

[290][von Braunmühl (1900, 1903), v. 1, p. 101]

[291][Zinner (1968), p. 148]

[292][Bendefy (1980), p. 248] Bendefy's statement seems based on Barna Szénássy's history of Hungarian mathematics (*A magyarországi matematika története*, 1970), but I was not able to check this source. Bendefy also cites Zinner's article on Regiomontanus in Hungary, published in Hungarian, and which does not seem to contain such a statement (Ernő Zinner, Regiomontanus Magyarországon, *Matematikai és Természettudományi Értesítő*, volume 55, 1936?, pp. 280-288).

4.6 Secant tables

Regiomontanus did not compute tables of secants, but the first tables of secants are based on his tables of sines. This is the case of Copernicus's table of secants, which might have been computed around 1530. Bianchini had computed a table of cosecants (§ 3.2). But I am not aware of earlier such tables, although, as mentioned before (§ 2.3), Abū al-Wafā᾿ introduced the notion of secant in Baghdad in the 10th century.

The tables of secants published by Rheticus in 1551 [Rheticus (1551)] and by Maurolico in 1558 [Maurolico (1558)] also ultimately derive from Regiomontanus.[293]

It was Viète[294] who in 1579 was the first to compute a table of secants with an interval of $1'$ albeit with a variable radius between $R = 10^5$ and $R = 10^9$.

And the first table of secants with an interval of $1'$ and $R = 10^7$ was published by Fincke in 1583.[295] Fincke was actually the one who named it secant. His secants were certainly computed from his tangents, which themselves go back to Regiomontanus, via Reinhold.[296]

[293] [Glowatzki and Göttsche (1990), p. 193] Incidentally, there have also been surprising statements, such as the one of Davis [Davis (1933), p. 21] who wrote that the first table of secants was that of Maurolico, and that Lansberge was wrong in ascribing this fact to Rheticus, when in fact Lansberge was right, and still is if one ignores manuscript tables.

[294] [Viète (1579)]

[295] [Fincke (1583)]

[296] See [Reinhold (1554)] and [Glowatzki and Göttsche (1990), p. 193].

5 An analysis of Regiomontanus's great tables

One of my purposes has been to find out how Regiomontanus computed his two large tables of sines. We know rather well how he computed the sines at intervals of $45'$, but we know little beyond that, and no one seems to have investigated this matter so far, not even Glowatzki and Göttsche.[297]

The first step in such an investigation is to clear the tables of the noise they contain, namely of the typos, both in the printed versions and in the manuscripts. Although I have not consulted manuscripts of Regiomontanus's table, I believe that it is possible to come very close to what Regiomontanus has actually computed.

5.1 Typos, accuracy and statistics

5.1.1 General principles

I have gone over each of the 2×5401 values of the sines (from 0° to 90° by steps of $1'$) in the 1541 printing, trying to detect obvious typos. This work has been done independently of that of Glowatzki and Göttsche who had already reported a number of typos.[298] I have consequently made two tables where I corrected a number of typos, such as wrong digits in the left figures, swapped figures, or swapped lines. In the resulting tables, I carefully examined all the cases where Regiomontanus's tables were in error by more than 2 units of the last place. Every such case which appeared isolated was removed. The justification for correcting these seemingly small errors was that they would have been very easy to detect by computing differences between consecutive terms, and that almost always these anomalies were isolated, and could not have been Regiomontanus's real values, at least not his intended values. These decisions may be objectionable, but I have only corrected errors which are easy to detect by anybody working on tables. I did not correct any more fundamental issue. And these corrections are necessary in order to get a better understanding of the underlying computations.

5.1.2 Corrections to the tables

Apart from the very conspicuous typos already reported by Glowatzki and Göttsche (mostly not repeated here), I made the following smaller

[297] [Glowatzki and Göttsche (1990)]

[298] [Glowatzki and Göttsche (1990), p. 46-47 and 145-147]

corrections to the sexagesimal table:

	values			values	
angle	table	corrected	angle	table	corrected
$1°29'$	155315	155317	$45°42'$	4294154	4294156
$1°44'$	181486	181487	$49°34'$	4566965	4566968
$2°38'$	275668	275665	$51°35'$	4701078	4701077
$4°58'$	519454	519456	$57°21'$	5051893	5051891
$5°\ 1'$	524674	524673	$61°26'$	5269565	5269567
$6°38'$	693009	693088	$66°33'$	5504447	5504445
$7°52'$	821219	821211	$70°\ 2'$	5639347	5639349
$12°\ 4'$	1254295	1254297	$70°37'$	5659910	5659916
$17°33'$	1809221	1809228	$73°50'$	5762737	5762735
$18°28'$	1900518	1900516	$74°51'$	5791465	5791470
$18°40'$	1920372	1920370	$76°51'$	5842661	5842667
$19°29'$	2001193	2001195	$77°17'$	5852821	5852823
$19°50'$	2035718	2035710	$79°58'$	5908230	5908238
$21°43'$	2220109	2220102	$81°10'$	5928833	5928835
$41°59'$	4013488	4013486	$81°29'$	5933835	5933837
$42°24'$	4045818	4045814	$82°46'$	5952258	5952250
$42°53'$	4083045	4083046	$86°17'$	5987385	5987382
$44°\ 1'$	4169203	4169206	$86°46'$	5990440	5990450
$45°\ 9'$	4253736	4253734	$87°56'$	5996094	5996097

Note that my corrections do not always replace the 1541 printed values by the exact ones, but by the values I believe should have been printed. For instance, for $6°38'$, Glowatzki and Göttsche replaced 693009 by 693090, which does make sense as a typo. However, the value 693090 does not make much sense in its context (*i.e.*, the surrounding values) and I believe that there was an error before that, and that Regiomontanus should have obtained 693088, which is the value I gave in my table. In this case, I believe that Regiomontanus accidently obtained the correct value 693090, and that the printer got it wrong by setting 693009.

The only values with a deviation of 3 units of the last place are those from $6°50'$ to $6°53'$. I believe that the pivot $6°50'$ was erroneously computed and should probably have been 713889 (with an error of 1). This has probably caused the sines of $6°51'$ to $6°53'$ to be also wrong by 3 units. I have however not fixed these deviations and these errors remain in the cleaned table, as they are not mere typos. But the truth is that these errors would not escape a close scrutiny by differencing.

The above table also contains corrections for a number of deviations of 2 units, when these were clearly isolated ($1°29'$, $1°44'$, $6°38'$, $17°33'$,

$41°59'$, $42°53'$, $45°42'$, $49°34'$, $51°35'$, $57°21'$, $70°2'$, $73°50'$, $77°17'$, $81°10'$, $81°29'$, $82°46'$).

I also corrected some suspicious transitions, where the error switched from 1 to -1 or from -1 to 1. These errors would have been very easy to detect by differencing and concern $5°1'$, $7°52'$, $18°28'$, $18°40'$, $66°33'$, and $79°58'$.

In the case of the decimal table, I also made a number of corrections, including

	values		angle	values	
angle	table	corrected	angle	table	corrected
$4°19'$	752688	752687	$39°50'$	6405569	6405566
$13°14'$	2289163	2289171	$40°21'$	6474556	6474550
$13°28'$	2328799	2328796	$54°15'$	8115746	8115740
$17°14'$	2962630	2962639	$54°29'$	8139469	8139466
$19°39'$	3362739	3362735	$58°43'$	8546096	8546099
$20°24'$	3485724	3485720	$59°59'$	8658793	8658799
$35°21'$	5785691	5785697	$60°17'$	8684873	8684875
$37°42'$	6115272	6115271	$61°15'$	8767267	8767269
$38°20'$	6202350	6202356	$61°50'$	8815783	8815781
$38°53'$	6277368	6277367	$87°32'$	9981731	9990734
$39°24'$	6347309	6347306			

Among these corrections, the small ones for $37°42'$ and $38°53'$ have been made because their deviations appeared to be isolated. And like in the sexagesimal table, I have also corrected some suspicious transitions, where the error switched from 1 to -1 or from -1 to 1. These errors concern the values $4°19'$, $13°14'$, $60°17'$, $61°15'$, and $61°50'$. Some of these corrections may appear larger than these small transitions, but that may be because there may have been both printer typos and earlier errors, and that I first corrected the large errors, for instance for $13°14'$ whose sine value in Regiomontanus's manuscript may have been 2289173, but which still can't have been the right one.

Some of these typos/errors were reported by Glowatzki and Göttsche, but not all of them, and, as I explained above, Glowatzki and Göttsche reported other errors which I have corrected, but not included in the above tables.[299] Moreover, my corrections do not always coincide with theirs, as I have tried to replace the incorrectly printed values by those that Regiomontanus has presumably computed, and not by the exact sines. I believe

[299]The errors which have not been reported can easily be found either by a careful comparison of my cleaned tables with Regiomontanus's tables, or by checking the tables given by Glowatzki and Göttsche.

however that all the typos reported by Glowatzki and Göttsche have been taken care of in my versions.

Eventually, we end up with two tables which must be very close to Regiomontanus's calculations, and which have been cleared of probably almost all typos, both in the printed versions and in the manuscripts. What I mean by this is that Regiomontanus would have found all these errors by mere differencing and that the resulting cleaned tables provide a better start for the analysis of Regiomontanus's actual computations.

These tables are provided separately[300] as text files for others to analyze, should they wish to.

5.1.3 The sexagesimal pivots

The cleaned tables now make it possible to have a closer look at computational errors and in particular at the accuracy of the pivots. It first appears that the sexagesimal table contains *about* 2223 errors of one unit or more, and none of more than 3 units. This does agree with the count given by Glowatzki and Göttsche who came up with 2232, but with slightly different corrections. I am of course writing "about," because in some cases I made adjustments which may or may not be correct. The same remark applies to the decimal table.

The pivots at $45'$ intervals (for $R = 6 \cdot 10^8$) for the sexagesimal table appear very accurate. There are only 17 values which are not correct, and among them all are off by one unit of the last place, except those for $45'$, 27°, $57^\circ 75'$, and $59^\circ 25'$. In the case of $57^\circ 75'$, there is an obvious typo, and the original value may have been correct. The value for $45'$ may also be a typo. In any case, none of these small errors have any serious impact on the values in the sexagesimal table.

Consequently, the $45'$ pivots in the sexagesimal table (for $R = 6 \cdot 10^6$) are mostly correct. In fact, they should even all be correct. But there are three exceptions. The $6^\circ 45'$ pivot is correctly given in the table for $R = 6.10^8$, but there is a different value in the final table. The neighboring values would make things worse if I gave the correct value to $\mathrm{Sin}\, 6^\circ 45'$, so that I suspect that Regiomontanus made an error when copying his own (correct) value of $\mathrm{Sin}\, 6^\circ 45'$. The same observations apply to $\mathrm{Sin}\, 8^\circ 15'$ and $\mathrm{Sin}\, 44^\circ 15'$. These three sines are off by one in the sexagesimal table.

As far as the other pivots are concerned, two $15'$ pivots are off by 2 and 78 are off by 1. 345 $5'$ pivots are off by 1, and 11 by 2 or 3.

[300] See the files `roegel2021regio6.txt` and `roegel2021regio10.txt`.

5.1.4 The decimal pivots

The decimal table contains about 1841 errors or one unit or more. Again, this is very close to Glowatzki and Göttsche's count which is 1833, but with slightly different corrections. There are also three $45'$ pivots which are incorrect, but not the same ones as for the sexagesimal table. 22 $15'$ pivots are off by 1, and none by 2. 282 $5'$ pivots are off by 1, and one is off by 2.

It does therefore appear that the decimal table is somewhat more accurate than the sexagesimal table, but not by an order of magnitude.

5.1.5 Some general statistics

We can also observe that in the sexagesimal table the longest sequence without errors is of length 52 and starting at $56^\circ 4'$: once the typos are corrected, all the sines from $56^\circ 4'$ to $56^\circ 55'$ are correct. The longest sequence with a constant error of one unit of the last place (in the same direction) is of length 30 and starts at $3^\circ 52'$. The longest sequence with a constant error of two units is of length 6.

Similar results are obtained with the decimal table and the longest sequence without errors is of length 50, starting at $27^\circ 45'$.

The average errors are -0.10 for the sexagesimal table and 0.12 for the decimal table, but it is difficult to analyze errors in more depth without taking into account the structure of the computations, namely the two trisections and the possibly final linear interpolation.

We can now try to answer a number of questions on the computation of the pivots:

- For instance, assuming two $45'$ pivots are correct, how often are the $15'$ pivots correct?

  The answer to this question is surprising, because there is a clear difference between the sexagesimal and decimal tables. In the first case, 70 $15'$ pivots (for 114 ranges out of 120) are incorrect, but in the second case only 17 are incorrect (also for 114 ranges). The $15'$ pivots of the decimal table appear clearly more accurate than in the sexagesimal table.

  An example of incorrect $15'$ pivot in the sexagesimal table is that of $45^\circ 15'$, where the sines of 45° and $45^\circ 45'$ are correct.

  One should actually distinguish the cases where the two twin/double $15'$ pivots (in a $45'$ interval) are wrong, and the cases where only one of them is wrong. Surprisingly, there are 24 cases of wrong twin pivots in the sexagesimal table, and none in the decimal table.

Morevover, in the decimal table, 13 out of 17 wrong (non twin) $15'$ pivots concern the second $15'$ pivots. Things are more even in the sexagesimal table, where 13 out of 22 wrong (non twin) $15'$ pivots concern the second $15'$ pivots.

An example of incorrect $15'$ pivot in the decimal table is that of 32°, where $31^\circ 30'$ and $32^\circ 15'$ are correct.

- And assuming two $15'$ pivots are correct, how often are the $5'$ pivots correct?

  We find that for the sexagesimal table, 119 $5'$ pivots are incorrect (for 231 ranges out of 360) when the $15'$ pivots are correct, and that there are 30 incorrect twin pivots.

  For the decimal table, 217 $5'$ pivots are incorrect (for 318 ranges out of 360), including 52 twin pivots. Under this perspective, the sexagesimal table appears more accurate than the decimal table.

  An example of an incorrect $5'$ pivot in the sexagesimal table is that of $75^\circ 5'$.

- Finally, how often are the $5'$ interpolations correct?

  Again, we restrict ourselves to the cases where the two pivots are correct, as such a restriction is still representative.[301] What is the most common outcome? Is it $0, 0, 0, 0, 0, 0$? In other words, if two adjacent $5'$ pivots are correct, are the four intermediate values also usually correct?

  The number of ranges to consider (where the two $5'$ pivots are correct) is similar in both tables: 576 ranges for the sexagesimal table and 604 ranges for the decimal table. Is the outcome the same? First, given that the table has been checked by differences, the only values which can appear between the two end 0s are 0 and ± 1. There are therefore $3^4 = 81$ different possible sequences, but the most common sequence is $(0, 0, 0, 0, 0)$ with 277 cases in the sexagesimal table and 258 cases in the decimal table. Again, under this perspective, the sexagesimal table is in fact slightly more accurate than the decimal one.

[301] We could also consider the computation of $5'$ pivots from incorrect $15'$ pivots, for instance by shifting these pivots, but I don't think we would reach significantly different results.

5.2 A tentative analysis of Regiomontanus's construction

The procedure used by Regiomontanus to construct his large tables is a bit vague, but I believe that it can be clarified. As far as I know, no attempt has been made so far to explain this process. As mentioned above, Regiomontanus basically describes a subtabulation process, where from sine values at $45'$ intervals he obtains values for every $15'$, then for every $5'$, and finally for every minute. Regiomontanus explicitely speaks of making the differences increase regularly, and it should be clear that the differences between values played a key role in this computation. It is also clear that what Regiomontanus has done was to interpolate values, more than merely to compute accurately thousands of sines.

Reading Regiomontanus's description, one can not avoid thinking of the works of Bürgi[302] and Briggs[303] and wonder if, perhaps, Regiomontanus had not anticipated them. I believe in fact that his computations were indeed forerunners of what Bürgi and Briggs did, a century or a century and a half later. Both Bürgi and Briggs analyzed how finite differences could be used not merely to find new values by adding differences, but also to subtabulate, and find intermediate values from larger differences. For instance, Briggs computed the logarithms of various integers as interpolations of logarithms given at larger intervals. Among the techniques he describes is the quinquisection, where he is able to divide an interval in five parts and obtain the intermediate logarithms.

5.2.1 The general setting

Here, I want briefly to test this hypothesis, which may be expanded later in the future. To be as general as possible, I will consider a sequence of sines $v_0, v_1, v_2, \ldots$, for angles $a_0, a_1, a_2, \ldots$, where $a_{i+1} - a_i$ is a constant interval, for instance $45'$. $v_i = \mathrm{Sin}(a_i)$, with some radius R, which I will take here as $6 \cdot 10^8$, but which could be different.

These values are used to define the finite differences $\Delta_0^1 = v_1 - v_0$, $\Delta_1^1 = v_2 - v_1, \ldots, \Delta_0^2 = \Delta_1^1 - \Delta_0^1, \Delta_1^2 = \Delta_2^1 - \Delta_1^1, \ldots, \Delta_0^3 = \Delta_1^2 - \Delta_0^2$, etc.

What Regiomontanus sought to do was to find the sines $v_{1/3}$, $v_{2/3}$, etc., of the intermediate angles $a_{1/3}$, $a_{2/3}$, $a_{4/3}$, etc. In other words, he was working on a trisection. For instance, if $a_0 = 3^\circ$, $a_1 = 3^\circ 45'$, etc., then $a_{1/3} = 3^\circ 15'$ and $v_{1/3} = \mathrm{Sin}\, 3^\circ 15'$.

The subtabulated differences are $\delta_0^1 = v_{1/3} - v_0$, $\delta_0^2 = \delta_{1/3}^1 - \delta_0^1$, and so on.

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[302] [Roegel (2016a)]

[303] [Roegel (2010a)]

I believe that during the first stage of his procedure, Regiomontanus tried to compute the smaller differences, that is the differences for intervals of $15'$, from the differences for intervals of $45'$. In other words, I believe that he tried to compute δ_0^1 and δ_0^2, and these two values would then be sufficient to compute $v_{1/3}$ and $v_{2/3}$.

I invite those who are unconvinced by this suggestion to consider for instance the trisection of the $45'$ interval between $\operatorname{Sin} 20^\circ$ and $\operatorname{Sin} 20^\circ 45'$. The radius could be taken as $R = 10^5$, and the sines to start with would then be 34202, 35429, 36650, 37865, etc. Merely manipulating these numbers without great thought leads to two approximations of the subtabulated first differences, namely 410 and 407. We would have three 410 differences and three 407 differences. This is of course not satisfactorily, it is not an even decrease, and looking at the second differences, we find 0,0,−3,0,0. This can be improved by starting with the first subtabulated difference 410 and spreading the -3 over five values, hence taking -0.6 instead of -3 for the second difference. We have here a very simple means to obtain the second differences. Adding up these differences, we end up with 36653 instead of 36650. It is not perfect, but it is not that bad. Since the second difference was correctly spred, we may want to improve the first difference 410, but it will actually be difficult to reach a better result with this radius. Such experiments are useful to convince oneself that it is practically unfeasible to get the differences to vary evenly merely by fiddling with the numbers, and at the same time they pave the way for the discovery of a relationship between certain values. And these are the key issues here.

The first key is to notice that the second differences are practically proportional to the sines. This had actually been discovered long before Regiomontanus, for instance in India by Āryabhaṭa in the 6th century[304] And Wagner and Hunziker recently suggested[305] that there was perhaps a transmission from India to Bürgi, although I am rather doubtful about such an assertion. In the above simplified example, one would readily find that the second differences are all equal to -6, at least around 20°, and if this operation is done for other values, one can't be far from discovering that the second differences are proportional to the sines.

I will therefore assume that Regiomontanus first noticed that $\Delta_0^2 \approx \frac{v_1}{C_{45}}$ where C_{45} is some constant, and that this is true on the entire sine table. I am also guessing that Regiomontanus knew that the constant depends on

[304] See [Hayashi (1997)], [Bressoud (2002)], [Raju (2007), p. 132], [Lefort (2007)] and [Gupta (2008)] for some references (among many others) describing Āryabhaṭa's computation of sines and how the second differences are used.

[305] [Wagner and Hunziker (2019)]

the size of the interval, hence my subscript. For intervals of $45'$, we have $C_{45} \approx -5836$. The exact expression behind this value matters little here,[306] but what is important is that by playing with differences, any serious table computer would eventually find out that there is some constant ratio involved, and perhaps think of using it backwards. By computing a few exact values of sines at $15'$ intervals, Regiomontanus may have found that another constant is involved:[307] $\delta_0^2 \approx \frac{v_{1/3}}{C_{15}}$ and that $C_{15} \approx -52525$. Regiomontanus may or may not have noticed that $C_{15}/C_{45} \approx 9$. But he must certainly have noticed that the third differences Δ^3 vary only very slowly and that their variations can be neglected on small ranges.

At this stage, Regiomontanus could have had a means to compute δ_0^2 using an approximation of $v_{1/3}$. Of course, $v_{1/3}$ is what we are looking for, but we can easily get an approximation of $v_{1/3}$ such as

$$v_{1/3} \approx v_0 + \frac{v_1 - v_0}{3}$$

and this is in fact sufficient to get a good approximation of δ_0^2.

What now remains is to obtain an approximation of δ_0^1. An obvious approximation is

$$\frac{\Delta_0^1}{3}$$

but Regiomontanus needed a better one.

The second key here is to see or guess that the second differences are involved in the approximations of the first differences. In any case, one may want to test whether

$$\delta_0^1 \approx \frac{\Delta_0^1}{3} + \alpha\delta_0^2$$

for some value of α. Although the constancy of δ^2 makes this actually obvious,[308] it is also possible to observe experimentally that $\alpha = -1$, and thus that

$$\delta_0^1 \approx \frac{\Delta_0^1}{3} - \delta_0^2$$

Again, in the simplified example given above, where the first differences are 1227, 1221, and 1215, and where the second differences are all about -6,

[306]See [Roegel (2010b), § 2.4] for the computation of the exact values of $\Delta \sin x$. The value 5836 is actually about $1/\sin^2 \Delta x$, that is $1/\sin^2 45'$.

[307]This constant is given by $1/\sin^2 15' \approx 52525$.

[308]The three first differences are then $x - \delta^2$, x, and $x + \delta^2$, and the average first difference is necessarily the median first difference.

it should not be difficult to notice that the first first difference is equal to the average first difference minus the second difference, $\frac{37865-34202}{3} + 6 = 1227$, or that the second (middle) first difference is also the mean first difference.

If these two observations are made, namely 1) the link between the second differences and the sines, and 2) the dependency of the subtabulated first differences on the subtabulated second differences, then it is possible to derive the values $v_{1/3}$ and $v_{2/3}$.

5.2.2 An example

Let me show how to put this in practice on a small example. Let's for instance interpolate the sines between 39° and $39^\circ 45'$. I will assume that all of Regiomontanus's values at $45'$ intervals were exact, which, as mentioned above, is true except in a few instances.[309] So, Regiomontanus must have had

$$\begin{aligned} \mathrm{Sin}\, 39^\circ &= 377592235 \\ \mathrm{Sin}\, 39^\circ 45' &= 383663401 \end{aligned}$$

Using the approximation $v_{1/3} \approx 379615957$ (the exact value is 379623197) we obtain

$$\begin{aligned} \delta_0^2 &\approx -\frac{379615957}{52525} = -7227 \\ \Delta_0^1 &= 383663401 - 377592235 = 6071166 \\ \delta_0^1 &\approx \frac{6071166}{3} + 7227 = 2030949 \\ \delta_1^1 &\approx \delta_0^1 + \delta_0^2 = 2023722 \\ v_{1/3} &= \mathrm{Sin}\, 39^\circ + 2030949 = 379623184 \\ v_{2/3} &= v_{1/3} + 2023722 = 381646906 \end{aligned}$$

and in fact Regiomontanus's table with $R = 6 \cdot 10^6$ does have the values 3796232 and 3816469 which would have been obtained from the above computation.

This procedure does unfortunately not work on all $45'$ intervals, and Regiomontanus's pivots sometimes differ from those obtained with this procedure, although the difference never exceeds one unit of the last place. This does not prove that Regiomontanus did not use such a procedure, but

[309] The two values I am using are in fact given in [von Peuerbach and Regiomontanus (1541)].

it may be that some computations were lacking uniformity, and also that some errors were introduced in the computations. I also believe that the two guard digits, viz. those added when computing with $R = 6 \cdot 10^8$, were used throughout the interpolation, and not merely for the pivotal values.

The same procedure used to obtain the sines at $15'$ intervals can be used to obtain the sines at $5'$ intervals. The only difference is that δ_0^2 involves a new constant, which may have been guessed or computed by Regiomontanus, namely

$$\delta_0^2 \approx -\frac{v_{1/3}}{52525 \times 9}$$

If for instance we want to compute $\operatorname{Sin} 39^\circ 5'$, we find

$$\begin{aligned}
\delta_0^2 &\approx -\frac{378269218}{52525 \times 9} = -800 \\
\delta_0^1 &\approx \frac{2030949}{3} + 800 = 677783 \\
\delta_1^1 &\approx 676983 \\
v_{1/3} &\approx 378270018 \\
v_{2/3} &\approx 378947001
\end{aligned}$$

and these two values $v_{1/3}$ and $v_{2/3}$, when rounded to $R = 6 \cdot 10^6$, are exactly the values given by Regiomontanus for the sines of $39^\circ 5'$ and $39^\circ 10'$. But again, I must stress that although this procedure works on this example, it does (slightly) fail to give Regiomontanus's values on others.

Anyway, if Regiomontanus proceeded along these lines, he now has obtained the sines for all multiples of $5'$, using relatively simple techniques. In fact, the computations involved here (except those for the pivots) are more a matter of being clever than of being hard working.

What now remains is to divide the $5'$ intervals in five parts. This is what Briggs called a quinquisection.

The same procedure could be applied here as for the trisection, but we would have

$$\delta_0^2 \approx -\frac{v_{1/5}}{52525 \times 9 \times 25}$$

and[310]

$$\delta_0^1 \approx \frac{\Delta_0^1}{5} - 2\delta_0^2$$

[310] This is in fact also pretty obvious, because like in the case of the trisection, we have a sequence of values of which the median is necessarily equal to the average, and the first first difference is obtained by subtracting twice the second difference from the median value.

When applying this procedure (which is left as an exercise) to the interval from 39° to $39^\circ 5'$, one obtains

$$\mathrm{Sin}\, 39^\circ 1' = 377727856$$
$$\mathrm{Sin}\, 39^\circ 2' = 377863445$$
$$\mathrm{Sin}\, 39^\circ 3' = 377999034$$
$$\mathrm{Sin}\, 39^\circ 4' = 378134623$$

and Regiomontanus's table has 3777278, 3778634, 3779990 and 3781345, that is two values differ by one unit of the last place.

This suggests that Regiomontanus may perhaps not have used such an interpolation. If one performs a mere linear interpolation, with $\delta^1 = 135557$, we end up with the values 377727792, 377863349, 377998906 and 378134463, also with two differing values.

But if instead we interpolate with only one guard digit, that is between 37759224 and 37827002 and with $\delta^1 = 13556$, we end up with the values 37772780, 37786336, 37799892 and 37813448, where only one value differs from that of Regiomontanus.

And if the guard digits are entirely discarded, we have an interpolation between 3775922 and 3782700, $\delta^1 = 1356$, and we end up with the values 3777278, 3778634, 3779990 and 3781346, and again only one value differs from that of Regiomontanus.

Finally, if we interpolate with only one guard digit, but with $\delta^1 = 13557$, then we end up with exactly Regiomontanus's values. This does not prove that Regiomontanus did such an interpolation in every case, but it does at least make it plausible that he proceeded that way in some cases.

5.3 Conclusion

Looking at Regiomontanus's tables, it is pretty clear that he had the means to compute the $45'$ pivots correctly. The $15'$ and $5'$ pivots are relatively accurate, but less than the $45'$ pivots. In the previous section, I have given a procedure which may be close to the one used by Regiomontanus to find his pivots.

For the $15'$ pivots, we have seen earlier that Regiomontanus's sexagesimal table has 70 pivot errors. Now, if we use my algorithm using finite differences, we end up with 42 errors on the all the $15'$ pivots. However, if we compare my pivot values with those of Regiomontanus, there appears to be about 85 differences. Regiomontanus's values do not perfectly agree with those of my algorithm for the first trisection, although the differences do not exceed one unit of the last place. This agreement can not be significantly improved even with a different constant C_{15}. It is still possible that Regiomontanus made use of an algorithm close to the one I sketched, but perhaps he did not always use two guard digits, in addition of having made a few computation errors here and there.

I also believe that the last step was a linear interpolation, but that glitches came into play and that the computations were not done totally uniformly and rigorously.

To sum up, and in the absence of other convincing theories, I believe that it is plausible that Regiomontanus applied two trisections, computed the first subtabulated first and second differences in each range, derived the missing values, and interpolated linearly in the $5'$ intervals, perhaps using only one guard digit, and eventually rounding all values to $R = 6 \cdot 10^6$. The same procedure could have been applied with the decimal table.

I believe that Regiomontanus's tables contain the germs of several innovations, and that it was the quality of workmanship underlying these tables which is the true reason why they endured so long. They did contain errors and typos, but they provided a solid foundation for others to build upon, and only Bürgi, Briggs and a few others were able to develop similar skills to renew the computation of tables.

6 After Regiomontanus

Most of the trigonometric tables printed in the 16th century actually use values or computations inherited from Regiomontanus's tables[311] (see figures 13 and 14). Rheticus (1514-1574) was the only one to compute really new values which were eventually published in 1596 by Otho[312] and in 1613 by Pitiscus.[313] Bürgi also computed sines anew, but his table was not published and was not used by others.

Among all these tables, Glowatzki and Göttsche distinguished those which retain the radius $R = 10^7$ and those for which $R = 10^5$.[314] But we should also consider separately the few sexagesimal tables based on Regiomontanus's tables, namely those of Engel, Fine, Schreckenfuchs and Bressieu.

The tables with radius 10^7 include those of Rheticus (1542 and 1551),[315] Reinhold (1554),[316] Eisenmenger (1562),[317] Viète (1579),[318] Fincke (1583),[319] Clavius (1586),[320] Lansberge (1591),[321] Magini (1592),[322] Blundeville (1594),[323] and Ceulen (1596).[324] Glowatzki and Göttsche also considered the 17th century tables of Sems/Dou (1600, 1612, 1616 and 1620), Stevin (1608 and 1628), Roomen (1609), Crüger (1612), Napier (1614, 1616 and 1620), Blebel (1616 and 1629), Ursinus (1618), Alsted (1620, 1630 and 1649), Muller (1621) and Tonski (1640 and 1645),[325] which all go back to Regiomontanus, but which are outside the limited scope of this survey. The fact that all these tables use Regiomontanus's values is asserted by several checks, including on the typographical and last digit errors stemming from Regiomontanus, but also on mere layout considerations. Many table makers did actually not

[311] [Glowatzki and Göttsche (1990), p. i]
[312] [Rheticus and Otho (1596)]
[313] [Pitiscus (1613)]
[314] [Glowatzki and Göttsche (1990), p. 148]
[315] [Copernicus (1542)] and [Rheticus (1551)].
[316] [Reinhold (1554)]
[317] [Eisenmenger (1562)]
[318] [Viète (1579)]
[319] [Fincke (1583)]
[320] [Clavius (1586)]
[321] [van Lansberge (1591)]
[322] [Magini (1592)]
[323] [Blundeville (1594)]
[324] [Ceulen (1596)]
[325] [Glowatzki and Göttsche (1990), pp. 161-168]

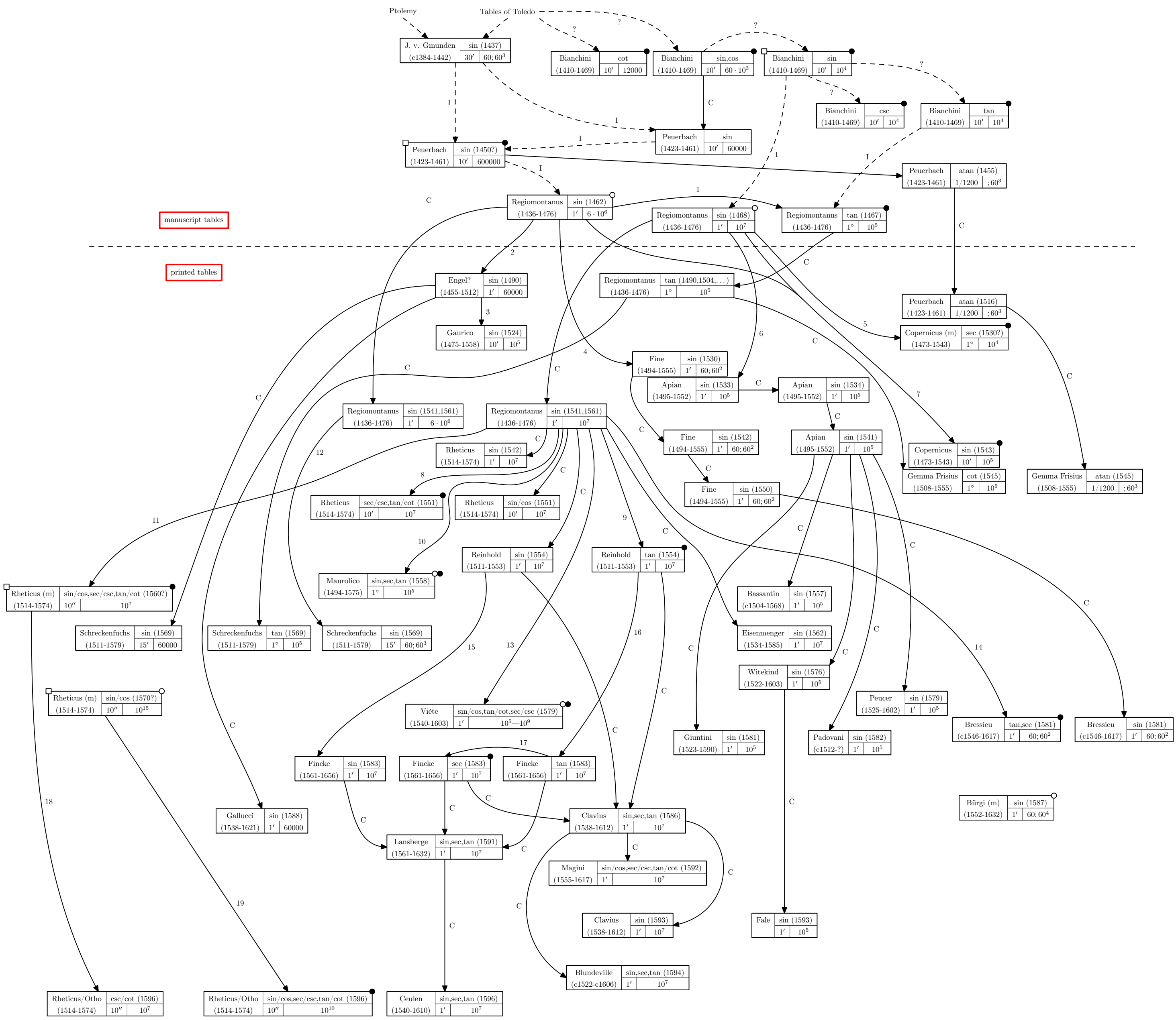

Figure 13: The interrelationships between the main 15th and 16th century fundamental trigonometric tables. Corner squares (□) indicate no longer extant tables, unfilled corner circles (○) indicate new computations, and filled corner circles (●) indicate computations based on earlier tables. Tables marked "(m)" in the lower part are manuscript tables. See figure 14 for details on the links.

C	means entirely copied from, or printed at a later date.
I	inspired or influenced by.
?	probable link.
1	Regiomontanus (tan, 1467) obtained by computation from the 1462 table.
2	Engel (1490) obtained from the 1462 table by truncation (and not rounding) (no computation).
3	Gaurico (1524): table obtained from Engel by multiplication by 10/6.
4	Fine (1530): computed from Regiomontanus's table with R = 6000000.
5	Copernicus (c1530): computed from Regiomontanus's table with R = 10^7.
6	Apian (1533): obtained from Regiomontanus (10^7) by mere truncation, without rounding.
7	Copernicus (1543): probably obtained from a combination of Regiomontanus's tables.
8	Rheticus (1551) (tan, sec): computed from Regiomontanus (or Rheticus 1542).
9	Reinhold (1554): tangents computed from the sines.
10	Maurolico (1558): computed from Regiomontanus.
11	Rheticus (c1560): computed from Regiomontanus.
12	Schreckenfuchs (1569): one table computed from the 1541/1561 table.
13	Viète (1579): computed from Regiomontanus.
14	Bressieu (1581) (tan, sec): from Regiomontanus and interpolation.
15	Fincke (1583): sines based on Reinhold's sines, but with slight adaptations.
16	Fincke (1583): tangents based on Reinhold, but with corrections.
17	Fincke (1583): secants computed from Fincke's tangents.
18	Rheticus (1596): 10^7 table was probably excerpted from an earlier table from c1560.
19	Rheticus (1596): 10^{10} table obtained from a 10^{15} table.

Figure 14: The interrelationships between the main 15th and 16th century fundamental trigonometric tables (cont'd, see figure 13). The number of places of sexagesimal tables is shown as $60; 60^n$, the first 60 being the value of R, and n being the number of additional sexagesimal places. Note that in Rheticus's 1551 table, the sines were copied from Regiomontanus (or Rheticus 1542); in Reinhold's table (1554), the sines were copied from Regiomontanus (or Rheticus 1551); in Schreckenfuchs's table (1569), one table was copied from Engel (in an edition of the *Tabulæ directionum profectionumque*) and another was copied from the 1490 table of tangents (or another edition of the *Tabulæ directionum profectionumque*); in Bressieu (1581), the sines were copied from Fine (1530 or 1550).

bother checking the values using the differences, and Clavius in 1586 was apparently the first to get rid of the typographical errors of earlier tables. Note however that Glowatzki and Göttsche do not always give the direct predecessor of a table, and do seldom consider the layouts of the tables as an indication for their source.

Among the tables with radius 10^5 are the tables of Bassantin (1557),[326] Witekind (1576),[327] Peucer (1579)[328], Giuntini (1581)[329], Padovani (1582)[330] and Fale (1593)[331] which were taken directly or indirectly from Apian (1533),[332] which itself goes back to Regiomontanus's table of 1468, merely by dropping two digits and no rounding.[333]

Engel's table with $R = 60000$ (figure 16) is derived from Regiomontanus's large sexagesimal table and was used by Schreckenfuchs in 1569 (see § 6.14).

Immediately following the table of sines for radius 60000 published in 1524,[334] there is an additional table of sines with radius 100000 and for every $10'$ (figure 17). As mentioned by Delambre,[335] this table was added by Gaurico (see § 6.2).

Fine's tables from 1530 and 1550 are not mentioned by Glowatzki and Göttsche.[336] Fine's table published in 1530 and reprinted in 1550 gives the sines for a radius $R = 60$, at intervals of $1'$ and to two sexagesimal places.

Fine's tables are the only fully sexagesimal tables based on Regiomontanus's tables, apart from those of Schreckenfuchs published in 1569 and of Bressieu published in 1581.

Mention should also be made of Bürgi's sexagesimal sine table from c1587, which seems to be a totally independent and very accurate recomputation of sines, paralleling to some extent Rheticus's efforts that led to the *Opus palatinum* (1596) and the *Thesaurus mathematicus* (1613).

To sum up, the main new computations based on Regiomontanus's values are the following, which are detailed in the subsequent sections:

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[326] See [Bassantin (1557)] and [Glowatzki and Göttsche (1990), p. 176].

[327] See [Witekind (1576)] and [Glowatzki and Göttsche (1990), p. 176]

[328] See [Peucer (1579)] and [Glowatzki and Göttsche (1990), p. 177].

[329] [Giuntini (1581)]

[330] [Padovani (1582)]

[331] See [Fale (1593)] and [Glowatzki and Göttsche (1990), p. 177].

[332] [Apian (1533)]

[333] [Glowatzki and Göttsche (1990), p. 169]

[334] [Regiomontanus (1524)]

[335] [Delambre (1819), p. 292]

[336] [Glowatzki and Göttsche (1990)]

- in 1551, Rheticus published his computations of tangents and secants at intervals of $10'$;[337]

- in 1554, Reinhold published his computations of tangents at intervals of $1'$ (and $10''$ for the last degree);[338]

- in 1579, Viète published his computations of tangents and secants at intervals of $1'$;[339]

- in 1583, Fincke published his computations of secants at intervals of $1'$.[340]

In the following sections, I go into more detail for each of these tables copied from Regiomontanus's tables, or based on them. This list tries to be as complete as possible, but it is possible that some lesser known work containing a sine table or a more complete canon still escaped my attention.

6.1 Engel (1490)

Johannes Engel (or Johannes Angelus) (1453-1512) was an astronomer and astrologer from Aichach, near Augsburg. He published many almanachs and astronomical tables.[341]

The 1490 edition of Regiomontanus's *Tabulæ directionum profectionumque*[342] contains corrections by Johannes Engel and in particular a 30 pages long sine table with $R = 60000$ giving the sines for every minute (figure 16).[343]

Folkerts[344] assumed that this table had been computed before 1463-1464, but in fact the table was certainly added by Johannes Engel who obtained it by truncating (not rounding) Regiomontanus's table for $R = 6 \cdot 10^6$.[345] Here is a sample of Regiomontanus's values (R) and Engel's values (E):

337 [Rheticus (1551)]

338 [Reinhold (1554)]

339 [Viète (1579)]

340 [Fincke (1583)]

341 On Johannes Engel, see [Knobloch (1983)] and [Dobrzycki and Kremer (1996)]. He is also mentioned by Gessner [Gessner and Simmler (1574), p. 336].

342 [Regiomontanus (1490)]

343 Not all editions seem to contain this sine table, and it is for instance absent from the copy at ULB Darmstadt (Inc II 357).

344 [Folkerts (1977), p. 234]

345 [Glowatzki and Göttsche (1990), pp. 48-49]

Angle	R	E
$60°0'$	5196152	51961
$60°1'$	5197024	51970
$60°2'$	5197896	51978
$60°3'$	5198768	51987
$60°4'$	5199639	51996
$60°5'$	5200510	52005
$60°6'$	5201380	52013
$60°7'$	5202350	52022
$60°8'$	5203119	52031

In this sample, we can also see that Engel introduced an additional error for $60°7'$.

Moreover, Regiomontanus's table contains a column of differences, whose values can be interpreted as the sixths of the differences of the sines. For instance, $(5197024 - 5196152)/6 = 145.333\ldots$ and that column starts with 145. Engel's table contains exactly the same value 145, although it is basically meaningless in that reduced table.

By analyzing the correspondence of Regiomontanus with Bianchini and others, Glowatzki and Göttsche have also shown that Regiomontanus actually did not himself use the table $R = 60000$ printed in 1490, and whenever he used this radius, he drew the values by dropping two digits from his large table and rounding the value.[346] This is an additional proof for the fact that the table for $R = 60000$ actually did not exist before it was prepared for printing in 1490. Instead, Glowatzki and Göttsche[347] assume that Regiomontanus had made a table with $R = 60000$ for himself, but different from the printed one. Such a table may have been held at the Seitenstetten Abbey until 1924, but it was then sold and had not been located by the authors. If this table surfaces again, one should check whether its values are truncated or rounded.

Engel's table (figure 16) is found again in the 1504 edition,[348] where the title of the work explicitely mentions this sine table. It is also found in later editions of the *Tabulæ directionum profectionumque*, where it is often attributed to Regiomontanus. My modern reconstruction[349] is based on the 1504 edition which has less idiosyncrasies than the 1490 version.

[346] [Glowatzki and Göttsche (1990), pp. 65-71]
[347] [Glowatzki and Göttsche (1990), p. 71]
[348] [Regiomontanus (1504)]
[349] [Roegel (2021d)]

Engel's table was used by Gaurico in 1524 (see below, § 6.2), in 1569 by Schreckenfuchs (§ 6.14) and reprinted in 1588 by Gallucci (§ 6.24).

6.2 Gaurico (1524)

Luca Gaurico (in Latin, Lucas Gauricus, in French Luc Gauric) (1475-1558) was an Italian astrologer, astronomer, and mathematician.[350]

In 1524, as an appendix to Regiomontanus's *Tabulæ directionum profectionumque*,[351] Gaurico published a table of sines with $R = 100000$ and at intervals of $10'$ (figure 17). This table was reprinted in 1557,[352] together with Regiomontanus's table of tangents but the latter only up to $50°$.

It is tempting to consider that Gaurico took his sines from a manuscript of Regiomontanus's table for $R = 10^7$ and truncated or rounded the values (this was also suggested by Glowatzki and Göttsche[353]), but this is actually not the case. Gaurico's values differ both from the truncated values and from the rounded ones of the 10^7 table.

In fact, it seems that Gaurico took the values in Engel's table for $R = 60000$, and merely multiplied them by $10/6$, although this procedure will in a few cases give values that differ from those in Gaurico's table.[354]

Gaurico's table was also certainly *not* the basis of Copernicus's table of sines (or semi-chords) published in 1543, although it uses the same radius and interval.

6.3 Copernicus (c1530?)

The earliest known decimal table of secants is a handwritten table by Nicolaus Copernicus (1473-1543),[355] included in his copy of Regiomontanus's *Tabulæ directionum profectionumque* published in 1490.[356] There Copernicus

[350]For a summary of Gaurico's life and works, see [Gessner and Simmler (1574), p. 455] as well as [Moréri (1733), p. 243-244].

[351][Regiomontanus (1524)]

[352][Gaurico (1557)]

[353][Glowatzki and Göttsche (1990), p. 178]

[354]This procedure anticipates what Copernicus has probably done in some places in the sine table included in his 1543 opus, although on the basis of Regiomontanus's full sexagesimal table.

[355]For a summary of Copernicus's life and works, see [Rosen (1971)]. Note in passing that in 1574 Gessner only briefly mentions Copernicus [Gessner and Simmler (1574), p. 518].

[356]See [Curtze (1875), pp. 34-37], [Glowatzki and Göttsche (1990), pp. 190-192] and [Folkerts et al. (2019)].

gave the values of the secant for each degree[357] and for $R = 10000$, overlayed to Regiomontanus's table of tangents. A reproduction of Copernicus's table was given by Glowatzki and Göttsche.[358]

Curtze considered that Copernicus had computed the secants from the cosines[359] but Birkenmajer[360] thought that the secants were computed using the formula $\mathrm{Sec}\, x = \sqrt{\mathrm{Tan}^x + R^2}$. Rosińska also observed that Copernicus's table of secants is not copied from Bianchini's table of cosecants.[361] Rosińska concluded therefore that Copernicus used neither Bianchini's nor Regiomontanus's tables for his table of secants.[362]

But still, Glowatzki and Göttsche observed that since Copernicus's values are very accurate, he must have used a manuscript of Regiomontanus's tables with $R = 6 \cdot 10^6$ or $R = 10^7$, later published in 1541.[363]

Now, according to my experiments, using either of Regiomontanus's tables, computing the exact secants, by mere division, as Curtze suggested, and rounding, will give almost always Copernicus's values, except for 88° and 89° where Copernicus probably tried to obtain more accurate values. The discrepancy of these two values is not, in my opinion, a sufficient reason to look for a different source or a different computation for Copernicus's entire table of secants.

6.4 Fine (1530)

Oronce Fine (1494-1555) was a French mathematician and cartographer. After having learned his first lessons of mathematics from his father in Briançon, he matriculated at the University of Paris and from about 1531 until his death he occupied the chair of mathematics of the Collège Royal in Paris.[364]

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[357] Stamm mistakenly wrote that the secants are given for every minute, but this is surely a typo [Stamm (1933)].

[358] [Glowatzki and Göttsche (1990), p. 191]

[359] See [Curtze (1875), pp. 34-37] and [Rosińska (2002), pp. 15-16].

[360] [Birkenmajer (1900), pp. 62-63]

[361] [Rosińska (2002), p. 16]

[362] [Rosińska (1987), p. 422]

[363] [Glowatzki and Göttsche (1990), p. 192]

[364] For summaries of Fine's life and works, see [Gallois (1890b)], Poulle [Poulle (1978)], [Marr (2009)], [Pantin (2013)] and [Axworthy (2016), Axworthy (2020)]. See also the accounts given by [Lindgren (2007)] (on land surveys) and [Fréchet (2009)], as well as the early notice by Gessner [Gessner and Simmler (1574), p. 534]. I have chosen to spell his name "Fine," in accordance with Poulle, but it is also sometimes spelled "Finé."

In 1530, Fine published his *De geometria*[365] which contains a sexagesimal table of sines. Fine's table gives the sines for a radius $R = 60$, at intervals of $1'$ and to two sexagesimal places (figure 18).

This table is not based on Engel's 1490 table[366] as one might think. Instead it must be based directly on a manuscript of Regiomontanus's large sexagesimal table, without truncating.

In 1542, Fine published his *De sinibus libri II* which was the first treatise solely on trigonometry to be printed in France.[367] This work is an appendix of Fine's *De mundi sphaera*.[368] Ross is very critical of this work and considers that it was unoriginal and out of date,[369] because it does not contain any contributions to trigonometric mathematics, and because it lags behind the developments soon introduced by Rheticus for the unification of sines, shadows, etc., in a same framework. Fine also appears to be unaware of Regiomontanus's *De triangulis omnimodis*[370] published in 1533 and which laid the foundations of modern trigonometry. But on the other hand, Fine's purpose with this book was pedagogical and he succeeded in contributing to the revival of mathematics in Paris.

The 1542 *De sinibus libri II* reprinted the table of sines published in 1530, with only minor variations. The layout is the same, although the table was obviously reset, as can be observed on the last lines of each page.

The second edition of *De sinibus libri II* was published in 1550,[371] but the sine table now uses a different layout (figure 30). In the two editions of this work (1542 and 1550), Fine's introduction gives the sines up to 90° at intervals of $3^\circ 45'$. In a second table, he gives the sines up to $7^\circ 30'$ at intervals of $15'$. These two tables were not given in the 1530 *De geometria*[372] and were presumably not used for the computation of the table published in 1530. None of these tables are mentioned by Glowatzki and Göttsche.[373]

Fine's table is the only sexagesimal table based on Regiomontanus's tables, apart from those of Schreckenfuchs[374] published in 1569 and of

[365] [Fine (1530)]
[366] [Regiomontanus (1490)]
[367] [Ross (1975), p. 379]
[368] [Fine (1542)]
[369] [Ross (1975), pp. 385-386]
[370] [Regiomontanus (1533)]
[371] [Fine (1550)]
[372] [Fine (1530)]
[373] [Glowatzki and Göttsche (1990)]
[374] [Schreckenfuchs (1569)]

Bressieu published in 1581.[375] Bressieu has actually copied Fine's table, from either of the three editions I have mentioned.

Certainly in order to help for the work with sexagesimal numbers, Fine had also published a sexagesimal multiplication table, his *tabula proportionalis*.[376]

I am giving separately a modern reconstruction of Fine's 1530 and 1550 tables.[377]

6.5 Apian (1533)

Peter Apian (1495-1552), also known as Petrus Apianus, was actually born Peter Bennewitz, or Peter Bienewitz, in Leisnig, Germany.[378] He was active in astronomy and geography and was a popularizer of astronomical and geographical instrumentation.[379] Apian studied at the University of Leipzig from 1516 to 1519 and then for two years in Vienna. His first major work was his *Cosmographia* (1524), which was later revised by Gemma Frisius (1508-1555), Apian's student.

Apian's second major work was his *Astronomicum Caesareum* (1540) which displayed an elaborate typography and the use of sophisticated volvelles.

From 1526 until his death he occupied the chair of mathematics and astronomy at the University of Ingoldstadt.

Apians's mathematical work is linked to Regiomontanus's writings.[380] He published his work on sines in 1533.

In his *Introductio geographica* published in 1533,[381] Apian provides a table of sines with $R = 10^5$ and for every minute of the quadrant (figure 19). The same table was reprinted in 1534 in Apian's *Instrvmentvm primi mobilis*[382] (figure 20) and in 1541 in his *Instrumentum sinuum*[383] (figure 21).

[375] [Bressieu (1581)]

[376] [Roegel (2021e), Roegel (2021f)]

[377] See [Roegel (2021g)] and [Roegel (2021h)].

[378] For surveys of Apian's life and works, see in particular [Günther (1882)], [Gallois (1890a), pp. 102-116], [North (1966)], [Kish (1970)] and [Röttel (1995)]. See also the early notice by Gessner [Gessner and Simmler (1574), p. 552].

[379] See in particular [Lindgren (2007)] for some background on land surveys.

[380] See [Kaunzner (1995)], [Folkerts (1995)] and [Lindgren (2007), p. 501].

[381] [Apian (1533)]

[382] [Apian (1534)]

[383] [Apian (1541)]

This table appears to have been obtained by merely dropping the last two digits of Regiomontanus's table for $R = 10^7$, without any rounding.[384] Regiomontanus's table having only been printed in 1541, Apian must have had access to one of the manuscripts of the 1468 table. Moreover, as observed by Glowatzki and Göttsche, the manuscript table used by Apian is in fact the same as the one used in the 1541 printed edition of Regiomontanus's table.[385]

And, as remarked by Kish,[386] Apian's sine table is the first printed table giving sines every minute and divided decimally, Regiomontanus's table having only been printed in 1541.

I am giving separately a modern reconstruction of Apian's 1533 table.[387]

Apian's tables seem to have been copied by directly or indirectly by Bassantin in 1557,[388] by Witekind in 1576,[389] by Peucer in 1579,[390], by Giuntini in 1581,[391], by Padovani in 1582,[392] and indirectly by Fale[393] in 1593.

6.6 Rheticus (1542)

Georg Joachim Rheticus (1514-1574) was born in Feldkirch (Austria).[394] In 1539, while he was professor of mathematics in Wittenberg, he set out to meet Copernicus in Frombork (Poland). He stayed with Copernicus for two years, published a first account of Copernicus's theory as *Narratio Prima* in Gdansk (1540), and was instrumental in the publication of Copernicus's *De revolutionibus orbium coelestium*[395] in 1543. When Rheticus returned from

[384] Delambre had written that the table was "computed by Apian," but this is a bit excessive [Delambre (1819), p. 395].

[385] See [von Peuerbach and Regiomontanus (1541)] and [Glowatzki and Göttsche (1990), pp. 173-174].

[386] [Kish (1970)]

[387] [Roegel (2021i)]

[388] [Bassantin (1557)]

[389] [Witekind (1576)]

[390] [Peucer (1579)]

[391] [Giuntini (1581)]

[392] [Padovani (1582)]

[393] [Fale (1593)]

[394] For summaries of Rheticus's life and works, see in particular [Kästner (1796), 561-564], [De Morgan (1841)], [Burmeister (1967–1968)], [Bernleithner (1973)], [Rosen (1975b)], [Kraai (2003)], [Danielson (2006)] [Wanner and Schöbi-Fink (2010)], [Schöbi-Fink and Sonderegger (2014)], and [van Brummelen (2021), pp. 7-9]. Note also Gessner's description of Rheticus's work [Gessner and Simmler (1574), p. 228].

[395] [Copernicus (1543)]

his visit, he also made it possible for Erasmus Reinhold to become closely acquainted with Copernicus's theory, leading to the publication of the *Prutenic tables* in 1551.

But in 1542, before the publication of Copernicus's *De revolutionibus*, Rheticus published its trigonometrical chapters under the title *De lateribus et angulis triangulorum*.[396] This work contains a table of sines at intervals of $1'$ and for a radius of 10^7 (figure 25). The sines were in fact not called sines, but half-chords. And the table is actually by Rheticus and not by Copernicus.[397] More precisely, Rheticus took the sines from Regiomontanus's table[398] published in 1541 (or from a common manuscript source). This is in particular confirmed by printing errors found in both editions.[399] Some of the typos that remain in Rheticus's table are in fact so conspicuous that they should have been corrected by Rheticus. Rheticus however did not carry over Regiomontanus's differences, but introduced the actual differences.

In her article on Copernicus's tables, Rosińska[400] hypothesizes that Copernicus had first planned to append to his work a table of sines with $R = 10^6$ but that this table was eventually replaced by Rheticus's one with $R = 10^7$.

In the past von Braunmühl,[401] Cantor,[402] Busard,[403] Rosen[404] and Folkerts[405] were of the opinion that Rheticus was the real author (computer) of the sine table. And Zinner thought that the author of the table was Copernicus himself and that he may have been inspired to construct a table with $R = 10^7$ by a glimpse of Regiomontanus's table.[406]

Rheticus's table seems to be the first table of sines where a value can easily and explicitely be read in two different ways. This novelty was

[396] [Copernicus (1542)] An edition of this work is given in [Folkerts et al. (2019)].

[397] See [Swerdlow and Neugebauer (1984), pp. 27-28] and [Rosińska (2002), pp. 18-20].

[398] See [von Peuerbach and Regiomontanus (1541)], [Zinner (1988), pp. 193-194], [Glowatzki and Göttsche (1990), p. 150] and [Rosińska (1994b)].

[399] [Rosińska (1987), p. 423]

[400] [Rosińska (2002)]

[401] [von Braunmühl (1900, 1903), v. 1, pp. 140-141]

[402] [Cantor (1900), p. 474]

[403] [Busard (1971a), p. 76]

[404] [Rosen (1975b), p. 396]

[405] [Folkerts (1977), p. 235]

[406] [Zinner (1990), p. 183]

observed by Stamm,[407] Rosen[408] and more recently by Husson.[409] For each value, there is both a reading using the first line and the first column (giving the sines), and a second reading using the last line and the last column (giving the cosines). Similar features are found again in the tables of Rheticus (1551), Reinhold (1554), Viète (1579), Clavius (1583), Magini (1592) and Rheticus/Otho (1596). Of course, earlier tables, including those of Regiomontanus, can also be read that way, but not explicitely, and it is necessary to perform a (simple) computation to find the cosine of an angle, for instance.

I am giving separately a modern reconstruction of Rheticus's table.[410]

6.7 Copernicus (1543)

As we have seen earlier, Copernicus (1473-1543)[411] had computed a small table of secants, perhaps around 1530, and in 1542 the trigonometric chapters of *De revolutionibus orbium coelestium* were published separately by Rheticus, together with a sine table. But in Copernicus's famous *De revolutionibus orbium coelestium* published in 1543 shortly before his death,[412] Copernicus included another table of sines, with an interval of $10'$ and a radius $R = 10^5$ (figure 26). Like in the excerpt published in 1542, the sines were actually not called sines, but half-chords.

Copernicus's table shows a few deviations from the values obtained from Regiomontanus's table with $R = 10^7$ when the values are rounded to $R = 10^5$. For instance there are three errors in the first 36 values (from 0° to 6°) and Copernicus gives $\mathrm{Sin}\,0^\circ 40' = 1163$ instead of 1164, $\mathrm{Sin}\,1^\circ 30' = 2617$ instead of 2618, and $\mathrm{Sin}\,4^\circ = 6975$ instead of 6976. However, there are more deviations when Copernicus's table is compared to Gaurico's table published in 1524 (also with $R = 10^5$), with eight errors in the same interval. The most likely basic explanation would then be that Copernicus used Regiomontanus's table for $R = 10^7$ and made a few rounding errors.

[407] [Stamm (1933), p. 2]

[408] [Rosen (1975b)]

[409] [Husson (2014)]

[410] [Roegel (2021j)]

[411] For a first summary of Copernicus's life and works, see [Rosen (1971)]. A recent biography of Copernicus is that by [Freely (2014)]. For further study, one might turn to [Swerdlow and Neugebauer (1984)], to Owen Gingerich's works as well as to Copernicus's complete works. On the connections between Italy and Krakow before Copernicus, see [Walsh (1996)]. For Copernicus's trigonometric tables, see [Rosińska (2002)].

[412] [Copernicus (1543)]

Glowatzki and Göttsche were of the same opinion and concluded that Copernicus must have used a manuscript version of Regiomontanus's table of sines with $R = 10^7$,[413] as he certainly did for his table of secants.[414]

Copernicus probably did not take the sines directly from the table published in 1542, although it is almost identical to that of Regiomontanus, because Copernicus's manuscript must have been ready long before its publication. In any case, if Copernicus took his values from Regiomontanus, he also made some corrections to that table, as Regiomontanus's typos for the sines of $36^\circ 10'$ and $51^\circ 20'$, reprinted in 1542, have been corrected in the *De revolutionibus orbium coelestium*.[415]

However, I think that it is possible to get a somewhat better understanding of the elaboration of Copernicus's table. I have said above that among the first 36 values of Copernicus's table, there are three obvious errors when comparing them to the rounded values from Regiomontanus's table with $R = 10^7$. For instance, for $0^\circ 40'$, Regiomontanus's table gives 116353, and Copernicus has 1163, which looks like a truncation, but for almost every other angle Copernicus's sine is the rounded and not truncated value obtained from Regiomontanus's table.

Now, if we start with Regiomontanus's sexagesimal table, that is, the table with $R = 6 \cdot 10^6$, the decimal values can be obtained by dividing Regiomontanus's values by 6. Considering only the first 36 values in Copernicus's table (from 0° to 6°), it appears that until $3^\circ 50'$, one obtains Copernicus's values by dropping one digit of Regiomontanus's table and rounding, then dividing by 6, then rounding.[416] For instance, for $0^\circ 50'$, one obtains $8726/6 = 1454.333\ldots$ which is rounded to 1454. In case the result is a half integer, the rounding occurs to the integer below, except if the first rounding was by default, although there may be exceptions (such as $2^\circ 40'$) taking account of how the first rounding was performed.

This procedure fails after $3^\circ 50'$ and it seems that a different operation was involved. In fact, between 4° and $5^\circ 20'$, there was apparently a truncation of the last two digits of Regiomontanus's table, the resulting value was multiplied by 10, divided by 6 and rounded. Between $5^\circ 40'$ and 6°, the initial procedure was again applied. These two procedures give a slightly better outcome than merely using Regiomontanus's table with $R = 10^7$.

413 [Glowatzki and Göttsche (1990), pp. 178-179]

414 [Glowatzki and Göttsche (1990), p. 192]

415 [Glowatzki and Göttsche (1990), p. 150]

416 This procedure is reminiscent from that probably used by Gaurico in 1524, although Gaurico started with Engel's table.

If we look at the last page of Copernicus's table, also containing 36 values, a comparison with Regiomontanus's table with $R = 10^7$ reveals five rounding errors ($84°30'$, $85°$, $86°40'$, $87°40'$ and $89°50'$). But if we start with the great sexagesimal table as above, there are in fact even more errors, 16 altogether. For instance, for $86°$ one obtains 99757, and not Copernicus's 99756.

I have of course only sampled the first and last 36 values of Copernicus's table, and this should be further investigated. It suggests however that different computations may have been involved in the making of Copernicus's sine table, and probably that some parts of Copernicus's table are based on Regiomontanus's sexagesimal table, whereas others are based on the table with $R = 10^7$, in addition of involving different rounding schemes from the same source. It is also possible that some values were based on other tables. But among the 16 values that are incorrectly rounded on the last page of Copernicus's table when starting with Regiomontanus's great sexagesimal table, only 10 of Copernicus's values are identical with those published by Apian in 1534.[417] It is therefore not possible to conclude that Copernicus used Apian's table. Perhaps for some comparisons, but not for all values.

Given this somewhat confused situation, it is understandable that Copernicus's table led to other opinions or conclusions. For instance, Stamm[418] wrote that Copernicus probably compared his values to those published by Apian in 1534,[419] but I have just shown that this is not conclusive. Folkerts[420] thought that Copernicus had computed the table himself, since he could not have been able to use Regiomontanus's table for $R = 10^7$ which was only published in 1541. Looking for Copernicus's source, Swerdlow and Neugebauer[421] excluded most sources, including Regiomontanus's tables printed in 1541, but they did not conclude further. And according to Rosińska,[422] Copernicus did not use Regiomontanus's table for his table of sines, although she did not provide another theory for the origin or calculation of the table.

[417] [Apian (1534)]
[418] [Stamm (1933)]
[419] [Apian (1534)]
[420] [Folkerts (1977), p. 234]
[421] [Swerdlow and Neugebauer (1984), pp. 100-101]
[422] [Rosińska (1987), p. 422]

6.8 Gemma Frisius (1545)

Gemma Frisius (1508-1555) (Jemme Reinerszoon, or Rainer Gemma) was a Dutch physician, mathematician, cartographer, philosopher, and instrument maker. He was born in Dokkum in the Netherlands.[423]

Gemma Frisius first practiced medicine in Louvain, but his real interests seem to have been geography and mathematics. In 1529 he revised Peter Apian's *Cosmographia*. He also designed globes and astronomical instruments. He died in Louvain.

In 1545, he published his *De radio astronomico et geometrico*[424] in which he included a table of cotangents (figure 27) which was copied from Regiomontanus.[425]

This work also contained a table of arctangents (figures 28 and 29) obviously copied from that of Peuerbach published in 1516.[426]

6.9 Rheticus (1551)

In 1551, Rheticus (1514-1574)[427] published his *Canon doctrinæ triangulorum*.[428] There, he gave the sines, cosines, tangents, cotangents, secants and cosecants at intervals of $10'$ and for a radius $R = 10^7$. Rheticus's table was in fact the first table to give all six possible ratios in a right triangle (figure 31).[429]

The sines in Rheticus's table were copied from Regiomontanus's table for $R = 10^7$.[430] Most of the values were not changed, but some of the typos were corrected, for instance the cosine of $38°40'$ whose value was still incorrect (as $\mathrm{Sin}\,51°20'$) in Rheticus's 1542 table.

Rheticus made new computations for the tangents and the secants using

[423] For summaries on Gemma Frisius's life and works, see [Cantor (1878)], [Hallyn (1996), Hallyn (1998), Hallyn (2004), Hallyn (2008)] and [Kish (1972)]. [Lindgren (2007)] gives some background on Gemma Frisius's work on land surveys. Note in passing that Gessner briefly mentions Gemma Frisius [Gessner and Simmler (1574), p. 221].

[424] [Gemma Frisius (1545)]

[425] [Glowatzki and Göttsche (1990), p. 181]

[426] [von Peuerbach (1516)]

[427] [Rosen (1975b)]

[428] [Rheticus (1551)] See [De Morgan (1845a), De Morgan (1845b)].

[429] However, in the first treatise of trigonometry independent of astronomical applications, the *Treatise on the Quadrilateral*, the Persian al-Tūsī (1201-1274), already in the 13th century, had used all six trigonometric functions [Archibald (1949), p. 31].

[430] See [Rosińska (1994b)] and [Glowatzki and Göttsche (1990), p. 152]

these sines. According to Glowatzki and Göttsche,[431] Rheticus merely computed the ratios for the tangents, but things are actually a bit more complicated.

First, it appears that the secants and cosecants were computed by dividing 1 (or rather 10^{14}) by the values of the cosines or sines, and truncating the results. This can readily be observed on the secants of $17^\circ 20'$, 29°, $29^\circ 50'$, 43°, etc., and practically every ratio whose decimal part is greater than 0.5. This is also true for the cosecants, an example being $35^\circ 30'$.

But the tangents and cotangents are another story. I don't know exactly how Rheticus computed these values, but a close examination of Rheticus's values reveals that the tangents are more accurate than the cotangents and consequently one cannot have been computed from the other. They must have been computed differently. The tangents may have been computed by dividing the sines by the cosines, but this cannot have been the case for the cotangents.[432]

It appears that the values of the cotangents are close to those obtained when computing

$$\mathrm{Cot}\, x = \sqrt{\mathrm{Csc}^2\, x - R^2}$$

but they are not totally identical. The agreement is however much better than that obtained by merely dividing the values of the cosines by the sines of Regiomontanus, and it may even be a little better if $\mathrm{Csc}^2\, x$ is rounded to seven or eight significant digits. This hypothesis may need to be tested further, but it parallels a suggestion by van Brummelen and Byrne for the computation of secants by Maurolico,[433] although I argue below that their suggestion is in fact not applicable to Maurolico's computations. However, I also suggest below that Fincke used a similar procedure to compute his secants in 1583.

In any case, Rheticus's work remains based on Regiomontanus's tables, and although he was the first to construct a table giving all six triangle ratios, he did not compute the cosecants and cotangents sufficiently accurately for small angles, and seems to have not yet understood that more accurate sines were needed. He had no problems giving cosecants and cotangents to 10 figures, when the sines were only given to 5 figures. This understanding of the need for more accurate sines only came later, and even the *Opus*

[431] [Glowatzki and Göttsche (1990), p. 185]

[432] It will be interesting to see to what conclusions came [Pritchard (2021)] who seems to have conducted a similar investigation, but whose result is not yet published at the time I am writing this.

[433] [van Brummelen and Byrne (2021)]

palatinum[434] published in 1596 is still marred by this problem which will only fully be solved by Pitiscus in the early 17th century.[435]

I am giving separately a modern reconstruction of Rheticus's table.[436]

6.10 Reinhold (1554)

Erasmus Reinhold (1511-1553) was a German astronomer and mathematician. He was born in Saalfeld, Germany. In 1536 he became professor of mathematics at the university of Wittenberg.[437] In 1542, Reinhold published a commentary on Peuerbach's *Theoricae Novae Planetarum*. When Rheticus came back from his visit to Copernicus, Reinhold studied Copernicus's theory closely and after the publication of Copernicus's *De revolutionibus orbium coelestium*, Reinhold made detailed annotations of this work.[438]

Between 1544 and 1551, Reinhold worked on recasting Copernicus's theory in handier tables and in 1551 he finally published his *Tabulæ prutenicæ cœlestium motuum* (Prutenic tables).

In his *Primus liber tabularum directionum* published in 1554 after his death,[439] Reinhold gave a table of sines (figure 34) and a table of tangents (figures 32 and 33), both with radius $R = 10^7$ and at intervals of $1'$.[440]

The sines were copied from Regiomontanus's sines,[441] probably from the 1541 printing, but the tangents were recomputed at intervals of $1'$, using these sines.[442] Moreover, in the range from 89° to $89^\circ 59' 50''$, Reinhold gave the tangents at intervals of $10''$.

The tangents seem to have been computed in a non systematic way. For the angles which are found in Regiomontanus's table, Reinhold has apparently mostly taken the ratio of the sines given by Regiomontanus, but sometimes the result was truncated (for instance for $\mathrm{Tan}\,1^\circ$ where Reinhold gives 174550 instead of 174551), and sometimes the sines were rounded to the tens (for instance for $\mathrm{Tan}\,10^\circ$ where 173648/984808 was computed

[434] [Rheticus and Otho (1596)]

[435] [Pitiscus (1613)]

[436] [Roegel (2010c)]

[437] For a summary of Reinhold's life and works, see [Gingerich (1975)]. Note in passing that Gessner briefly mentions Reinhold [Gessner and Simmler (1574), p. 184].

[438] [Gingerich (1973)]

[439] [Reinhold (1554)]

[440] See [van Brummelen (2021), pp. 5-7].

[441] [Glowatzki and Göttsche (1990), pp. 152-153]

[442] [Glowatzki and Göttsche (1990), p. 185]

instead of 1736482/9848078). In the case of $\mathrm{Tan}\,80^\circ$, there must have been a computation error, as Reinhold has actually computed 9848085/1736482 instead of 9848078/1736482. This error is not present in Rheticus's table.[443] My samples may or may not be representative of the entire table, and it would be useful to conduct a thorough analysis of Reinhold's table of tangents.[444] It seems in particular that Reinhold did not copy Rheticus's values given in 1551 at intervals of $10'$. Reinhold's error on $\mathrm{Tan}\,80^\circ$, incidentally, is found again in Fincke's table,[445] as well as in Clavius's table.[446]

In the last part of the table, Reinhold added the tangents at intervals of $10''$. The values themselves are not as accurate as one might wish, but this matters little here. What does interest us is to find out how Reinhold computed these values. These computations seem so far not to have been analyzed, not even by Glowatzki and Göttsche.[447] At first, this part of the table suggests a new computation of the sines and cosines of $10''$, $20''$, etc., up to $59'50''$, but this was most certainly not the case. It is in fact very easy to see what Reinhold has done, because the ratios behind each tangent value can be reconstructed. I am giving here only some samples:

Angle	Fraction	Value
$89^\circ\ 0'10''$	$\frac{9998370}{174038}$	$57.44935014\ldots$
$89^\circ\ 0'30''$	$\frac{9998502}{173070}$	$57.77143352\ldots$
$89^\circ\ 0'50''$	$\frac{9998519}{172101}$	$58.09680943\ldots$
$89^\circ30'10''$	$\frac{9999625}{86780}$	$115.22960359\ldots$
$89^\circ30'20''$	$\frac{9999628}{86296}$	$115.87591543\ldots$
$89^\circ59'10''$	$\frac{9999999}{2424}$	$4125.41212871\ldots$
$89^\circ59'20''$	$\frac{9999999}{1939}$	$5157.29706034\ldots$
$89^\circ59'30''$	$\frac{10000000}{1455}$	$6872.85223367\ldots$
$89^\circ59'40''$	$\frac{10000000}{970}$	$10309.27835051\ldots$
$89^\circ59'50''$	$\frac{10000000}{485}$	$20618.55670103\ldots$

443 [Rheticus (1551)]

444 The forthcoming study [Pritchard (2021)] may contain some interesting clues on this matter.

445 [Fincke (1583)]

446 [Clavius (1586)]

447 [Glowatzki and Göttsche (1990), p. 185]

The "Fraction" column gives the ratios actually used by Reinhold for the tangents, and the column on the right gives the values of these fractions. These can be compared with those in Reinhold's table.

It turns out that the values given by these fractions are almost exactly those of Reinhold, with the occasional rounding errors or typos. For instance, Reinhold table has $\mathrm{Tan}\,89°0'10'' = 574493507$ which must be a typo for 574493501. In most cases, the fractions seem to have been truncated (and for instance for $89°30'10''$ this resulted in the incorrect value), but in some cases they were rounded (for instance for $89°59'30''$).

We can see in this sample that Reinhold uses

$$\mathrm{Sin}\,10'' = 485 \qquad \mathrm{Sin}\,20'' = 970 \qquad \mathrm{Sin}\,30'' = 1455$$
$$\mathrm{Sin}\,40'' = 1939 \qquad \mathrm{Sin}\,50'' = 2424 \qquad \ldots$$

and these values are obtained by linear interpolation of Regiomontanus's sines. There may again be some slight inaccuracies, and the value of $\mathrm{Sin}\,59'50''$ would for instance have been better at 174039 than 174038.

The numerators used by Reinhold were also obtained by interpolation from Regiomontanus's values. For instance, for $89°30''$, 9998502 is just halfway between Regiomontanus's sine values 9998477 and 9998527. But for $\mathrm{Tan}\,89°10''$, something obviously went wrong, because the value of $\mathrm{Sin}\,89°10''$ used is smaller than that for $\mathrm{Sin}\,89°$, although the resulting value is still acceptable. Of course, given the limited number of significant digits for the sines, especially at the end of the range, most of the figures in the tangents end up being meaningless. It doesn't make much sense to give $\mathrm{Tan}\,89°59'50''$ to 12 places, when the value of $\mathrm{Sin}\,10''$ used only has three places...

We can also see that at the end of the range Reinhold moved to cosine values of 10^7, but that he did not try to do a finer interpolation. In any case, such an analysis can be made for all $60 \times 5 = 300$ values which are not multiples of $1'$, but this is left as an exercise. There may be other errors such as the one mentioned for $89°10''$ and it might be interesting to do some detailed statistics about these errors.

It is precisely for these reasons that Viète's tangents and secants published in 1579 are much more accurate than those of Reinhold, because Viète used sufficiently accurate sines for the number of figures he was trying to compute for the tangents and secants.

Reinhold's table of tangents was the first table of tangents at intervals of $1'$. Secants at this interval would only be published 25 years later by

Viète.[448] Reinhold's tangents were reused by Fincke in 1583, and Fincke added the secants.

A modern reconstruction of Reinhold's tables is provided separately.[449]

6.11 Bassantin (1557)

James Bassantin (c1504-1568) was a Scottish astronomer and mathematician who came to France under the reign of Henri II who was King of France from 1547 to 1559.[450]

Bassantin owes his fame to the publication of his *Astronomique Discours*[451] in 1557 in Lyon. This book contains many volvelles based on the system of Ptolemy.[452] It also contains a table of sines with $R = 10^5$ and at intervals of $1'$ (figure 35) which was presumably copied from Apian, since both the values and the layout agree with Apian's 1533,[453] 1534[454] and 1541[455] tables. Glowatzki and Göttsche came to the same conclusion.[456] Possibly the only altered value is the sine of $89^\circ 59'$ which Apian had put at 100000, but which Bassantin put at its correct value, 99999. However, many other values are wrong, since Apian truncated and did not round Regiomontanus's values.

6.12 Maurolico (1558)

Francesco Maurolico (1494-1575) was a mathematician and astronomer born in Messina, Sicily, but of Greek lineage. He lived almost all of his life in Sicily and made contributions to the fields of geometry, optics, conics, mechanics, music, and astronomy.[457] He was ordained a priest in 1521 and

[448] [Viète (1579)]

[449] [Roegel (2021k)]

[450] See [de Chaufepié (1750), p. 112], [Delambre (1821), v. 1, p. 308-309], [Hoefer (1873), p. 314] and [Henderson (1885)] for some biographical elements on Bassantin. In France, he was called Jacques Bassantin.

[451] [Bassantin (1557)]

[452] For some interesting information on the physical structure of this work, see [Vaucher (2020)].

[453] [Apian (1533)]

[454] [Apian (1534)]

[455] [Apian (1541)]

[456] [Glowatzki and Göttsche (1990), p. 169]

[457] For a summary of Maurolico's life and works, see [Masotti (1974)] and [van Brummelen (2021), pp. 12-13]. Note in passing that Gessner briefly mentions Maurolico [Gessner

later became a Benedictine.

Maurolico edited the works of classical authors including Archimedes, Apollonius, Autolycus, Theodosius and Serenus. He also composed his own unique treatises on mathematics and mathematical science.

In 1558, Maurolico published his commentary on the spherics of Theodosius.[458] It contains short tables of sines (figure 39), tangents (figure 40) and secants (figure 41), all using the radius $R = 10^5$ and only giving values every degree. Maurolico also gave the tangents and secants for $89°15'$, $89°30'$, $89°45'$, $89°55'$ and $89°59'$.

The values of the sines differ from those of the earlier tables with $R = 10^5$, namely those of Gaurico (1524)[459] and Apian (1533).[460] Instead, Maurolico seems to have taken his values from Regiomontanus's table by dropping two digits and rounding.[461] And, contrary to what von Braunmühl wrote,[462] Maurolico was unaware of Rheticus's *Canon doctrinæ triangulorum*[463] published in 1551.[464]

As far as the tangents are concerned, Glowatzki and Göttsche wrote that Maurolico took his values from Regiomontanus's *Tabulæ directionum profectionumque*[465] and recomputed those above $45°$ using Regiomontanus's sines.[466]

But according to Brummelen,[467] the tangents were copied from Regiomontanus's 1490 table up to about $60°$. Above $60°$, the values of the tangents seem to have been recomputed from Regiomontanus's sines and Brummelen implies (his table 3) that they have been recomputed that way until $89°15'$ inclusive. Beyond $89°15'$ the difference between Maurolico's values and those computed from Regiomontanus's sines becomes much larger. The last four values are more accurate than the values that could have been obtained from Regiomontanus's table with $R = 10^7$. Van Brum-

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and Simmler (1574), p. 204].

458[Maurolico (1558)]

459[Regiomontanus (1524)]

460[Apian (1533)]

461[Glowatzki and Göttsche (1990), pp. 178-179]

462[von Braunmühl (1900, 1903), vol. 1, p. 151]

463[Rheticus (1551)]

464See [van Brummelen and Byrne (2021), p. 200]. In 1944, Zeller had already considered the different opinions of Fincke and Magini, but without settling with any [Zeller (1944), p. 72].

465[Regiomontanus (1490)]

466[Glowatzki and Göttsche (1990), pp. 181, 185]

467[van Brummelen and Byrne (2021), p. 202] melen therefore suggested some kind of independent computation.[468]

I believe however that the threshhold for that "independent" computation occurs earlier than $89°15'$. In fact, it is easy to see what Maurolico has done for the last values of the table. He basically recomputed the required sines with one more digit and used them for the tangents. For instance,

- for $89°30'$, Maurolico used $\frac{9999619.2}{87265.3} = 114.588721\ldots$ (and printed 11458872), when Regiomontanus had only given $\text{Sin}\, 89°30' = 9999619$ and $\text{Sin}\, 30' = 87265$;
- for $89°45'$, Maurolico used $\frac{9999904.8}{43633.1} = 229.1816258\ldots$ (and printed 22918163), when Regiomontanus had only given $\text{Sin}\, 89°45' = 9999904$ and $\text{Sin}\, 15' = 43632$ (and not 43633);
- for $89°55'$, Maurolico used $\frac{9999989.4}{14544.5} = 687.544391\ldots$ (and printed 68754439), when Regiomontanus had only given $\text{Sin}\, 89°55' = 9999989$ and $\text{Sin}\, 5' = 14544$;
- for $89°59'$, Maurolico used $\frac{9999999.6}{2908.9} = 3437.725463\ldots$ (and printed 343772546), when Regiomontanus had only given $\text{Sin}\, 89°59' = 9999999$ and $\text{Sin}\, 1' = 2909$.

The values taken by Maurolico are all correct, except for $\text{Sin}\, 5'$ which should have been 14544.4 and $\text{Sin}\, 30'$ which should have been 87265.4. In summary, Maurolico must have recomputed the sines of eight angles ($1'$, $5'$, $15'$, $30'$, $89°30'$, $89°45'$, $89°55'$ and $89°59'$), perhaps using $\text{Sin}\, 1'$ as a basis. In fact, if Maurolico has started with $\text{Sin}\, 1' = 2908.9$, he could have computed $\text{Sin}\, 5'$ and found 14544.5 or even 14544.4, depending how he computed the value. Continuing with 14544.4, Maurolico could have found $\text{Sin}\, 15'$ to be 43633.1, and eventually $\text{Sin}\, 30'$ to be 87265.3.

This procedure was applied at least as early as $85°$ and we can see for instance that the ratio $9961947/871557.4$ gives exactly Maurolico's value, but that $9961947/871557$ (Regiomontanus's values) does not.

Maurolico's accurate computation of the last tangents then boils down to a one digit more accurate value of $\sin 1'$ than that provided by Regiomontanus. I don't know how Maurolico obtained that value, but there is a very simple way, which is to observe that the sine of a small angle measured on the circumference (in radians) is almost equal to the angle itself. Therefore, with $R = 10^7$,

$$\text{Sin}\, 1' \approx \frac{\pi}{180 \cdot 60} \cdot 10^7 = 2908.882\ldots$$

[468] [van Brummelen and Byrne (2021), p. 205]

and all that Maurolico needed was to take 2908.9 instead of Regiomontanus's 2909. There was no need to resort to bisections, trisections, etc., and to recompute the sines of small angles. There was also no need to interpolate as Regiomontanus did to construct his tables. And proceeding the above way does not require a very accurate value of π, as Ptolemy's value $3.1416\ldots$ is sufficient.

Maurolico's third table is his table of secants, which he called *tabella benefica*.[469] It was actually Fincke who first named that function secant.[470] Fincke thought that Maurolico had copied his secants from Rheticus.[471] But Brummelen recently gave an edition of Maurolico's short manual of his *tabella benefica*.[472] There Maurolico claims that he worked on this matter in 1550, and it would then very likely be a work independent of that of Rheticus. Magini had also assumed that Maurolico's work was an independent one, not influenced by Rheticus.[473] One might therefore assume that it was directly computed from Regiomontanus, probably from the 1541 edition.

Maurolico's table of secants is in fact very accurate, and more accurate than what would have been obtained by a mere use of Regiomontanus's tables.[474] In order to explain this accuracy, Brummelen and Byrne claim that Maurolico computed the secants from the tangents, and not directly from Regiomontanus's sines, as claimed by Glowatzki and Göttsche.[475]

According to van Brummelen and Byrne, Maurolico used the relation

$$\sec^2\theta = \tan^2\theta + 1$$

which transcribes into

$$\mathrm{Sec}^2\theta = \mathrm{Tan}^2\theta + R^2$$

when the radius is R. For instance, for $\theta = 81^\circ$, $\mathrm{Tan}\,\theta = 631375$ and $\sqrt{631375^2 + 100000^2} = 639245.172\ldots$ and Maurolico gives $\mathrm{Sec}\,\theta = 639245$. This appears to work for 81°, but merely computing $1/1564345$ would have given the correct result too.

[469] [Delambre (1819), p. 440]

[470] [Fincke (1583)]

[471] [van Brummelen and Byrne (2021), p. 200]

[472] See [van Brummelen and Byrne (2021)] as well as [van Brummelen (2021), p. 22].

[473] See the second page of the preface of Magini's *Primum mobile* [Magini (1609)], of which an excerpt is translated in [van Brummelen and Byrne (2021)], but mistakenly attributed to Magini's *De planis triangulis*.

[474] [van Brummelen and Byrne (2021), p. 206]

[475] [Glowatzki and Göttsche (1990), p. 193]

In fact, the procedure suggested by van Brummelen and Byrne (which echoes Birkenmajer's suggestion for Copernicus's table of secants) does not always work, and fails to give Maurolico's values for 86°, 87°, or $89^\circ 30'$. The solution is actually much simpler, and Maurolico must have proceeded like for the tangents, using an additional digit for a number of sines. Doing so from 85° to the last angle gives the values in Maurolico's table, the only exception being 86°. But in that case, computing the secant from the tangent also does not yield the value in Maurolico's table. It is possible that Maurolico only used one additional digit for the secants from 87°, or that he made a mistake, or perhaps that the table contains a typo there.

Incidentally, Fincke seems to have used the procedure suggested by van Brummelen and Byrne in order to compute his secants in 1583 (and in fact van Brummelen and Byrne claim so,[476] but with no references).

I have also given a modern reconstruction of Maurolico's tables in a separate document.[477]

6.13 Eisenmenger (1562)

Samuel Eisenmenger (1534-1585), known as Siderocrates, was a German physician, theologian and astronomer. He was professor of astronomy at the University of Tübingen in 1557-1568.

In 1562, Eisenmenger published his *Libellus geographicus*[478] in which he gave a table of sines with $R = 10^7$ and at intervals of $1'$ (figure 42). This table was certainly also copied from Regiomontanus,[479] and probably from the 1541 printing.[480] The layout and headings of Eisenmenger's table are practically identical to those of Regiomontanus's published table, except that Eisenmenger put only half a degree in each column.

6.14 Schreckenfuchs (1569)

Erasmus Oswald Schreckenfuchs (1511-1579) was an Austrian humanist, astronomer and Hebraist.[481] In 1551 he produced a commentary to the Al-

[476] [van Brummelen and Byrne (2021), p. 206]

[477] [Roegel (2021l)]

[478] [Eisenmenger (1562)]

[479] [Glowatzki and Göttsche (1990), pp. 153-154]

[480] [von Peuerbach and Regiomontanus (1541)]

[481] For a summary of Schreckenfuchs's life and works, see [von Khauz (1755), pp. 184-203]. Note in passing that Gessner briefly mentions Schreckenfuchs [Gessner and Simmler (1574), p. 184].

magest of Ptolemy and in 1556, he published a commentary on Peuerbach's *Theoricae Novae Planetarum*.[482]

In 1569, in his *Commentaria in Sphaeram Ioannis de Sacrobusto*,[483] he reprinted Regiomontanus's table of tangents from one of the editions of the *Tabulæ directionum profectionumque* (figure 43).

But Schreckenfuchs also gave two tables of sines. His first table of sines covers two pages and uses the radius $R = 60000$ and an interval of $15'$ (figure 44). This table is likely based on the table of sines found in Regiomontanus's *Tabulæ directionum profectionumque* (1490, 1504, 1550, 1552 or 1559), namely Johannes Engel's table (figure 16), as the values are truncated and not rounded from the large table with $R = 6 \cdot 10^6$.

Schreckenfuchs's second table spans six pages, also with $15'$ intervals, and uses a radius $R = 60$ and three sexagesimal places (figure 45). However, the last place is always given as 0 or 30. This second table cannot have been obtained from the first one. For instance, for $15'$, Schreckenfuchs gives the sine as 261 ($60000 \times \sin 15' = 261.7\ldots$), but 261 would give a sine of $0^{\text{p}}15'39''36'''$, not $0^{\text{p}}15'42''30'''$. It is therefore to assume that now Schreckenfuchs used the sines in Regiomontanus 1541 (or 1561). For $15'$, Regiomontanus gave 26180, and this then would lead to $0^{\text{p}}15'42''28.8'''$ that Schreckenfuchs could have rounded to $0^{\text{p}}15'42''30'''$.

6.15 Witekind (1576)

Hermann Witekind (or Wilken) (1522-1603), a student of Melanchthon, was a German humanist and mathematician. In 1585, under the pseudonym of Augustine Lercheimer he published a book against the persecution of witches.[484]

In 1576, he published his work *Conformatio horologiorum sciotericorum etc.*[485] in which he included a table of sines for a radius of 100000 and for every minute of the quadrant (figure 46). This table was presumably copied from one of Apian's tables (1533, 1534 or 1541),[486] as the values all seem to agree.[487] The layout, however, is different. Each page has six columns for degrees and 30 rows for 30 minutes. Six degrees therefore span two pages.

[482] [Malpangotto (2020), pp. 221-232]
[483] [Schreckenfuchs (1569)]
[484] [Binz (1888)]
[485] [Witekind (1576)]
[486] [Apian (1533)], [Apian (1534)] and [Apian (1541)].
[487] [Glowatzki and Göttsche (1990), p. 169]

Moreover, repeated leading digits are not printed (for instance the value 145 following 116 is shown as 45).

Among Witekind's other scientific publications, I mention only his *De sphaera mundi* published in 1574 (second edition in 1590).

6.16 Peucer (1579)

Caspar Peucer (1525-1602) was a German reformer, physician, and scholar from Bautzen, Germany.[488] He wrote on mathematics, astronomy, geometry, and medicine, and edited some of Philip Melanchthon (1497-1560)'s letters, having married one of his daughters. He also became professor of mathematics in Wittenberg in 1554, the successor of Erasmus Reinhold after his untimely death in 1553. In 1560, he was appointed to the medical faculty of Wittenberg.

In his *De dimensione terræ etc.*[489] published in 1579, and reprinted in 1587, Peucer included a table of sines for $R = 10^5$ and at intervals of $1'$ (figure 47). The situation parallels that of Witekind, and Peucer's table was presumably also copied from Apian (1533, 1534 or 1541),[490] since the values agree,[491] with only minor alterations. However Peucer adapted Apian's layout and put only five degrees and 30 minutes per page. Therefore one page of Apian's table corresponds to four pages of Peucer's table.

6.17 Viète (1579)

François Viète (1540-1603) was a French mathematician whose work on the new algebra was an important step towards modern algebra. According to Zeller, Viète "was the foremost mathematician of France in the sixteenth century."[492] Viète received a bachelor's degree in law in 1560 and held a number of official positions. In 1573, the King Charles IX made him counselor to the *parlement* of Brittany. He came back to Paris in 1580. Among his many works is his *In artem analyticem isagoge*, the earliest work on symbolic algebra (1591).

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[488] For a summary of Peucer's life and works, see [Kolb (1976)].

[489] [Peucer (1579)]. Earlier editions from 1550 [Chassagnette (2006)] and 1554 do not include the table of sines.

[490] [Apian (1533)], [Apian (1534)] and [Apian (1541)].

[491] [Glowatzki and Göttsche (1990), p. 169]

[492] [Zeller (1944), p. 73]. For summaries of Viète's life and works, see [De Morgan (1843)], [Ritter (1895)], [Busard (1976)], and [van Brummelen (2021), pp. 9-11].

Also inspired by Rheticus' 1551 *Canon doctrinæ triangulorum*,[493] Viète constructed a new table, which he called the *Canon mathematicus*.[494] This work contained a typographically sophisticated table of the six trigonometric functions for every minute of the quadrant and with a radius of 100000, with sometimes one or more additional figures[495] (see figure 48). The printing of the table was started in 1571 but it was only completed in 1579.[496]

This was the first published canon giving the trigonometric functions every minute, but on the other hand it gave them to less places than Rheticus' 1551 table (which however only had an interval of $10'$).

The sines from 0° to 45° and the cosines from $0^\circ30'$ to 45° were taken from Regiomontanus (or Reinhold) and rounded or not, depending on the range of the table. Glowatzki and Göttsche had observed that Viète had recomputed the sines from $89^\circ61'$ to 90°,[497] but in fact the cosines from 0° to $30'$ have been computed from Regiomontanus's (or Reinhold's) sines, probably with $\cos x \approx 1 - \frac{\sin^2 x}{2}$, which, with a radius R other than 1, becomes $\mathrm{Cos}\, x \approx R - \frac{\mathrm{Sin}^2 x}{2R}$. For instance, for $6'$, $R = 10^8$ and Regiomontanus's sine value 17453:

$$\mathrm{Cos}\, 6' \approx 10^8 - \frac{174530^2}{2 \cdot 10^8} = 10^8 - \frac{17453^2}{2 \cdot 10^6} = 99999847.69\ldots$$

and Viète gives the value 99999848. Viète did not use the values of the sines in his table for this purpose, and using 175 (Viète's value for $\mathrm{Sin}\, 6'$) in the previous example would not produce a sufficiently accurate cosine.

For the tangents and secants, Glowatzki and Göttsche wrote that they were recomputed from Regiomontanus's values.[498] But we can actually tell a bit more.

First, we can see that Viète computed the secants from 0° to 45° by inverting his cosines (and not those of Regiomontanus). The tangents between 0° to 45° were computed by using Regiomontanus's full values.

[493]See [Rheticus (1551)] and [Hunrath (1899)].

[494][Viète (1579)]

[495]See [Hunrath (1899)] and [Delambre (1819), pp. 455-456]. The cosines and secants are given more accurately than the other values throughout the table. They are given to four more places from 0° to $0^\circ2'$, to three more places from $0^\circ3'$ to $0^\circ24'$, to two more places from $0^\circ25'$ to $4^\circ5'$, and to one more place from $4^\circ6'$ to 45°.

[496][Tannery (1896), p. 205]

[497][Glowatzki and Göttsche (1990), pp. 154-155]

[498][Glowatzki and Göttsche (1990), pp. 189, 196]

Then, the cotangents from about 5° to 45° were also computed from the ratios $\cos$ / $\sin$ using Viète's values (and not by inverting the tangents). The cosecants from about 5° to 45° were computed by inverting Viète's values of the sines.

But for the cotangents and cosecants between 0° and about 5°, Viète used more accurate values of the sines than those printed in his table, with 1 to 5 more figures than Regiomontanus. For instance, for $\mathrm{Cot}\,1'$, Viète used $\mathrm{Sin}\,1' = 29.0888204$ (here with $R = 10^5$), that is 5 more figures than Regiomontanus. This enabled him to obtain (reducing the cosine and sine to $R = 10^9$)

$$\cot 1' = \frac{999999958}{290888.204} = 3437.74668$$

or (with $R = 10^5$)

$$\mathrm{Cot}\,1' = \frac{999999958}{290888.204} \times 10^5 = 343774668$$

and this is precisely the value given in the *Canon mathematicus* (the exact value being 343774667).

The same applies for $\mathrm{Csc}\,1'$. For $\mathrm{Sin}\,2'$, Viète took 58.1776385, whereas Regiomontanus only has 5818. And so on.

These values are much more accurate than the tangents and secants given by Reinhold in 1554 and Fincke in 1583 for large angles, and obviously Viète had a much better understanding of the requirements for exact computations.

In his treatise on angular sections,[499] Viète describes a way to compute the sine of $1'$ and other values he needed. This sine can be obtained as follows. First, like Ptolemy before, one can compute the sines of 18° and 60°. Trisecting 60° twice, we obtain $\mathrm{Sin}\,20^\circ$ and then $\mathrm{Sin}\,6^\circ 40'$. Using quinquisection with 18°, we obtain $\mathrm{Sin}\,3^\circ 36'$. Bisecting $6^\circ 40'$ we find $\mathrm{Sin}\,3^\circ 20'$. Using the two values $\mathrm{Sin}\,3^\circ 36'$ and $\mathrm{Sin}\,3^\circ 20'$, we obtain the sine of the difference of the angles, namely $\sin 16'$. And bisecting $16'$ four times, we obtain $\mathrm{Sin}\,1'$. But as a matter of fact Viète seems to have proceeded slightly differently for his table. He actually found two approximations of $\mathrm{Sin}\,1'$, one greater and one smaller than the sought value. An interpolation between these two values then gave a better approximation of $\mathrm{Sin}\,1'$.[500]

As observed by Tannery, Viète's tables are rare because of the success of Rheticus' *Opus palatinum* (1596),[501] of Pitiscus' *Thesaurus mathematicus*

[499] See [Viète (1615)] and [Zeller (1944), pp. 79-80].

[500] See [Viète (1579), pp. 62-67] and [van Brummelen (2021), pp. 22-23].

[501] [Rheticus and Otho (1596)]

(1613),[502] and because of the introduction of logarithms in 1614. They all made Viète's tables obsolete.

A persistent legend is also that the *Canon mathematicus* contained many errors, and that Viète consequently withdrew or re-purchased all the copies he could find and had them destroyed. This would then explain why this book is of great rarity.[503] But according to Ritter's biography of Viète,[504] this legend rests on the editor of Viète's 1646 *Opera* omitting the *Canon mathematicus*, on the grounds that the computations would have to be redone.[505] Moreover, as I have shown, Viète's table is actually very accurate.[506]

The *Canon mathematicus* was also published with a London imprint in 1589 (*Opera mathematica*, London: Bouvier) and there is an edition dated 1609, but Bosmans showed that it is not a reprint. It is the 1579 edition rebound.[507] Cantor and von Braunmühl had mistakenly thought that it was a new edition,[508] probably after Eneström led them to think so.[509]

A modern reconstruction of Viète's table is given separately.[510]

6.18 Bressieu (1581)

Maurice Bressieu (c1546-1617) was a French mathematician and humanist.[511] In 1576, Bressieu won a position of mathematics professor founded in Paris by Petrus Ramus (1515-1572), which he kept until 1608.[512]

In his *Metrices astronomicæ* published in 1581,[513] Bressieu first gives a sexagesimal table of sines, with an unusual layout (figure 49). The sines and sines of the complementary angle (cosines) are given in two adjacent columns and the table therefore only runs up to 45°. But Bressieu's layout is in fact very unusual, in that it doesn't use a footer line. In figure 49, the first column (headed 18) gives from top to bottom the sines from 18° to 19°. The second column (headed 71) gives the sines from 71° to 72°, but from

[502] [Pitiscus (1613)]

[503] See [Eneström (1892)] and [Cantor (1900), pp. 583-584].

[504] [Ritter (1895), p. 54]

[505] See [Tannery (1896), p. 208] and [Tannery (1900)].

[506] [Roegel (2011)]

[507] See [Bosmans (1901-1902), pp. 111-114] and [Bosmans (1901), pp. 297-298].

[508] See [von Braunmühl (1900, 1903), v. 1, p. 158] and [Cantor (1900), p. 583].

[509] [Eneström (1892)]

[510] [Roegel (2011)]

[511] See [de Mérez (1880)] for a summary of Bressieu's life and works.

[512] [Waddington (1855), p. 337]

[513] [Bressieu (1581)]

bottom to top. Consequently, the second column actually also gives the cosines from 18° to 19°, from top to bottom.

The values are given in degrees (or parts) with a radius of 60. For instance, the sine of 45° is given as $42; 25, 35$ as $\mathrm{Sin}\,45^\circ = 42 + 25/60 + 35/60^2 + \cdots$. This table contains exactly the same values as in Fine's tables.[514]

Bressieu also gives a second table (figure 50), which actually contains values of the tangents and secants, also with a radius of 60. For instance,

$\mathrm{Tan}\,45^\circ = 60$	is given as	$1, 0; 0, 0$
$\mathrm{Sec}\,60^\circ = 120$	is given as	$2, 0; 0, 0$
$\mathrm{Tan}\,89^\circ 3' = 60^2 + 18 + 20/60 + \cdots$	is given as	$1, 0, 18; 20$

and so on. Bressieu's tables of tangents and secants are the only known printed fully sexagesimal tables of tangents and secants. Their layout follows the style used by Rheticus in 1542 and not that used in Bressieu's table of sines.

It appears that these tangents and secants have not been computed from Bressieu's table of sines which is not sufficiently accurate. Bressieu could have taken another table giving the tangents and secants for every minute, but the only such table available in 1581 was Viète's table[515] and the last values of Bressieu's tangents do not agree with Viète's values. Yet another possibility is that Bressieu used Reinhold's values for the tangents.[516] But this appears again not to be the case.

I believe instead that Bressieu used Regiomontanus's table of sines (or a derivative thereof) for $R = 10^7$ and computed a number of sexagesimal values of the tangents and secants in his second table, but not all of them. For the last values of the table, Bressieu may have done a number of special computations, but for the other gaps, I believe that Bressieu interpolated the missing values. In fact, if Bressieu would have used Regiomontanus's values in each case, he would have obtained more accurate values for the tangents and secants. The deviations do not occur only for the last values around 90°, but also for smaller values. For instance, for $\mathrm{Tan}\,75^\circ$, Bressieu gives $3, 43; 55, 18$, and working with Regiomontanus's values would have given him $3, 43; 55, 23$ which is the correct value. For $\mathrm{Tan}\,89^\circ 59'$, Bressieu gives $57, 17, 42; 26$, when the correct value is $57, 17, 44; 48, 1, \ldots$, which he would have obtained using Viète's table. Regiomontanus's values instead would have given $57, 17, 36; 25$.

[514] [Fine (1530), Fine (1550)]

[515] [Viète (1579)]

[516] [Reinhold (1554)]

In figure 13, the accuracy of Bressieu's tables is indicated as $60; 60^2$, by which I mean a radius of 60 and two sexagesimal places. However, for 89° and above, the values of the tangents and secants are given to only one sexagesimal place.

Bressieu is mentioned by Zeller[517] but not by Glowatzki and Göttsche.

6.19 Giuntini (1581)

Francesco Giuntini (1523-1590) was an Italian theologian and one of the most famous astrologer of the second half of the 16th century.[518]

In his *Speculum astrologiæ*[519] published in 1581, Giuntini included a table of sines with $R = 10^5$ and an interval of $1'$ (figure 51). This table was most certainly copied from one of Apian's tables (1533, 1534 or 1541),[520] or perhaps from one of its derivatives.

The 1573 edition of the *Speculum astrologiæ* does not contain this sine table.

6.20 Padovani (1582)

Giovanni Padovani (b. c1512) was an Italian mathematician and astronomer.[521] He was from Verona and a student of the astronomer and mathematician Pietro Pitati.

In his *De compositione, & vsu multiformium horologiorum solarium*,[522] a work on sundials published in 1582, Padovani included a table of sines with $R = 10^5$ and an interval of $1'$ (figure 52). Like Giuntini's table (1581), Padovani's table was also most certainly copied from one of Apian's tables (1533, 1534 or 1541),[523] or perhaps from one of its derivatives, but Giuntini and Padovani's tables do not share the same layout.

An earlier edition of Padovani's work on sundials was published in 1570, but I have not seen it. It possibly lacks the table of sines.

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517 [Zeller (1944), pp. 86-88]

518 For a summary of Giuntini's life and works, see [Ernst (2001)].

519 [Giuntini (1581)]

520 [Apian (1533)], [Apian (1534)] and [Apian (1541)].

521 For a summary of Padovani's life and works, see [Pizzamiglio (2004), p. 58-59].

522 [Padovani (1582)]

523 [Apian (1533)], [Apian (1534)] and [Apian (1541)].

6.21 Fincke (1583)

Thomas Fincke (1561-1656) was born in Flensburg, Germany, now at the border with Denmark. From 1577 to 1582, he studied mathematics, astrology, rhetoric and philosophy, in particular with Conrad Dasypodius, a teacher at the Strasbourg University and one of the authors of the second astronomical clock of the Strasbourg cathedral.[524]

In 1581, Fincke published an ephemeris based on the prutenic tables (*Ephemeris coelestium motuum anni 1582, supputata ex Tabulis Prutenicis*). He returned from Strasbourg to Heidelberg and Leipzig, and moved to Basel in 1583. This is where, at the age of 22, he published his most famous work, his *Geometriæ Rotundi*,[525] an influential work on plane and spherical trigonometry based on Ramus's *Geometria* (1569).[526]

This book does in particular contain tables of sines (figure 54), tangents (figure 53) and secants (figure 55) with $R = 10^7$ and intervals of $1'$.[527] And it was precisely Fincke who coined the names "tangent" and "secant" which had not been used before. Incidentally, Viète did apparently not approve of these names.[528]

Fincke's sines do slightly differ from those of Reinhold, hence from those of Regiomontanus. It seems that Fincke made a number of small last figure adjustments to either Reinhold's or Regiomontanus's tables.[529] Given that the tangents were certainly taken from Reinhold (1554),[530] I assume that this was also the case for the sines.

As far as the tangents are concerned, we can see for instance that the last values agree with those of Reinhold, except for $89°53'$, $89°56'$, and $89°57'$. In the first case, Fincke's tangent is less accurate than Reinhold's, but in the two other cases the tangents are slightly more accurate.

Finally, Fincke's secants are the result of new computations. The val-

[524] For summaries of Fincke's life and works, see [Thorndike (1958), p. 140], [Verdonk (1971)], [Moesgaard (1972), p. 119-120] and [van Brummelen (2021), pp. 13-16]. Some authors have wrongly attributed some works of Kaspar Finck (1578-1631), who was a German theologian, to Thomas Fincke. In particular, the *Methodica tractatio doctrinae sphaericae* published in 1626, and cited by Moesgaard, is not by Thomas Fincke.

[525] [Fincke (1583)]

[526] See [Schönbeck (2004)] and [Zeller (1944), pp. 88-90].

[527] See also [Glaisher (1873), p. 42].

[528] [Zeller (1944), p. 88]

[529] [Glowatzki and Göttsche (1990), pp. 157-158]

[530] [Reinhold (1554)]

ues differ from those of Rheticus's *Canon doctrinæ triangulorum*.[531] Fincke, however, did not use Regiomontanus's sines, nor his own version to compute the secants. Instead, it seems that he computed the secants using his tangents. Fincke most certainly used the formula

$$\mathrm{Sec}\, x = \sqrt{\mathrm{Tan}^2 x + R^2}$$

to compute the secants, and when computing $\mathrm{Tan}^2 x$, he must have kept only seven or eight significant figures and rounded, although the procedure may not have been systematic. This is reminiscent of the computation of cotangents by Rheticus in 1551, and also echoes a recent suggestion by van Brummelen and Byrne for the computation of secants by Maurolico.[532] In fact, it is only after I concluded the above that I noticed that van Brummelen and Byrne claimed that Fincke used this formula to compute the secant.[533]

Fincke's tangents and secants, as well as Reinhold's tangents, are less accurate than those published by Viète in 1579. For instance, Fincke and Reinhold gave $\mathrm{Tan}\, 89°59' = \mathrm{Cot}\, 1' = 34376070815$ (for $R = 10^7$) where only the first four figures are correct. Instead, Viète gives a value whose error is about 10000 times smaller. This is so because Viète took more accurate values for the sines and understood that this was necessary in order to obtain tangents with such an accuracy.

I am giving separately a modern reconstruction of Fincke's tables.[534]

After the publication of his *Geometriæ Rotundi*, Fincke began to study medicine in Basel, Padua, Siena and Pisa. He became MD in 1587. He then returned to Denmark where he held the chair of mathematics at the University of Copenhagen from 1591 until 1602, but afterwards was more active as a physician and his mathematical activity never reached again the level of his 1583 book.

6.22 Clavius (1586)

Christopher Clavius (1537 or 1538-1612) was a German mathematician and astronomer. He was born in Bamberg and entered the Jesuit order in Rome in 1555.[535] He published his *Euclidis elementorum libri XV* (*The elements of*

531 [Rheticus (1551)]
532 [van Brummelen and Byrne (2021)]
533 [van Brummelen and Byrne (2021), p. 206]
534 [Roegel (2021m)]
535 For summaries of Clavius's life and works, see [Busard (1971b)], [Naux (1983)], [Knobloch (1988)], [Lattis (1994)] and [Sasaki (2003), p. 45-93].

Euclid) in 1574 and was a supporter of the Ptolemaic system, and at the same time a friend of Galileo. He also helped develop algebra in Italy and introduced Stifel's symbols "+" and "−."

He was also a member of the Vatican commission that accepted the proposed calendar invented by Aloysius Lilius, that is known as Gregorian calendar.

In his last years he was probably the most respected astronomer in Europe and his textbooks were used for astronomical education for over fifty years in and even out of Europe.

In 1586, Clavius published an edition of Theodosius's sphaerics,[536] in which he included tables of sines (figure 56), tangents (figure 57) and secants (figures 58 and 59) with $R = 10^7$ and at intervals of $1'$.[537]

Clavius's sines and tangents were taken from Reinhold (1554),[538] as they do not show the alterations made by Fincke.[539] But the secants instead were taken from Fincke's work (1583).[540] And in fact Clavius used the new names "tangent" and "secant" coined by Fincke. Clavius, however, corrected all the typos in the earlier editions (but not the last digit deviations).[541]

On the other hand, Clavius's table has at least one typo, namely for $\sin 89^\circ 30'$ which he gives as 9999616 instead of the correct 9999619. This error was corrected by Magini in 1592, and by Clavius himself in 1593.

Clavius's 1586 table, without the corrections of the typos, was copied by Blundeville in 1594.

6.23 Bürgi (1587)

This survey of 15th and 16th century trigonometrical tables based on Regiomontanus's work would not be complete without mentioning Jost Bürgi (1552-1632). Bürgi is well known as a (very) skillful mechanician, clockmaker and instrument maker, and also as an inventor of a table of progressions which could be used for the same purpose as logarithms.[542]

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[536] [Clavius (1586)]
[537] [Zeller (1944), pp. 91-94]
[538] [Reinhold (1554)]
[539] [Fincke (1583)]
[540] [Fincke (1583)]
[541] [Glowatzki and Göttsche (1990), p. 158]
[542] On Bürgi's table of progressions, see [Roegel (2010d)]. The most recent overview of Bürgi's work, which contains many other references, is [Staudacher (2018)]. For reasons explained in [Roegel (2017)], I do not view Bürgi as a coinventor of logarithms.

Around 1587, Bürgi devised a new way (his so-called "*Kunstweg*") to compute sines iteratively, without any geometrical construction[543] and he constructed at least two tables, one giving the sines at intervals of $2''$ and another giving the sines at intervals of $1'$. However, I believe that Bürgi did not use his new algorithm to construct these tables, and instead built up the tables by finite differences. Although Bürgi's work represents a new computation of sines, it is therefore possible that he reinvented some techniques already used by Regiomontanus, and even before in India, as mentioned earlier (see § 5).

The $2''$ table does not seem to have survived, but the $1'$ table resurfaced a few years ago. At that time, I made modern reconstructions of both tables.[544]

Bürgi's surviving sine table (figure 60) gives the sines at intervals of $1'$, with a radius $R = 60$, and to four sexagesimal places, except for the last two degrees where they are given to five and six sexagesimal places. These four sexagesimal places correspond to a radius of 10^9 with a sine usually correct to 9 decimal places.

For instance, $\mathrm{Sin}\,75^\circ$ is given as $57; 57, 19, 58, 43$ which corresponds to the decimal value 0.965925827, the correct value being $0.96592582628906\ldots$.

In contrast, Rheticus and Otho's *Opus palatinum* (1596)[545] gives the value 9659258263 for $\mathrm{Sin}\,75^\circ$, and this is correct to 10 places. Rheticus also gives the sines every $10''$.

And in 1613 Pitiscus[546] gave 96592,58262,89067, instead of the correct 96592,58262,89068. Rheticus must have had such accurate values already in the 1570s, before Bürgi, but with the exception of Rheticus, Bürgi's table was probably the most accurate sine table constructed at the end of the 16th century.

Bürgi's table can be compared to those of Fine, Schreckenfuchs and Bressieu which are also sexagesimal tables, but which are less accurate and based on Regiomontanus's sines.

543 [Roegel (2015), Roegel (2016b), Roegel (2016a)]

544 [Roegel (2016c), Roegel (2016d)]

545 [Rheticus and Otho (1596)]

546 [Pitiscus (1613)]

6.24 Gallucci (1588)

Giovanni Paolo Gallucci (1538-1621) was an Italian astronomer and translator.[547] Among his notable translations, Gallucci published in 1591 his *Della simmetria dei corpi humani*, a translation of Dürer's "Four books on human proportion" (*Vier bücher von menschlicher Proportion*, 1528). He was also a private teacher to the Venetian nobility and a founding member of the second Venetian Academy.

Gallucci's most famous works are probably his *Theatrum mundi, et temporis*[548] published in 1588, and his *Speculum Uranicum* published in 1593, both featuring some volvelles. In the *Theatrum mundi* Gallucci also included a table of sines with a radius $R = 60000$ and an interval of $1'$ (figure 61). This table was most certainly copied from Engel's table (§ 6.1), in one of the editions of the *Tabulæ directionum profectionumque* where it appears, not necessarily the 1490 edition. Gallucci uses exactly the same layout, with six half-degrees per page, but he has dropped the differences. The values seem to agree, with the exception of a few transcription errors.

6.25 Lansberge (1591)

Philip van Lansberge (1561-1632) was born in Ghent, Belgium, but in 1566 his parents moved to France and then to England, because of the religious troubles. There, he studied mathematics and theology.[549] He became a protestant minister in Antwerp in 1580 and then established himself in the Netherlands.

In 1591 he published his *Triangulorum geometriæ*[550] which is closely based on Fincke's *Geometriæ Rotundi*.[551] Lansberge did in particular include Fincke's tables of sines (figure 62), tangents (figure 63) and secants (figure 64) with $R = 10^7$ and at intervals of $1'$.[552] These tables are therefore ultimately based on those of Regiomontanus.[553] Lansberge also used the new names "tangent" and "secant" coined by Fincke. I am giving

[547] For a description of some of Gallucci's works, see [Delambre (1821), v. 1, pp. 711-714] and [Ernst (1998)].

[548] [Gallucci (1588)]

[549] For a summary of Lansberge's life and works, see [Busard (1973)].

[550] [van Lansberge (1591)]

[551] [Fincke (1583)]

[552] [Zeller (1944), pp. 94-97]

[553] [Glowatzki and Göttsche (1990), p. 159]

separately a modern reconstruction of Lansberge's tables.[554]

In 1632, Lansberge published his best known work, his *Tabulae motuum coelestium perpetuæ*, for the prediction of planetary positions. Lansberge was a follower of Copernicus and his work is based on an epicyclic theory, but he did not accept Kepler's theories.

Lansberge died that same year in Middelburg in the Netherlands.

6.26 Magini (1592)

Giovanni Antonio Magini (1555-1617) was an Italian astronomer, astrologer, cartographer, and mathematician. He was born in Padua and studied in Bologna where in 1588 he obtained one of the chairs of mathematics.[555]

Magini's chief scholarly interest was astrology and he adhered to the Ptolemaic principles. He was much more skilled in calculations than in theory and his ephemerides were useful. In 1592, he published his work *De planis triangulis*.[556] This work also contained a *Tabula tetragonica*[557] which could be used to compute the products of two numbers.

The *De planis triangulis* also contains tables of sines (figure 65), tangents (figure 66) and secants (figure 67) with $R = 10^7$ and at $1'$ intervals.[558] These tables are copied from those of Clavius (and borrow Fincke's new names),[559] and thus are ultimately based on those of Regiomontanus.[560] But contrary to Clavius, Magini has adopted a semi-quadrantal arrangement and only runs the angles up to 45°. The sines are the *sinus primus*, the cosines are the *sinus secundus*, and similarly with the tangents and secants. The value of $\mathrm{Sin}\,89^\circ30'$ is given by Magini as 9999619, which is correct, but Clavius had 9999616. It therefore appears that Magini has corrected Clavius's typo.

Magini's *De planis triangulis* also contains a *Tabula gnomonica* which is a table of arctangents similar to that of Peuerbach,[561] but where the entries vary between 0 and 1000.

In his *Primum mobile duodecim libris contentum*[562] published in 1609,

[554] [Roegel (2021n)]
[555] For a summary of Magini's life and works, see [Campedelli (1974)].
[556] [Magini (1592)]
[557] [Magini (1593)]
[558] [Zeller (1944), pp. 97-100]
[559] [Clavius (1586)]
[560] [Glowatzki and Göttsche (1990), pp. 159-160]
[561] [von Peuerbach (1516)]
[562] [Magini (1609)]

Magini gives another table with $R = 10^7$ and at $1'$ intervals, with sines, versines, tangents and secants, but the values are not those of the 1592 table. Instead, Magini took the values from Rheticus and Othos' *Opus palatinum* (1596).[563]

The later years of Magini's life were devoted to cartography and geography. He worked in particular on an atlas of Italy.

6.27 Clavius (1593)

In 1593, Clavius published his work *Astrolabium*[564] which contained a sine table with $R = 10^7$ and at intervals of $1'$ (figure 68). This table was copied from Clavius's earlier tables published in 1586,[565] but with some corrections. For instance, as mentioned previously, the value of $\mathrm{Sin}\,89°30'$ was given incorrectly in Clavius's 1586 table, and was corrected here, perhaps after the discovery of the typo by Magini.

6.28 Fale (1593)

Thomas Fale (born c1560?) was an English mathematician. Very little is known of him.[566]

In 1593, Fale published his *Horologiographia*.[567] This work, which is the only one known of him, appears to be the first book in English on sundials.[568] It contains in particular a table of sines (figure 69) which was presumably copied from Witekind's *Conformatio horologiorum sciotericorum etc.*[569] published in 1576 and with which it shares the values and the layout.[570] There are however some slight differences, and Fale gives for instance $\mathrm{Sin}\,5°3' = 8803$, when Witekind gave the correct 8802 (compare figures 46 and 69).

As observed by De Morgan and Goodwin, Fale's table may be the earliest sine table printed in England.[571]

[563] See [Rheticus and Otho (1596)] and [Glowatzki and Göttsche (1990), p. 160].

[564] [Clavius (1593)]

[565] [Clavius (1586)]

[566] [Goodwin (1889)]

[567] [Fale (1593)] Later editions were printed in 1626, 1627, 1633, 1652 and perhaps other years. A facsimile was published in 1971.

[568] [Turner (1989)]

[569] [Witekind (1576)]

[570] [Glowatzki and Göttsche (1990), p. 177]

[571] See [De Morgan (1851), p. 598] and [Goodwin (1889)].

6.29 Blundeville (1594)

Thomas Blundeville (c1522-c1606) was an English writer and mathematician, who wrote in particular on horsemanship and cartography.[572]

In 1594, he published his *Exercises, containing sixe Treatises, etc.*[573] which contain tables of sines, tangents and secants for $R = 10^7$ and at intervals of $1'$.[574] These tables are based on those published by Clavius[575] in 1586 and Blundeville explicitely mentions Clavius. They use the new names coined by Fincke in 1583. It is possible that Blundeville's tables are the first complete (that is not merely of sines) trigonometric tables published in England.[576]

Interestingly, Blundeville carried Clavius's incorrect value for $\sin 89°30'$ given in the 1586 table.

Thus Blundeville's sines are ultimately based on Regiomontanus's tables.[577]

6.30 Ceulen (1596)

Ludolph van Ceulen (1540-1610) was a German-Dutch mathematician born in Hildesheim. At some point he settled in Holland. In the 1580s and 1590s he was a fencing master as well as a mathematics teacher. He died in 1610 in Leiden.[578]

In 1596 he published his main work, *Vanden circkel etc.*[579] where he gave among other things a 20-place approximation of π.

Ceulen's book also contains tables of sines, tangents and secants for $R = 10^7$ and at intervals of $1'$ (figure 71). Ceulen's tables are certainly based on those of Lansberge[580] who is mentioned by Ceulen.[581] Ceulen uses the

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[572] For summaries of Blundeville's life and works, see [Bullen (1886)], [Jacquot (1953)], [Taylor (1954), p. 173, 331], and [de Smet (1979)].

[573] [Blundeville (1594)]

[574] [Zeller (1944), p.101]

[575] [Clavius (1586)]

[576] See [De Morgan (1851), p. 598] and [van Brummelen (2021), p. 53].

[577] [Glowatzki and Göttsche (1990), p. 160]

[578] For a summary of Ceulen's life and works, see [Struik (1971)].

[579] [Ceulen (1596)]

[580] [van Lansberge (1591)]

[581] See [Ceulen (1596), f° 25] which mentions Regiomontanus, Reinhold, Rheticus, Clavius and Lansberge, but not Fincke. Glowatzki and Göttsche only relate Ceulen's tables to Regiomontanus [Glowatzki and Göttsche (1990), pp. 160-161].

new names introduced by Fincke in 1583.

Ceulen departed somewhat from the previous tables, in that he did not separate sines, tangents and secants in different tables, but put them together, for a range of two degrees, on each page.

6.31 Rheticus/Otho (1596)

After the publication of his *Canon doctrinæ triangulorum* in 1551 which was based on Regiomontanus's tables,[582] Rheticus (1514-1574)[583] continued to work on a more extensive project, where the six trigonometric functions would be given every $10''$ and for a larger radius. As observed by Zeller,[584] "Rheticus built his trigonometry on the foundation established by Regiomontanus."

Rheticus embarked on totally new computations, but his work was only completed after his death by Lucius Valentinus Otho (c1545-1603) and published in 1596 in the *Opus palatinum*[585] (figure 72). Otho had met Rheticus in 1573 and Rheticus had asked him to complete his work.

With the exception of Bürgi's work, this was the first new computation of trigonometric values in the 16th century, since most of the trigonometric tables printed in the 16th century actually use values or computations inherited from Regiomontanus's tables[586] (see figure 13).

However, even a cursory examination of the *Opus palatinum* reveals that it contains two overlapping tables. On one hand, there is a table giving all six functions with a radius $R = 10^{10}$ and an interval of $10''$. This table spans 540 pages. On the other hand, there is a table giving only the cosecants and cotangents, with a radius $R = 10^7$ and the same interval of $10''$. This second table spans 180 pages. One might expect the second table to be an abridgement of the first, but this is not the case, as is apparent when comparing the first values of the cosecants and cotangents. These two tables obviously correspond to two different computations. This has actually been noticed before, and Glaisher wrote that "there seems no reason why it should have been printed at all, as the great ten-decimal canon completely supersedes it."[587]

[582] [Rheticus (1551)]

[583] [Rosen (1975b)]

[584] [Zeller (1944), p. 62]

[585] See [Rheticus and Otho (1596)] and [Roegel (2010e)]. See also [Glaisher (1873), p. 43] and [van Brummelen (2009), pp. 273-282] for descriptions of the *Opus palatinum*.

[586] [Glowatzki and Göttsche (1990), p. i]

[587] [Glaisher (1873), p. 43]

I have therefore assumed that the shorter table is in fact an older table, perhaps computed by Rheticus around 1560.[588] I believe that after his *Canon doctrinæ triangulorum* (1551), which already used $R = 10^7$ and an interval of $10'$, Rheticus decided first to compute the functions with an interval 60 times smaller, that is $10''$, but with the same radius. This is what I have shown in figure 13.

It is in fact easy to see what were the computations in this first attempt at a $10''$ table (and it would consequently be rather straightforward to complete this table with those for the sines, cosines, tangents and secants, which presumably existed). We can observe that the cosecants at $1'$ intervals were merely obtained by the fractions $10^{14}/2909$, $10^{14}/5818$, $10^{14}/8727$, $10^{14}/11635$, $10^{14}/14544$, etc. In other words, Rheticus merely used the sines found in Regiomontanus's table, apparently sometimes with slight adjustments (as for $\mathrm{Csc}\,4'$ or $\mathrm{Csc}\,10'$), but adjustments that did not always produce more accurate results (as for $\mathrm{Csc}\,4'$). It is possible that some of these "adjustments" were in fact typos. Rheticus did the same for the cotangents, taking the sines from Regiomontanus. It seems that the adjustments made for the sines in the case of cosecants were also used for the cotangents, but this should be checked throughout the table.

For the $10''$ intervals, Rheticus merely interpolated the sines. For instance, $\mathrm{Csc}\,10''$ is obtained using $\mathrm{Sin}\,10'' = 485$, $\mathrm{Csc}\,20''$ uses $\mathrm{Sin}\,20'' = 970$, and so on. There may be the usual typos, such as for $\mathrm{Cot}\,10''$ which is given as 206085546390, but should be 206185546392, and was merely obtained by dividing 9999999 by 485.

Sometime after that first computation, Rheticus must have realized that the cosecants and cotangents could not be computed accurately with such a scheme, because Regiomontanus's sines were not accurate enough for small angles. He must therefore have decided to construct a larger table, and he computed this time the sines and cosines with a radius of 10^{15} and an interval of $10''$. This was probably done around 1570. This work was used to produce the table for $R = 10^{10}$ published in 1596. However, the cosecants and cotangents were not computed using these accurate values of the sines, but those from the *Opus palatinum* itself. For instance, for $\mathrm{Csc}\,1'$, Rheticus (or Otho) used the sine value 2908882, instead of Regiomontanus's 2909, but not the more accurate 290888204563 in the $R = 10^{15}$ table.

When the *Opus palatinum* was published, Otho must have decided to

[588] This concurs with De Morgan who considered that "it is clearly nothing but a previous attempt made before the larger plan was resolved on."([De Morgan (1851), p. 599] and [Glaisher (1873), p. 43])

include Rheticus's earlier computation of cosecants and cotangents, but the reason for publishing it remains unclear, as Otho must have realized that these first calculations were inadequate. On the other hand, it was much more difficult for him than for us to realize it, and he perhaps decided to include these tables in case they contained some valuable results.

Of course, computing the cosecants and cotangents with the sines given in the *Opus palatinum* is still not enough for small angles, as the sines are still not sufficiently accurate. This led Bartholomaeus Pitiscus (1561-1613) to correct the *Opus palatinum* and to publish Rheticus's sine table with $R = 10^{15}$ (figure 73) as well as other tables that he computed himself in his *Thesaurus mathematicus*.[589] Incidentally, Pitiscus was the one who first coined the word "trigonometry."

7 Conclusion

This marks the end of our journey through 15th and 16th century fundamental trigonometric tables. But this end is also a beginning. Rheticus's *Opus palatinum* and its amendments by Pitiscus were the start of a new era and these tables would themselves last until the 20th century. And the first years of the 17th century were the place of a bifurcation. On one hand trigonometric tables would continue their path, with little changes beyond Rheticus's masterpiece,[590] and on the other hand they made their foray into the world of logarithms, as if logarithms naturally absorbed the trigonometric functions.[591]

Logarithms first appeared in public in 1614, and they started in association with sines. Indeed, when Napier published[592] the first table of logarithms in 1614, it was a table of logarithms of sines, and these sines were either those of Fincke[593] or those of Lansberge.[594] Napier's work was therefore based again on that of Regiomontanus, and not yet on Rheticus's work.

[589] See [Pitiscus (1613)] and [Roegel (2010f)].

[590] De Morgan wrote that "There have been no trigonometrical tables of note published since the invention of logarithms, except those which contain logarithms" [De Morgan (1842), p. 497].

[591] See [van Brummelen (2021), pp. 62-109] for a recent survey of the development of logarithms as a continuation of trigonometry.

[592] See [Napier (1614)] and [Roegel (2010g)].

[593] [Fincke (1583)]

[594] [van Lansberge (1591)]

Three years later, the decimal logarithms were introduced by Briggs,[595] and expanded in 1624 and 1628.[596] They were however unrelated to trigonometric tables.

Edmund Gunter was the first to compute and publish tables of decimal logarithms of sines and tangents in 1620.[597] His tables gave the logarithms to 8 places and were probably based on Rheticus's *Opus palatinum* or Pitiscus's *Thesaurus mathematicus*.

In 1633, Henry Gellibrand completed and published Henry Briggs's *Trigonometria Britannica*[598] which was a large table of trigonometric functions and decimal logarithms of trigonometric functions. Briggs's table was in fact the result of a new computation of sines, tangents and secants,[599] in which he divided the degree in 100 parts. The sines were computed with $R = 10^{15}$ and the tangents and secants with $R = 10^{10}$. Briggs's trigonometric functions are not based on earlier tables, not even on those of Rheticus's *Opus palatinum*.

The same year 1633, Adriaan Vlacq independently published his *Trigonometria artificialis*.[600] This work gives only the logarithms of sines, cosines, tangents and cotangents, and not the trigonometric functions themselves. But contrary to Briggs, Vlacq computed his logarithms using the values given by Rheticus in his *Opus palatinum*. It was Vlacq's table and not Briggs's table which had the greatest offspring, and was many times reprinted, simplified and adapted until the 20th century.

[595] [Briggs (1617)]
[596] [Briggs (1624), Vlacq (1628)]
[597] [Gunter (1620)]
[598] [Briggs and Gellibrand (1633)]
[599] [Glowatzki and Göttsche (1990), p. ii]
[600] [Vlacq (1633)]

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Tabula secunda

G	Numerus	G	Numerus	G	Numerus
0	00000	31	60086	61	180402
1	1745	32	62486	62	188075
2	3492	33	64940	63	196263
3	5240	34	67452	64	205034
4	6992	35	70022	65	214450
5	8748	36	72654	66	224607
6	10511	37	75356	67	235583
7	12278	38	78129	68	247513
8	14053	39	80978	69	260511
9	15838	40	83909	70	274753
10	17633	41	86929	71	290422
11	19439	42	90040	72	307767
12	21256	43	93254	73	327088
13	23087	44	96571	74	348748
14	24932	45	100000	75	373211
15	26794	46	103551	76	401089
16	28674	47	107236	77	433148
17	30573	48	111062	78	470453
18	32492	49	115037	79	514438
19	34433	50	119177	80	567118
20	36396	51	123491	81	631377
21	38387	52	127994	82	711569
22	40402	53	132704	83	814456
23	42448	54	137639	84	951387
24	44522	55	142813	85	1143131
25	46631	56	148253	86	1430203
26	48772	57	153987	87	1908217
27	50952	58	160035	88	2863563
28	53170	59	166429	89	5729796
29	55432	60	173207	90	Infinitū
30	57734				

Figure 15: Excerpt of Regiomontanus's table of tangents [Regiomontanus (1490)] (source: The Budapest University of Technology and Economics, 85.211, www.manuscriptorium.com).

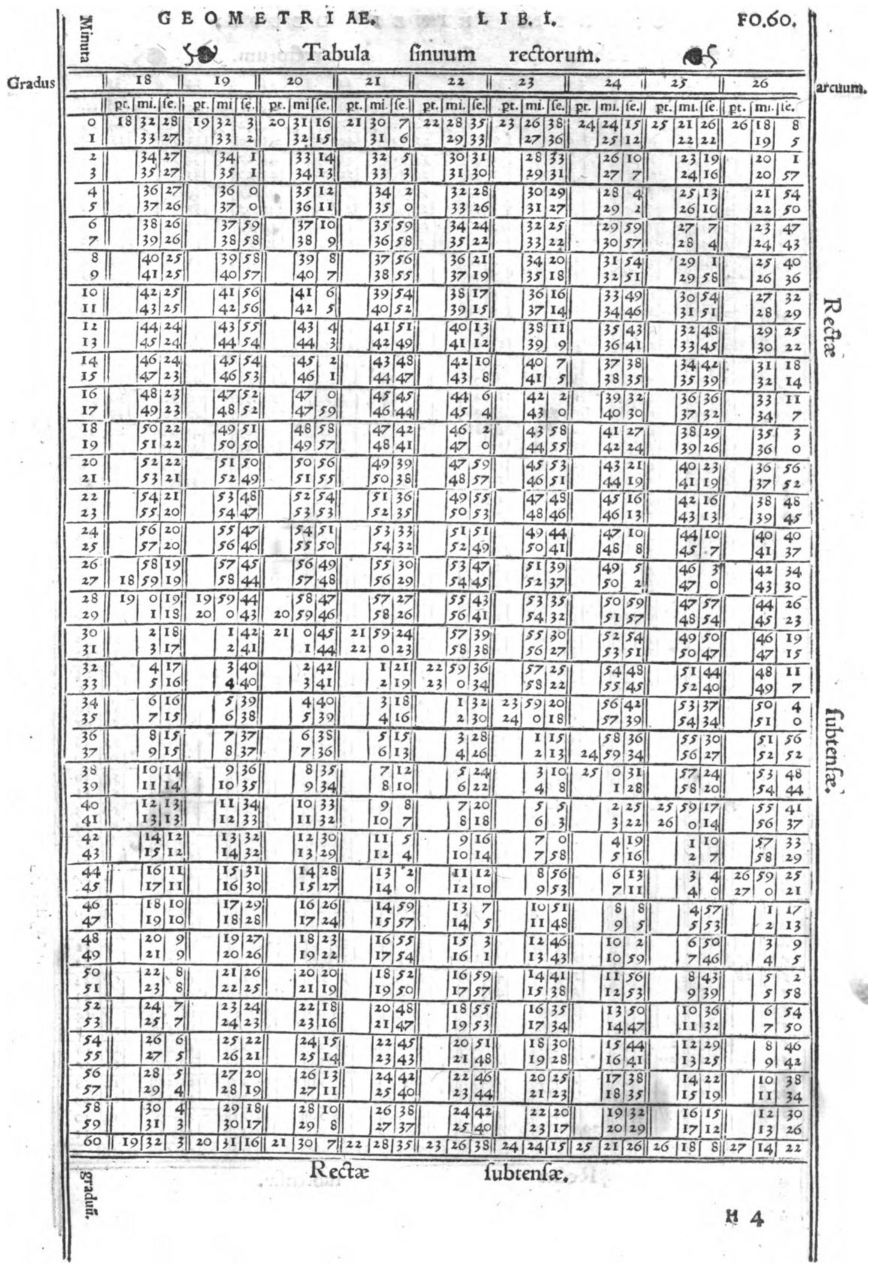
Tabella.

g	0		1		2		3		4		5	
m	ptes		ptes		ptes		ptes		ptes		ptes	
1	17	291	1064	291	2111	291	3157	290	4202	290	5246	290
2	34		82		28		75		20		64	
3	52		99		46		92		37		5281	
4	69		1116		2163		3209		4255		98	
5	87		34		81		27		72		5316	
6	104		1151		98		44		89		33	
7	22		69		2216		3262		4307		51	
8	39		86		33		79		24		5368	
9	157		1204		50		97		42		85	
10	74		21		2268		3314		4359		5403	
11	91		39		85		31		76		20	
12	209		1256		2303		49		94		37	
13	26		74		20		3366		4411		5455	
14	44		91		38		84		29		72	
15	61		1308		2355		3401		46		90	
16	279		26		73		18		4463		5507	
17	96		43		90		36		81		24	
18	314		1361		2407		3453		98		42	
19	31		78		25		71		4516		5559	
20	49		96		42		88		33		77	
21	366		1413		2460		3506		50		94	
22	83		31		77		23		4568		5611	
23	401		48		95		40		85		29	
24	18		1465		2512		3558		4603		46	
25	36		83		29		75		20		5663	
26	453		1500		47		93		37		81	
27	71		18		2564		3610		4655		98	
28	88		35		82		28		72		5716	
29	506		1553		99		45		90		33	
30	23		70		2617		62		4707		50	

Figure 16: Excerpt of Engel's table of sines with $R = 60000$ [Regiomontanus (1490)] (source: The Budapest University of Technology and Economics, 85.211, www.manuscriptorium.com).

Arcus G.	M.	Sinus Partes	Arcus G.	M.	Sinus Partes	Arcus G.	M.	Sinus Partes
45	20	71120	50	0	76604	55	0	81915
	30	71325		10	76790		10	82082
	40	71529		20	76977		20	82247
45	50	71730		30	77162		30	82412
46	0	71934		40	77347		40	82577
	10	72135	50	50	77530	55	50	82740
	20	72337	51	0	77714	56	0	82904
	30	72537		10	77897		10	83065
	40	72737		20	78079		20	83227
46	50	72934		30	78260		30	83389
47	0	73135		40	78440		40	83549
	10	73334	51	50	78620	56	50	83707
	20	73530	52	0	78800	57	0	83867
	30	73727		10	78979		10	84025
	40	73934		20	79157		20	84119
47	50	74119		30	79334		30	84339
48	0	74314		40	79512		40	84495
	10	74509	52	50	79687	57	50	84650
	20	74702	53	0	79864	58	0	84804
	30	74895		10	80037		10	84959
	40	75087		20	80212		20	85112
48	50	75279		30	80385		30	85264
49	0	75432		40	80559		40	85415
	10	75660	53	50	80730	58	50	85565
	20	75850	54	0	80902	59	0	85717
	30	76040		10	81072		10	85865
	40	76229		20	81242		20	86014
49	50	76417		30	81410		30	86162
				40	81580		40	86310
			54	50	81747	59	50	86457

Figure 17: Excerpt of Gaurico's table of sines, with $R = 100000$ [Regiomontanus (1524)] (source: Google books).



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Tabula ſinuum rectorum.

Minuta / Gradus	18			19			20			21			22			23			24			25			26		
	pt.	mi.	ſe.	pt.	mi	ſe.	pt.	mi	ſe.	pt.	mi.	ſe.	pt.	mi.	ſe.	pt.	mi.	ſe.	pt.	mi.	ſe.	pt.	mi.	ſe.	pt.	mi.	ſe.
0	18	32	28	19	32	3	20	31	16	21	30	7	22	28	35	23	26	38	24	24	15	25	21	26	26	18	8
1		33	27		33	2		32	15		31	6		29	33		27	36		25	12		22	22		19	5
2		34	27		34	1		33	14		32	5		30	31		28	33		26	10		23	19		20	1
3		35	27		35	1		34	13		33	3		31	30		29	31		27	7		24	16		20	57
4		36	27		36	0		35	12		34	2		32	28		30	29		28	4		25	13		21	54
5		37	26		37	0		36	11		35	0		33	26		31	27		29	2		26	10		22	50
6		38	26		37	59		37	10		35	59		34	24		32	25		29	59		27	7		23	47
7		39	26		38	58		38	9		36	58		35	22		33	22		30	57		28	4		24	43
8		40	25		39	58		39	8		37	56		36	21		34	20		31	54		29	1		25	40
9		41	25		40	57		40	7		38	55		37	19		35	18		32	51		29	58		26	36
10		42	25		41	56		41	6		39	54		38	17		36	16		33	49		30	54		27	32
11		43	25		42	56		42	5		40	52		39	15		37	14		34	46		31	51		28	29
12		44	24		43	55		43	4		41	51		40	13		38	11		35	43		32	48		29	25
13		45	24		44	54		44	3		42	49		41	12		39	9		36	41		33	45		30	22
14		46	24		45	54		45	2		43	48		42	10		40	7		37	38		34	42		31	18
15		47	23		46	53		46	1		44	47		43	8		41	5		38	35		35	39		32	14
16		48	23		47	52		47	0		45	45		44	6		42	2		39	32		36	36		33	11
17		49	23		48	52		47	59		46	44		45	4		43	0		40	30		37	32		34	7
18		50	22		49	51		48	58		47	42		46	2		43	58		41	27		38	29		35	3
19		51	22		50	50		49	57		48	41		47	0		44	55		42	24		39	26		36	0
20		52	22		51	50		50	56		49	39		47	59		45	53		43	21		40	23		36	56
21		53	21		52	49		51	55		50	38		48	57		46	51		44	19		41	19		37	52
22		54	21		53	48		52	54		51	36		49	55		47	48		45	16		42	16		38	48
23		55	20		54	47		53	53		52	35		50	53		48	46		46	13		43	13		39	45
24		56	20		55	47		54	51		53	33		51	51		49	44		47	10		44	10		40	40
25		57	20		56	46		55	50		54	32		52	49		50	41		48	8		45	7		41	37
26		58	19		57	45		56	49		55	30		53	47		51	39		49	5		46	3		42	34
27	18	59	19		58	44		57	48		56	29		54	45		52	37		50	2		47	0		43	30
28	19	0	19	19	59	44		58	47		57	27		55	43		53	35		50	59		47	57		44	26
29		1	18	20	0	43	20	59	46		58	26		56	41		54	32		51	57		48	54		45	23
30		2	18		1	42	21	0	45	21	59	24		57	39		55	30		52	54		49	50		46	19
31		3	17		2	41		1	44	22	0	23		58	38		56	27		53	51		50	47		47	15
32		4	17		3	40		2	42		1	21	22	59	36		57	25		54	48		51	44		48	11
33		5	16		4	40		3	41		2	19	23	0	34		58	22		55	45		52	40		49	7
34		6	16		5	39		4	40		3	18		1	32	23	59	20		56	42		53	37		50	4
35		7	15		6	38		5	39		4	16		2	30	24	0	18		57	39		54	34		51	0
36		8	15		7	37		6	38		5	15		3	28		1	15		58	36		55	30		51	56
37		9	15		8	37		7	36		6	13		4	26		2	13	24	59	34		56	27		52	52
38		10	14		9	36		8	35		7	12		5	24		3	10	25	0	31		57	24		53	48
39		11	14		10	35		9	34		8	10		6	22		4	8		1	28		58	20		54	44
40		12	13		11	34		10	33		9	8		7	20		5	5		2	25	25	59	17		55	41
41		13	13		12	33		11	32		10	7		8	18		6	3		3	22	26	0	14		56	37
42		14	12		13	32		12	30		11	5		9	16		7	0		4	19		1	10		57	33
43		15	12		14	32		13	29		12	4		10	14		7	58		5	16		2	7		58	29
44		16	11		15	31		14	28		13	2		11	12		8	56		6	13		3	4	26	59	25
45		17	11		16	30		15	27		14	0		12	10		9	53		7	11		4	0	27	0	21
46		18	10		17	29		16	26		14	59		13	7		10	51		8	8		4	57		1	17
47		19	10		18	28		17	24		15	57		14	5		11	48		9	5		5	53		2	13
48		20	9		19	27		18	23		16	55		15	3		12	46		10	2		6	50		3	9
49		21	9		20	26		19	22		17	54		16	1		13	43		10	59		7	46		4	5
50		22	8		21	26		20	20		18	52		16	59		14	41		11	56		8	43		5	2
51		23	8		22	25		21	19		19	50		17	57		15	38		12	53		9	39		5	58
52		24	7		23	24		22	18		20	48		18	55		16	35		13	50		10	36		6	54
53		25	7		24	23		23	16		21	47		19	53		17	34		14	47		11	32		7	50
54		26	6		25	22		24	15		22	45		20	51		18	30		15	44		12	29		8	46
55		27	5		26	21		25	14		23	43		21	48		19	28		16	41		13	25		9	42
56		28	5		27	20		26	13		24	42		22	46		20	25		17	38		14	22		10	38
57		29	4		28	19		27	11		25	40		23	44		21	23		18	35		15	19		11	34
58		30	4		29	18		28	10		26	38		24	42		22	20		19	32		16	15		12	30
59		31	3		30	17		29	8		27	37		25	40		23	17		20	29		17	12		13	26
60	19	32	3	20	31	16	21	30	7	22	28	35	23	26	38	24	24	15	25	21	26	26	18	8	27	14	22

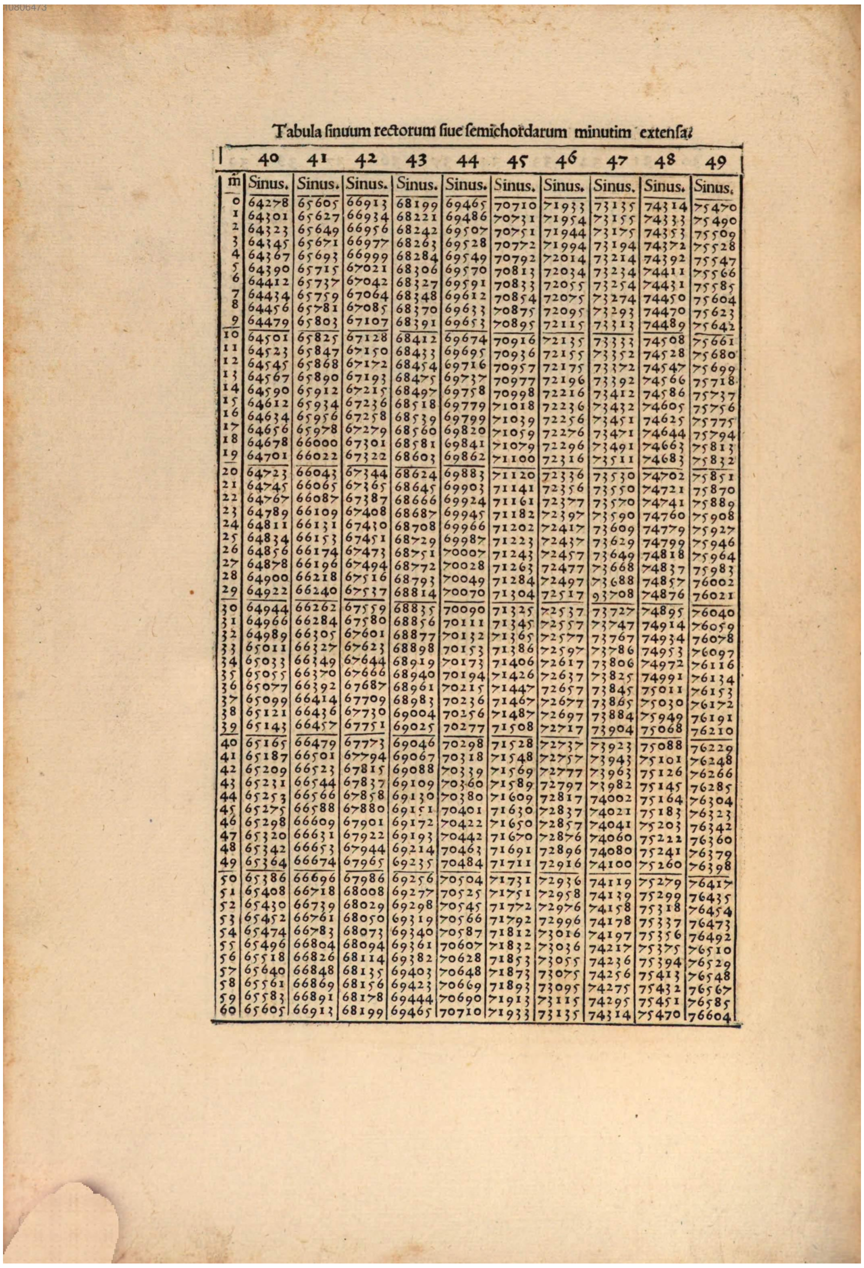
arcuum. Rectæ ſubtenſæ.

Rectæ ſubtenſæ.

graduũ.

H 4

Figure 18: Excerpt of Fine's table of sines [Fine (1530)] (source: Google books).



Tabula sinuum rectorum siue semichordarum minutim extensa.

m	40 Sinus.	41 Sinus.	42 Sinus.	43 Sinus.	44 Sinus.	45 Sinus.	46 Sinus.	47 Sinus.	48 Sinus.	49 Sinus.
0	64278	65605	66913	68199	69465	70710	71933	73135	74314	75470
1	64301	65627	66934	68221	69486	70731	71954	73155	74333	75490
2	64323	65649	66956	68242	69507	70751	71944	73175	74353	75509
3	64345	65671	66977	68263	69528	70772	71994	73194	74372	75528
4	64367	65693	66999	68284	69549	70792	72014	73214	74392	75547
5	64390	65715	67021	68306	69570	70813	72034	73234	74411	75566
6	64412	65737	67042	68327	69591	70833	72055	73254	74431	75585
7	64434	65759	67064	68348	69612	70854	72075	73274	74450	75604
8	64456	65781	67085	68370	69633	70875	72095	73293	74470	75623
9	64479	65803	67107	68391	69653	70895	72115	73313	74489	75642
10	64501	65825	67128	68412	69674	70916	72135	73333	74508	75661
11	64523	65847	67150	68433	69695	70936	72155	73352	74528	75680
12	64545	65868	67172	68454	69716	70957	72175	73372	74547	75699
13	64567	65890	67193	68475	69737	70977	72196	73392	74566	75718
14	64590	65912	67215	68497	69758	70998	72216	73412	74586	75737
15	64612	65934	67236	68518	69779	71018	72236	73432	74605	75756
16	64634	65956	67258	68539	69799	71039	72256	73451	74625	75775
17	64656	65978	67279	68560	69820	71059	72276	73471	74644	75794
18	64678	66000	67301	68581	69841	71079	72296	73491	74663	75813
19	64701	66022	67322	68603	69862	71100	72316	73511	74683	75832
20	64723	66043	67344	68624	69883	71120	72336	73530	74702	75851
21	64745	66065	67365	68645	69903	71141	72356	73550	74721	75870
22	64767	66087	67387	68666	69924	71161	72377	73570	74741	75889
23	64789	66109	67408	68687	69945	71182	72397	73590	74760	75908
24	64811	66131	67430	68708	69966	71202	72417	73609	74779	75927
25	64834	66153	67451	68729	69987	71223	72437	73629	74799	75946
26	64856	66174	67473	68751	70007	71243	72457	73649	74818	75964
27	64878	66196	67494	68772	70028	71263	72477	73668	74837	75983
28	64900	66218	67516	68793	70049	71284	72497	73688	74857	76002
29	64922	66240	67537	68814	70070	71304	72517	93708	74876	76021
30	64944	66262	67559	68835	70090	71325	72537	73727	74895	76040
31	64966	66284	67580	68856	70111	71345	72557	73747	74914	76059
32	64989	66305	67601	68877	70132	71365	72577	73767	74934	76078
33	65011	66327	67623	68898	70153	71386	72597	73786	74953	76097
34	65033	66349	67644	68919	70173	71406	72617	73806	74972	76116
35	65055	66370	67666	68940	70194	71426	72637	73825	74991	76134
36	65077	66392	67687	68961	70215	71447	72657	73845	75011	76153
37	65099	66414	67709	68983	70236	71467	72677	73865	75030	76172
38	65121	66436	67730	69004	70256	71487	72697	73884	75949	76191
39	65143	66457	67751	69025	70277	71508	72717	73904	75068	76210
40	65165	66479	67773	69046	70298	71528	72737	73923	75088	76229
41	65187	66501	67794	69067	70318	71548	72757	73943	75101	76248
42	65209	66523	67815	69088	70339	71569	72777	73963	75126	76266
43	65231	66544	67837	69109	70360	71589	72797	73982	75145	76285
44	65253	66566	67858	69130	70380	71609	72817	74002	75164	76304
45	65275	66588	67880	69151	70401	71630	72837	74021	75183	76323
46	65298	66609	67901	69172	70422	71650	72857	74041	75203	76342
47	65320	66631	67922	69193	70442	71670	72876	74060	75222	76360
48	65342	66653	67944	69214	70463	71691	72896	74080	75241	76379
49	65364	66674	67965	69235	70484	71711	72916	74100	75260	76398
50	65386	66696	67986	69256	70504	71731	72936	74119	75279	76417
51	65408	66718	68008	69277	70525	71751	72958	74139	75299	76435
52	65430	66739	68029	69298	70545	71772	72976	74158	75318	76454
53	65452	66761	68050	69319	70566	71792	72996	74178	75337	76473
54	65474	66783	68073	69340	70587	71812	73016	74197	75356	76492
55	65496	66804	68094	69361	70607	71832	73036	74217	75375	76510
56	65518	66826	68114	69382	70628	71853	73055	74236	75394	76529
57	65640	66848	68135	69403	70648	71873	73075	74256	75413	76548
58	65561	66869	68156	69423	70669	71893	73095	74275	75432	76567
59	65583	66891	68178	69444	70690	71913	73115	74295	75451	76585
60	65605	66913	68199	69465	70710	71933	73135	74314	75470	76604

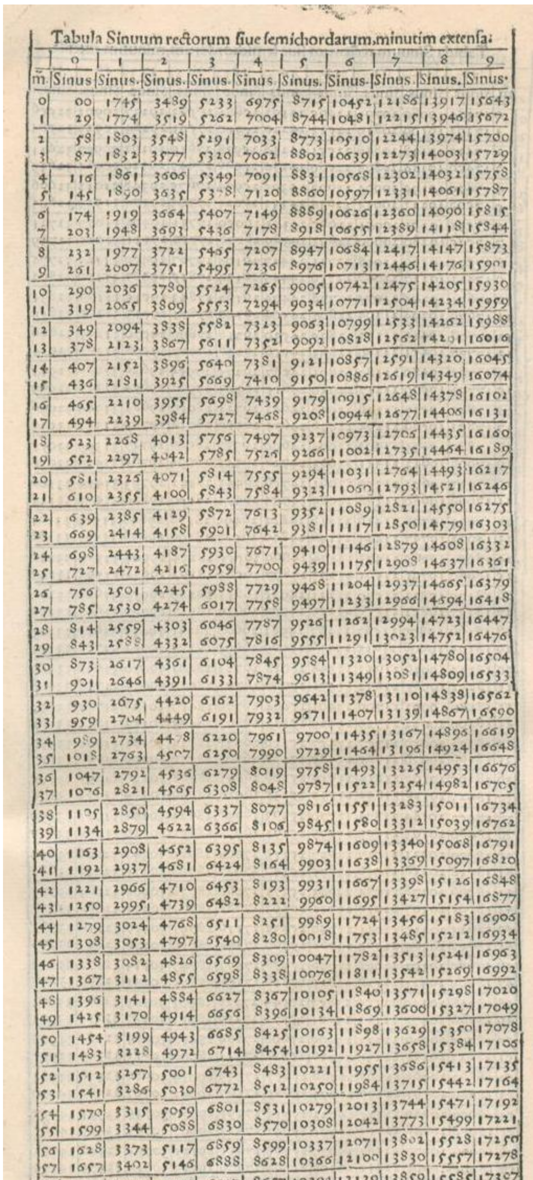
Figure 19: An excerpt of Apian's table of sines [Apian (1533)] (source: Bayerische Staatsbibliothek).

Tabula Sinuū rectorū siue semichordarū minutim extensa.

	0	1	2	3	4	5	6	7	8	9
m.	Sin9.	Sin9.	Sin9.	Sin9	Sin9.	Sinus.	Sinus.	Sinus.	Sinus.	Sinus.
0	00	1745	3489	5233	6975	8715	10452	12186	13917	15643
1	29	1774	3519	5262	7004	8744	10481	12215	13946	15672
2	58	1803	3548	5291	7033	8773	10510	12244	13974	15700
3	87	1832	3577	5320	7062	8802	10539	12273	14003	15729
4	116	1861	3606	5349	7091	8831	10568	12302	14032	15758
5	145	1890	2635	5378	7120	8860	10597	12331	14061	15787
6	174	1919	3664	5407	7149	8889	10626	12360	14090	15815
7	203	1948	3693	5436	7178	8918	10655	12389	14118	15844
8	232	1977	3722	5465	7207	8947	10684	12417	14147	15873
9	261	2007	3751	5495	7236	8976	10713	12446	14176	15901
10	290	2036	3780	5524	7265	9005	10742	12475	14205	15930
11	319	2065	3809	5553	7294	9034	10771	12504	14234	15959
12	349	2094	3838	5582	7323	9063	10799	12533	14262	15988
13	378	2123	3867	5611	7352	9092	10828	12562	14291	16016
14	407	2152	3896	5640	7381	9121	10857	12591	14320	16045
15	436	2181	3925	5669	7410	9150	10886	12619	14349	16074
16	465	2210	3955	5698	7439	9179	10915	12648	14378	16102
17	494	2239	3984	5727	7468	9208	10944	12677	14406	16131
18	523	2268	4013	5756	7497	9237	10973	12706	14435	16160
19	552	2297	4042	5785	7526	9266	11002	12735	14464	16189
20	581	2326	4071	5814	7555	9294	11031	12764	14493	16217
21	610	2355	4100	5843	7584	9323	11060	12793	14521	16246
22	639	2385	4129	5872	7613	9352	11089	12821	14550	16275
23	669	2414	4158	5901	7642	9381	11117	12850	14579	16303
24	698	2443	4187	5930	7671	9410	11146	12879	14608	16332
25	727	2472	4216	5959	7700	9439	11175	12908	14637	16361
26	756	2501	4245	5988	7729	9468	11204	12937	14665	16389
27	785	2530	4274	6017	7758	9497	11233	12966	14694	16418
28	814	2559	4303	6046	7787	9526	11262	12994	14723	16447
29	843	2588	4332	6075	7816	9555	11291	13023	14752	16476
30	873	2617	4361	6104	7845	9584	11320	13052	14780	16504
31	901	2646	4391	6133	7874	9613	11349	13081	14809	16533
32	930	2675	4420	6162	7903	9642	11378	13110	14838	16562
33	959	2704	4449	6191	7932	9671	11407	13139	14867	16590
34	989	2734	4478	6220	7961	9700	11435	13167	14896	16619
35	1018	2763	4507	6250	7990	9729	11464	13196	14924	16648
36	1047	2792	4536	6279	8019	9758	11493	13225	14953	16676
37	1076	2821	4565	6308	8048	9787	11522	13254	14982	16705
38	1105	2850	4594	6337	8077	9816	11551	13283	15011	16734
39	1134	2879	4622	6366	8106	9845	11580	13312	15039	16762
40	1163	2908	4652	6395	8135	9874	11609	13340	15068	16791
41	1192	2937	4681	6424	8164	9903	11638	13369	15067	16820
42	1221	2966	4710	6453	8193	9931	11667	13398	15126	16848
43	1250	2995	4739	6482	8222	9960	11695	13427	15154	16877
44	1279	3024	4768	6511	8251	9989	11724	13456	15183	16906
45	1308	3053	4797	6540	8280	10018	11753	13485	15212	16934
46	1338	3082	4826	6569	8309	10047	11782	13513	15241	16963
47	1367	3112	4855	6598	8338	10076	11811	13542	15269	16992
48	1496	3141	4884	6627	8367	10105	11840	13571	15298	17020
49	1425	3170	4914	6656	8396	10134	11869	13600	15327	17049
50	1454	3199	4943	6685	8425	10163	11898	13629	15356	17078
51	1483	3228	4972	6714	8454	10192	11927	13658	15384	17105
52	1512	3257	5001	6743	8483	10221	11955	13686	15413	17135
53	1541	3286	5030	6772	8512	10250	11984	13715	15442	17164
54	1570	3315	5059	6801	8531	10279	12013	13744	15471	17192
55	1599	3344	5088	6830	8570	10308	12042	13773	15499	17221
56	1628	3373	5117	6859	8599	10337	12071	13802	15528	17250
57	1657	3402	5146	6888	8628	10366	12100	13830	15557	17278
58	1687	3431	5175	6917	8657	10394	12129	13859	15585	17307
59	1716	3460	5204	6946	8686	10423	12158	13888	15614	17336
60	1745	3489	5233	6975	8715	10452	12186	13917	15643	17364

b 2

Figure 20: An excerpt of Apian's table of sines [Apian (1534)].



Tabula Sinuum rectorum siue semichordarum, minutim extensa:

m.	0 Sinus	1 Sinus	2 Sinus	3 Sinus	4 Sinus	5 Sinus	6 Sinus	7 Sinus	8 Sinus	9 Sinus
0	00	1745	3489	5233	6975	8715	10452	12186	13917	15643
1	29	1774	3519	5262	7004	8744	10481	12215	13946	15672
2	58	1803	3548	5291	7033	8773	10510	12244	13974	15700
3	87	1832	3577	5320	7062	8802	10639	12273	14003	15729
4	116	1861	3606	5349	7091	8831	10568	12302	14032	15758
5	145	1890	3635	5378	7120	8860	10597	12331	14061	15787
6	174	1919	3664	5407	7149	8889	10626	12360	14090	15815
7	203	1948	3693	5436	7178	8918	10655	12389	14118	15844
8	232	1977	3722	5465	7207	8947	10684	12417	14147	15873
9	261	2007	3751	5495	7236	8976	10713	12446	14176	15901
10	290	2036	3780	5524	7265	9005	10742	12475	14205	15930
11	319	2065	3809	5553	7294	9034	10771	12504	14234	15959
12	349	2094	3838	5582	7323	9063	10799	12533	14262	15988
13	378	2123	3867	5611	7352	9092	10828	12562	14291	16016
14	407	2152	3896	5640	7381	9121	10857	12591	14320	16045
15	436	2181	3925	5669	7410	9150	10886	12619	14349	16074
16	465	2210	3955	5698	7439	9179	10915	12648	14378	16102
17	494	2239	3984	5727	7468	9208	10944	12677	14406	16131
18	523	2268	4013	5756	7497	9237	10973	12706	14435	16160
19	552	2297	4042	5785	7526	9266	11002	12735	14464	16189
20	581	2325	4071	5814	7555	9294	11031	12764	14493	16217
21	610	2355	4100	5843	7584	9323	11060	12793	14521	16246
22	639	2385	4129	5872	7613	9352	11089	12821	14550	16275
23	669	2414	4158	5901	7642	9381	11117	12850	14579	16303
24	698	2443	4187	5930	7671	9410	11146	12879	14608	16332
25	727	2472	4216	5959	7700	9439	11175	12908	14637	16361
26	756	2501	4245	5988	7729	9468	11204	12937	14665	16379
27	785	2530	4274	6017	7758	9497	11233	12966	14594	16418
28	814	2559	4303	6046	7787	9526	11262	12994	14723	16447
29	843	2588	4332	6075	7816	9555	11291	13023	14752	16476
30	873	2617	4361	6104	7845	9584	11320	13052	14780	16504
31	901	2646	4391	6133	7874	9613	11349	13081	14809	16533
32	930	2675	4420	6162	7903	9642	11378	13110	14838	16562
33	959	2704	4449	6191	7932	9571	11407	13139	14867	16590
34	989	2734	4478	6220	7961	9700	11435	13167	14896	16619
35	1018	2763	4507	6250	7990	9729	11464	13196	14924	16648
36	1047	2792	4536	6279	8019	9758	11493	13225	14953	16676
37	1076	2821	4565	6308	8048	9787	11522	13254	14982	16705
38	1105	2850	4594	6337	8077	9816	11551	13283	15011	16734
39	1134	2879	4622	6366	8106	9845	11580	13312	15039	16762
40	1163	2908	4552	6395	8135	9874	11609	13340	15068	16791
41	1192	2937	4681	6424	8164	9903	11638	13369	15097	16820
42	1221	2966	4710	6453	8193	9931	11667	13398	15126	16848
43	1250	2995	4739	6482	8222	9960	11695	13427	15154	16877
44	1279	3024	4768	6511	8251	9989	11724	13456	15183	16906
45	1308	3053	4797	5540	8280	10018	11753	13485	15212	16934
46	1338	3082	4826	6569	8309	10047	11782	13513	15241	16963
47	1367	3112	4855	6598	8338	10076	11811	13542	15269	16992
48	1396	3141	4884	6627	8367	10105	11840	13571	15298	17020
49	1425	3170	4914	6656	8396	10134	11869	13600	15327	17049
50	1454	3199	4943	6685	8425	10163	11898	13629	15350	17078
51	1483	3228	4972	6714	8454	10192	11927	13658	15384	17106
52	1512	3257	5001	6743	8483	10221	11955	13686	15413	17135
53	1541	3286	5030	6772	8512	10250	11984	13715	15442	17164
54	1570	3315	5059	6801	8531	10279	12013	13744	15471	17192
55	1599	3344	5088	6830	8570	10308	12042	13773	15499	17221
56	1628	3373	5117	6859	8599	10337	12071	13802	15528	17250
57	1657	3402	5146	6888	8628	10366	12100	13830	15557	17278
58	1687	3431	5175	6917	8657	10394	12129	13859	15585	17307
59	1716	3460	5204	6946	8686	10423	12158	13888	15614	17336
60	1745	3489	5233	6975	8715	10452	12186	13917	15643	17364

b 3

Figure 21: An excerpt of Apian's table of sines [Apian (1541)].

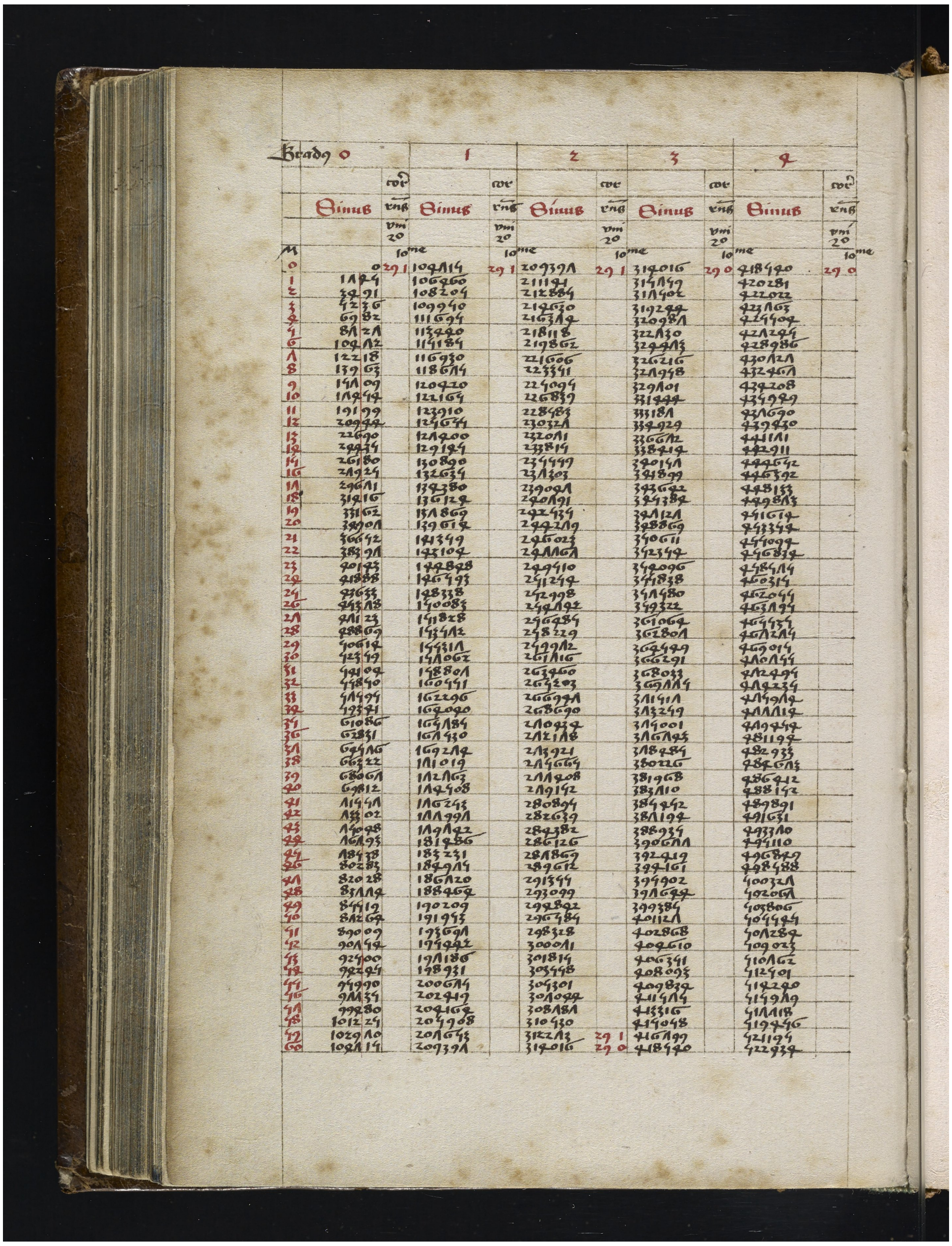
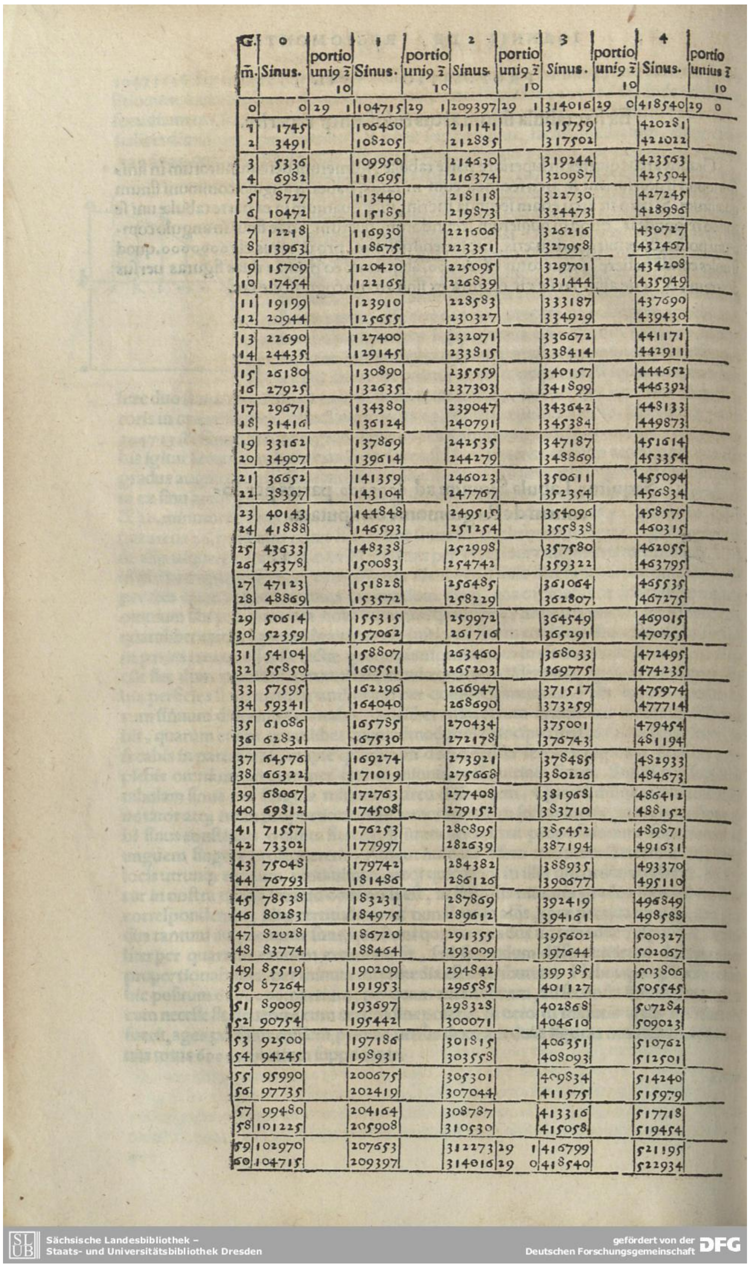
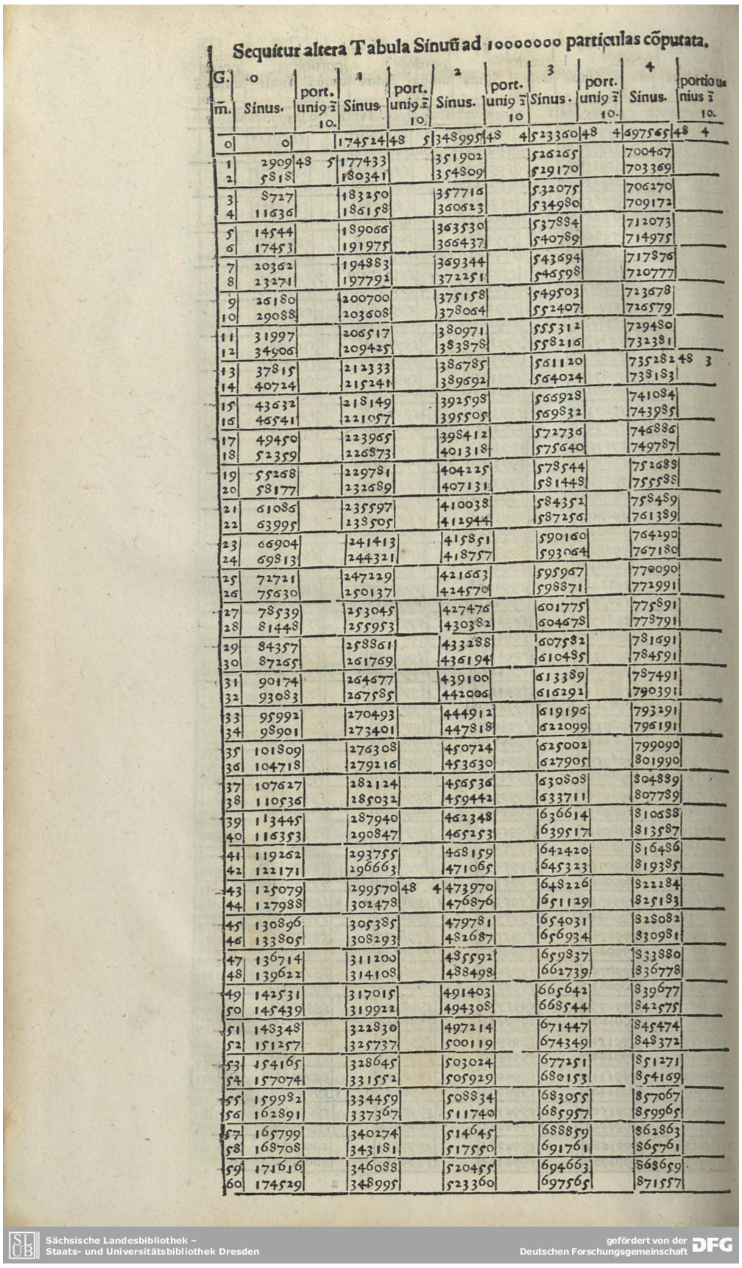


Figure 22: The first page of a manuscript of Regiomontanus's first great table of sines ($R = 6 \cdot 10^6$) (source: Kislak Center for Special Collections, Rare Books and Manuscripts, University of Pennsylvania, LJS 172, ca1476).



G. m.	0 Sinus.	portio uni9 ž 10	1 Sinus.	portio uni9 ž 10	2 Sinus.	portio uni9 ž 10	3 Sinus.	portio uni9 ž 10	4 Sinus.	portio unius ž 10
0	0	29 1	104715	29 1	209397	29 1	314016	29 0	418540	29 0
1	1745		106460		211141		315759		420281	
2	3491		108205		212885		317502		422022	
3	5336		109950		214630		319244		423563	
4	6982		111695		216374		320987		425504	
5	8727		113440		218118		322730		427245	
6	10472		115185		219873		324473		428986	
7	12218		116930		221606		326216		430727	
8	13963		118675		223351		327958		432467	
9	15709		120420		225095		329701		434208	
10	17454		122165		226839		331444		435949	
11	19199		123910		228583		333187		437690	
12	20944		125655		230327		334929		439430	
13	22690		127400		232071		336672		441171	
14	24435		129145		233815		338414		442911	
15	26180		130890		235559		340157		444652	
16	27925		132635		237303		341899		446392	
17	29671		134380		239047		343642		448133	
18	31416		136124		240791		345384		449873	
19	33162		137869		242535		347187		451614	
20	34907		139614		244279		348869		453354	
21	36652		141359		246023		350611		455094	
22	38397		143104		247767		352354		456834	
23	40143		144848		249510		354096		458575	
24	41888		146593		251254		355838		460315	
25	43633		148338		252998		357580		462055	
26	45378		150083		254742		359322		463795	
27	47123		151828		256485		361064		465535	
28	48869		153572		258229		362807		467275	
29	50614		155315		259972		364549		469015	
30	52359		157062		261716		365291		470755	
31	54104		158807		263460		368033		472495	
32	55850		160551		265203		369775		474235	
33	57595		162296		266947		371517		475974	
34	59341		164040		268690		373259		477714	
35	61086		165785		270434		375001		479454	
36	62831		167530		272178		376743		481194	
37	64576		169274		273921		378485		482933	
38	66322		171019		275668		380226		484673	
39	68067		172763		277408		381968		486412	
40	69812		174508		279152		383710		488152	
41	71557		176253		280895		385452		489871	
42	73302		177997		282639		387194		491631	
43	75048		179742		284382		388935		493370	
44	76793		181486		286126		390677		495110	
45	78538		183231		287869		392419		496849	
46	80283		184975		289612		394161		498588	
47	82028		186720		291355		395602		500327	
48	83774		188464		293009		397644		502067	
49	85519		190209		294842		399385		503806	
50	87264		191953		296585		401127		505545	
51	89009		193697		298328		402868		507284	
52	90754		195442		300071		404610		509023	
53	92500		197186		301815		406351		510762	
54	94245		198931		303558		408093		512501	
55	95990		200675		305301		409834		514240	
56	97735		202419		307044		411575		515979	
57	99480		204164		308787		413316		517718	
58	101225		205908		310530		415058		519454	
59	102970		207653		312273	29 1	416799		521195	
60	104715		209397		314016	29 0	418540		522934	

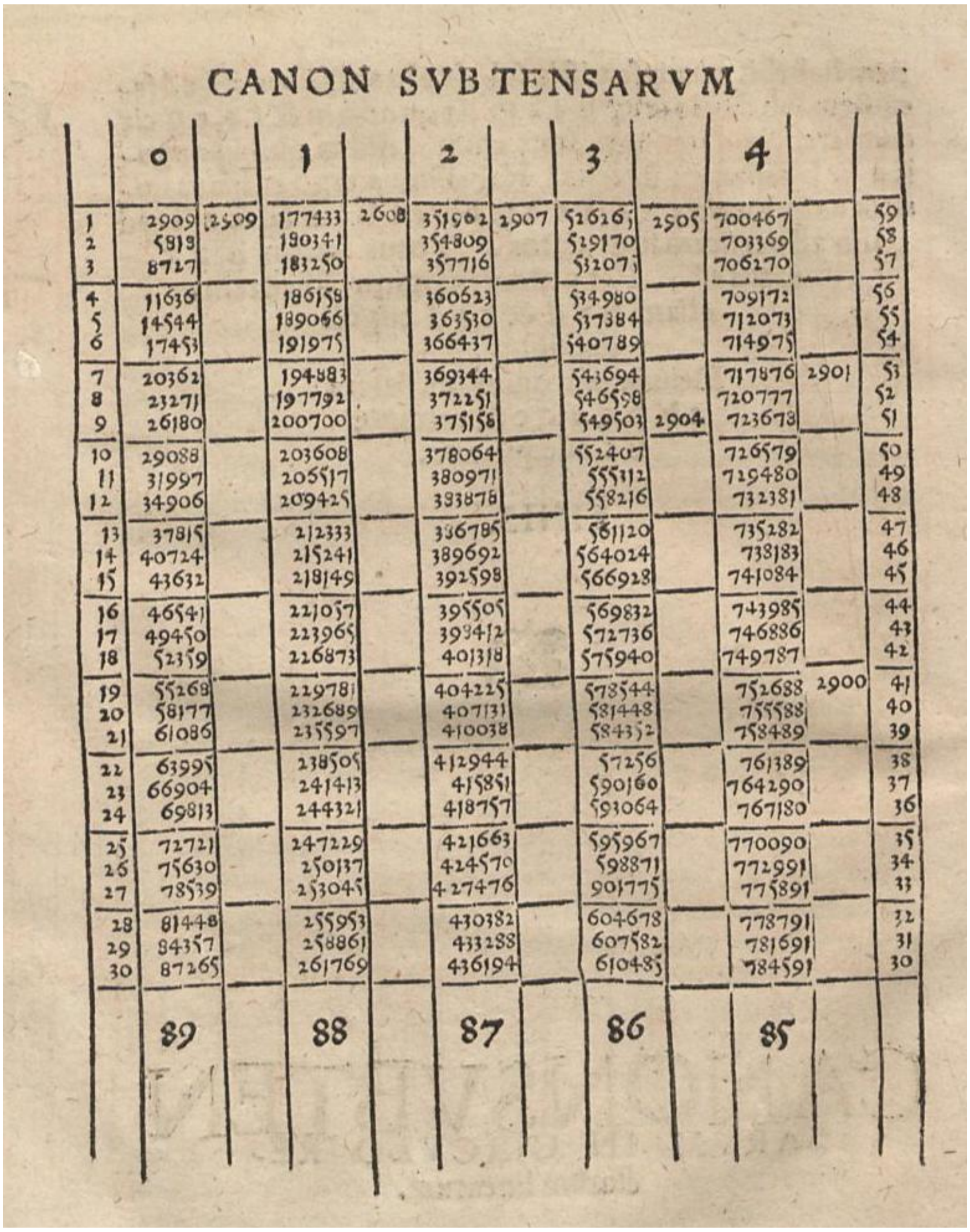
Figure 23: The first page of the first printing of Regiomontanus's first great table of sines ($R = 6 \cdot 10^6$) [von Peuerbach and Regiomontanus (1541)] (source: Dresden).



Sequitur altera Tabula Sinuū ad 10000000 particulas cōputata.

G. / m.	0 Sinus.	port. uni9 ī 10.	1 Sinus	port. uni9 ī 10.	2 Sinus.	port. uni9 ī 10	3 Sinus.	port. uni9 ī 10.	4 Sinus.	portio unius ī 10.
0	0		174524	48 5	348995	48 4	523360	48 4	697565	48 4
1	2909	48 5	177433		351902		526265		700467	
2	5818		180341		354809		529170		703369	
3	8727		183250		357716		532075		706270	
4	11636		186158		360623		534980		709172	
5	14544		189066		363530		537884		712073	
6	17453		191975		366437		540789		714975	
7	20362		194883		369344		543694		717876	
8	23271		197792		372251		546598		720777	
9	26180		200700		375158		549503		723678	
10	29088		203608		378064		552407		726579	
11	31997		206517		380971		555312		729480	
12	34906		209425		383878		558216		732381	
13	37815		212333		386785		561120		735282	48 3
14	40724		215241		389692		564024		738183	
15	43632		218149		392598		566928		741084	
16	46541		221057		395505		569832		743985	
17	49450		223965		398412		572736		746886	
18	52359		226873		401318		575640		749787	
19	55268		229781		404225		578544		752688	
20	58177		232689		407131		581448		755588	
21	61086		235597		410038		584352		758489	
22	63995		238505		412944		587256		761389	
23	66904		241413		415851		590160		764290	
24	69813		244321		418757		593064		767180	
25	72721		247229		421663		595967		770090	
26	75630		250137		424570		598871		772991	
27	78539		253045		427476		601775		775891	
28	81448		255953		430382		604678		778791	
29	84357		258861		433288		607582		781691	
30	87265		261769		436194		610485		784591	
31	90174		264677		439100		613389		787491	
32	93083		267585		442006		616292		790391	
33	95992		270493		444912		619196		793291	
34	98901		273401		447818		622099		796191	
35	101809		276308		450724		625002		799090	
36	104718		279216		453630		627905		801990	
37	107627		282124		456536		630808		804889	
38	110536		285032		459442		633711		807789	
39	113445		287940		462348		636614		810688	
40	116353		290847		465253		639517		813587	
41	119262		293755		468159		642420		816486	
42	122171		296663		471065		645323		819385	
43	125079		299570	48 4	473970		648226		822284	
44	127988		302478		476876		651129		825183	
45	130896		305385		479781		654031		828082	
46	133805		308293		482687		656934		830981	
47	136714		311200		485592		659837		833880	
48	139622		314108		488498		662739		836778	
49	142531		317015		491403		665642		839677	
50	145439		319922		494308		668544		842575	
51	148348		322830		497214		671447		845474	
52	151257		325737		500119		674349		848372	
53	154165		328645		503024		677251		851271	
54	157074		331552		505929		680153		854169	
55	159982		334459		508834		683055		857067	
56	162891		337367		511740		685957		859965	
57	165799		340274		514645		688859		862863	
58	168708		343181		517550		691761		865761	
59	171616		346088		520455		694663		868659	
60	174529		348995		523360		697565		871557	

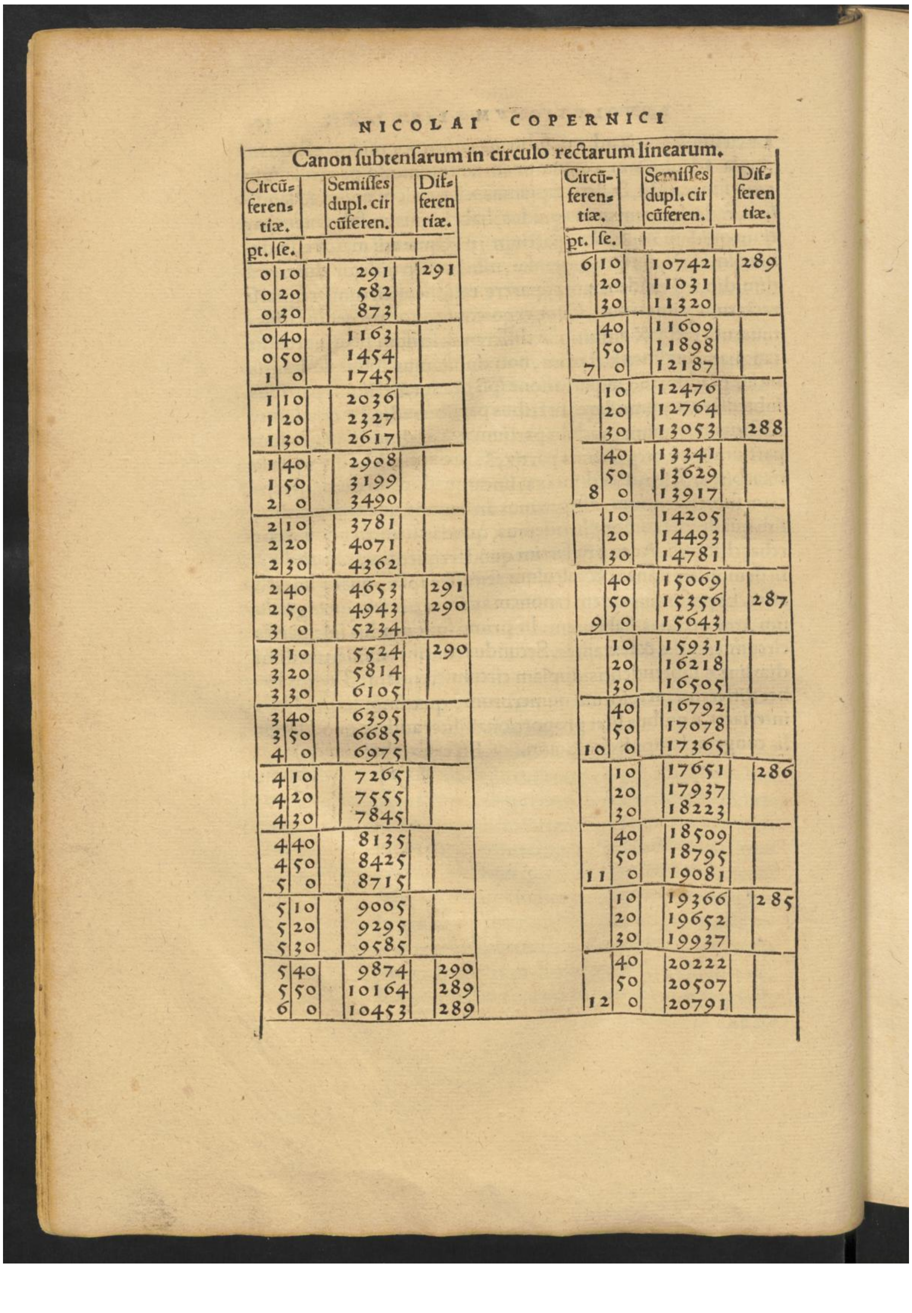
Figure 24: The first page of the first printing of Regiomontanus's second great table of sines ($R = 10^7$) [von Peuerbach and Regiomontanus (1541)] (source: Dresden).



CANON SVBTENSARVM

	0		1		2		3		4		
1	2909	2909	177433	2908	351902	2907	526265	2905	700467		59
2	5818		180341		354809		529170		703369		58
3	8727		183250		357716		532075		706270		57
4	11636		186158		360623		534980		709172		56
5	14544		189066		363530		537884		712073		55
6	17453		191975		366437		540789		714975		54
7	20362		194883		369344		543694		717876	2901	53
8	23271		197792		372251		546598		720777		52
9	26180		200700		375158		549503	2904	723678		51
10	29088		203608		378064		552407		726579		50
11	31997		205517		380971		555312		729480		49
12	34906		209425		383878		558216		732381		48
13	37815		212333		386785		561120		735282		47
14	40724		215241		389692		564024		738183		46
15	43632		218149		392598		566928		741084		45
16	46541		221057		395505		569832		743985		44
17	49450		223965		393412		572736		746886		43
18	52359		226873		401318		575940		749787		42
19	55268		229781		404225		578544		752688	2900	41
20	58177		232689		407131		581448		755588		40
21	61086		235597		410038		584352		758489		39
22	63995		238505		412944		57256		761389		38
23	66904		241413		415851		590160		764290		37
24	69813		244321		418757		593064		767180		36
25	72721		247229		421663		595967		770090		35
26	75630		250137		424570		598871		772991		34
27	78539		253045		427476		901775		775891		33
28	81448		255953		430382		604678		778791		32
29	84357		258861		433288		607582		781691		31
30	87265		261769		436194		610485		784591		30
	89		88		87		86		85		

Figure 25: The first page of Rheticus's table of sines in Copernicus's *De lateribus* [Copernicus (1542)] (source: Dresden).



NICOLAI COPERNICI

Canon ſubtenſarum in circulo rectarum linearum.

Circūferentiæ pt.	ſe.	Semiſſes dupl. circūferen.	Differentiæ	Circūferentiæ pt.	ſe.	Semiſſes dupl. circūferen.	Differentiæ
0	10	291	291	6	10	10742	289
0	20	582			20	11031	
0	30	873			30	11320	
0	40	1163			40	11609	
0	50	1454			50	11898	
1	0	1745		7	0	12187	
1	10	2036			10	12476	
1	20	2327			20	12764	
1	30	2617			30	13053	288
1	40	2908			40	13341	
1	50	3199			50	13629	
2	0	3490		8	0	13917	
2	10	3781			10	14205	
2	20	4071			20	14493	
2	30	4362			30	14781	
2	40	4653	291		40	15069	
2	50	4943	290		50	15356	287
3	0	5234		9	0	15643	
3	10	5524	290		10	15931	
3	20	5814			20	16218	
3	30	6105			30	16505	
3	40	6395			40	16792	
3	50	6685			50	17078	
4	0	6975		10	0	17365	
4	10	7265			10	17651	286
4	20	7555			20	17937	
4	30	7845			30	18223	
4	40	8135			40	18509	
4	50	8425			50	18795	
5	0	8715		11	0	19081	
5	10	9005			10	19366	285
5	20	9295			20	19652	
5	30	9585			30	19937	
5	40	9874	290		40	20222	
5	50	10164	289		50	20507	
6	0	10453	289	12	0	20791	

Figure 26: The first page of Copernicus's table of sines in the *De revolutionibus* [Copernicus (1543)] (source: e-rara).

¶ Tabula fœcunda.

Gra.	Part.æq̄.	Gra.	Par.æq̄.	Gra.	Par.æq̄.
1	5729799	31	166429	61	55432
2	2863563	32	160035	62	53170
3	1908217	33	153987	63	50952
4	1430203	34	148253	64	48772
5	1143131	35	142813	65	46631
6	951387	36	137639	66	44522
7	814456	37	132704	67	42448
8	711569	38	127994	68	40402
9	631377	39	123491	69	38387
10	567118	40	119197	70	36396
11	514438	41	115037	71	34433
12	470453	42	111062	72	32492
13	433148	43	107236	73	30573
14	401089	44	103551	74	28674
15	373211	45	100000	75	26794
16	348748	46	96571	76	24932
17	327088	47	93254	77	23087
18	307767	48	90040	78	21256
19	290422	49	86929	79	19439
20	274753	50	83909	80	17633
21	260511	51	80978	81	15838
22	247513	52	78129	82	14053
23	235583	53	75356	83	12278
24	224607	54	72654	84	10511
25	214450	55	70022	85	8748
26	205034	56	67452	86	6992
27	196263	57	64940	87	5240
28	188075	58	62486	88	3492
29	180402	59	60086	89	1745
30	173207	60	57734	90	0

Figure 27: Excerpt of Frisius's table of cotangents [Gemma Frisius (1545)].

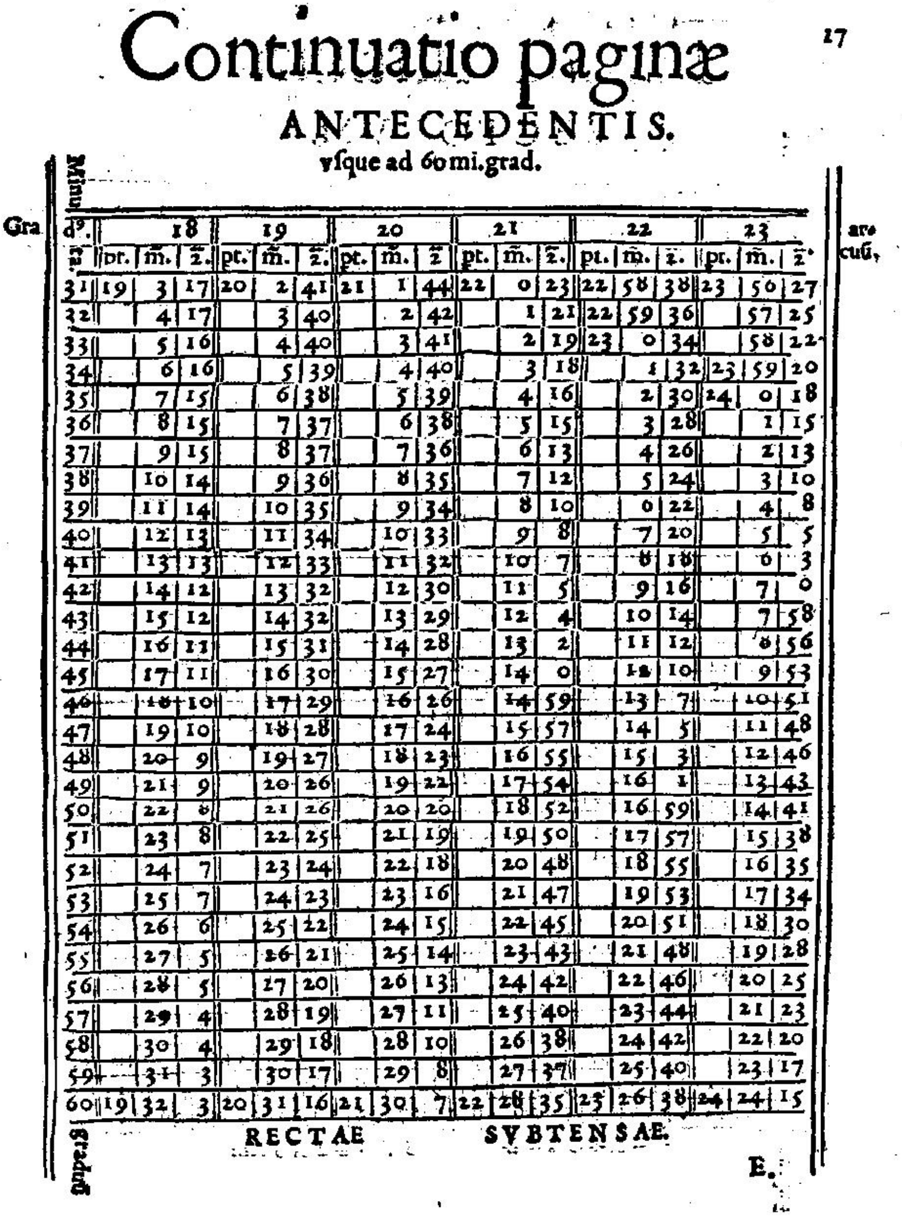
Tabula Gnomonica

	0			200			200			300			400			500		
	G.	m.	ſe.	G.	m.	ſe.	G.	m.	ſe.	G.	m.	ſe.	G.	m.	ſc.	G.	m.	ſe.
0	0	0	0	4	45	49	9	27	44	14	2	10	18	26	7	22	37	12
1	0	2	52	4	48	40	9	30	32	14	4	52	18	28	42	22	39	38
2	0	5	44	4	51	30	9	33	19	14	7	34	18	31	17	22	42	4
3	0	8	36	4	54	21	9	36	6	14	10	16	18	33	51	22	44	30
4	0	11	28	4	57	12	9	38	53	14	12	58	18	36	25	22	46	56
5	0	14	20	5	0	2	9	41	40	14	15	39	18	38	59	22	49	22
6	0	17	12	5	2	53	9	44	27	14	18	20	18	41	33	22	51	47
7	0	20	3	5	5	44	9	47	14	14	21	1	18	44	7	22	54	13
8	0	22	55	5	8	34	9	50	0	14	23	42	18	46	41	22	56	39
9	0	25	47	5	11	24	9	52	47	14	26	23	18	49	15	22	59	4
10	0	28	39	5	14	15	9	55	34	14	29	4	18	51	49	23	1	30
11	0	31	31	5	17	5	9	58	21	14	31	45	18	54	23	23	3	56
12	0	34	23	5	19	55	10	1	7	14	34	26	18	56	57	23	6	21
13	0	37	15	5	22	46	10	3	54	14	37	7	18	59	31	23	8	47
14	0	40	7	5	25	36	10	6	41	14	39	48	19	2	5	23	11	12
15	0	42	59	5	28	26	10	9	28	14	42	29	19	4	39	23	13	38
16	0	45	50	5	31	17	10	12	14	14	45	10	19	7	12	23	16	4
17	0	48	42	5	34	7	00	15	0	14	47	51	19	9	45	23	18	29
18	0	51	34	5	36	57	10	17	47	14	50	32	19	12	18	23	20	53
19	0	54	26	5	39	48	10	20	33	14	53	13	19	14	51	23	23	18
20	0	57	18	5	42	38	10	23	19	14	55	54	19	17	24	23	25	42
21	1	0	10	5	45	28	10	26	5	14	58	34	19	19	57	23	28	7
22	1	3	1	5	48	18	10	28	52	15	1	14	19	22	30	23	30	32
23	1	5	53	5	51	8	10	31	38	15	3	54	19	25	3	23	32	56
24	1	8	45	5	53	58	10	34	24	15	6	34	19	27	36	23	35	20
25	1	11	37	5	56	48	10	37	10	15	9	14	19	30	9	23	37	45
26	1	14	29	5	59	38	10	39	57	15	11	54	19	32	42	23	40	9

Figure 28: Excerpt of Frisius's table of arctangents [Gemma Frisius (1545)]. (continued on next page)

	0			100			200			300			400			500		
	G.	m.	se.	G.	m.	se.	G.	m.	se.	G.	m.	se.	G.	m.	se.	G.	m.	se.
27	1	17	20	6	2	28	10	42	43	15	14	34	19	35	15	23	42	34
28	1	20	12	6	5	18	10	45	29	15	17	14	19	37	48	23	44	58
29	1	23	4	6	8	8	10	48	15	15	19	54	19	40	20	23	47	22
30	1	25	56	6	10	58	10	51	1	15	22	34	19	42	52	23	49	45
31	1	28	47	6	13	48	10	53	47	15	25	14	19	45	24	23	52	9
32	1	31	39	6	16	38	10	56	33	15	27	54	19	47	56	23	54	32
33	1	34	31	6	19	28	10	59	19	15	30	34	19	50	28	23	56	56
34	1	37	23	6	22	17	11	2	5	15	33	14	19	53	0	23	59	19
35	1	40	14	6	25	7	11	4	50	15	35	53	19	55	32	24	1	43
36	1	43	6	6	27	57	11	7	36	15	38	32	19	58	4	24	4	6
37	1	45	58	6	30	46	11	10	21	15	41	11	20	0	36	24	6	30
38	1	48	49	6	33	36	11	13	6	15	43	50	20	3	8	24	8	53
39	1	51	41	6	36	26	11	15	51	15	46	29	20	5	40	24	11	17
40	1	54	34	6	39	15	11	18	36	15	49	8	20	8	12	24	13	40
41	1	57	25	6	42	5	11	21	21	15	51	47	20	10	43	24	16	2
42	2	0	17	6	44	55	11	24	6	15	54	26	20	13	14	24	18	25
43	2	3	9	6	47	44	11	26	51	15	57	5	20	15	45	24	20	47
44	2	6	0	6	50	34	11	29	36	15	59	44	20	18	16	24	23	10
45	2	8	51	6	53	24	11	32	21	16	2	23	20	20	47	24	25	32
46	2	11	43	6	56	13	11	35	6	16	5	0	20	23	18	24	27	55
47	2	14	34	6	59	2	11	37	51	16	7	41	20	25	49	24	30	17
48	2	17	26	7	1	52	11	40	36	16	10	20	20	28	20	24	32	39
49	2	20	18	7	4	41	11	43	21	16	12	59	20	30	51	24	35	2
50	2	23	9	7	7	30	11	46	6	16	15	37	20	33	22	24	37	24

Figure 29: Excerpt of Frisius's table of arctangents [Gemma Frisius (1545)] (cont'd).



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Gra d°. / Minu m.	18 pt.	18 m.	18 2.	19 pt.	19 m.	19 2.	20 pt.	20 m.	20 2.	21 pt.	21 m.	21 2.	22 pt.	22 m.	22 2.	23 pt.	23 m.	23 2.
31	19	3	17	20	2	41	21	1	44	22	0	23	22	58	38	23	50	27
32		4	17		3	40		2	42		1	21	22	59	36		57	25
33		5	16		4	40		3	41		2	19	23	0	34		58	22
34		6	16		5	39		4	40		3	18		1	32	23	59	20
35		7	15		6	38		5	39		4	16		2	30	24	0	18
36		8	15		7	37		6	38		5	15		3	28		1	15
37		9	15		8	37		7	36		6	13		4	26		2	13
38		10	14		9	36		8	35		7	12		5	24		3	10
39		11	14		10	35		9	34		8	10		0	22		4	8
40		12	13		11	34		10	33		9	8		7	20		5	5
41		13	13		12	33		11	32		10	7		8	18		6	3
42		14	12		13	32		12	30		11	5		9	16		7	0
43		15	12		14	32		13	29		12	4		10	14		7	58
44		16	11		15	31		14	28		13	2		11	12		8	56
45		17	11		16	30		15	27		14	0		12	10		9	53
46		18	10		17	29		16	26		14	59		13	7		10	51
47		19	10		18	28		17	24		15	57		14	5		11	48
48		20	9		19	27		18	23		16	55		15	3		12	46
49		21	9		20	26		19	22		17	54		16	1		13	43
50		22	8		21	26		20	20		18	52		16	59		14	41
51		23	8		22	25		21	19		19	50		17	57		15	38
52		24	7		23	24		22	18		20	48		18	55		16	35
53		25	7		24	23		23	16		21	47		19	53		17	34
54		26	6		25	22		24	15		22	45		20	51		18	30
55		27	5		26	21		25	14		23	43		21	48		19	28
56		28	5		27	20		26	13		24	42		22	46		20	25
57		29	4		28	19		27	11		25	40		23	44		21	23
58		30	4		29	18		28	10		26	38		24	42		22	20
59		31	3		30	17		29	8		27	37		25	40		23	17
60	19	32	3	20	31	16	21	30	7	22	28	35	23	26	38	24	24	15

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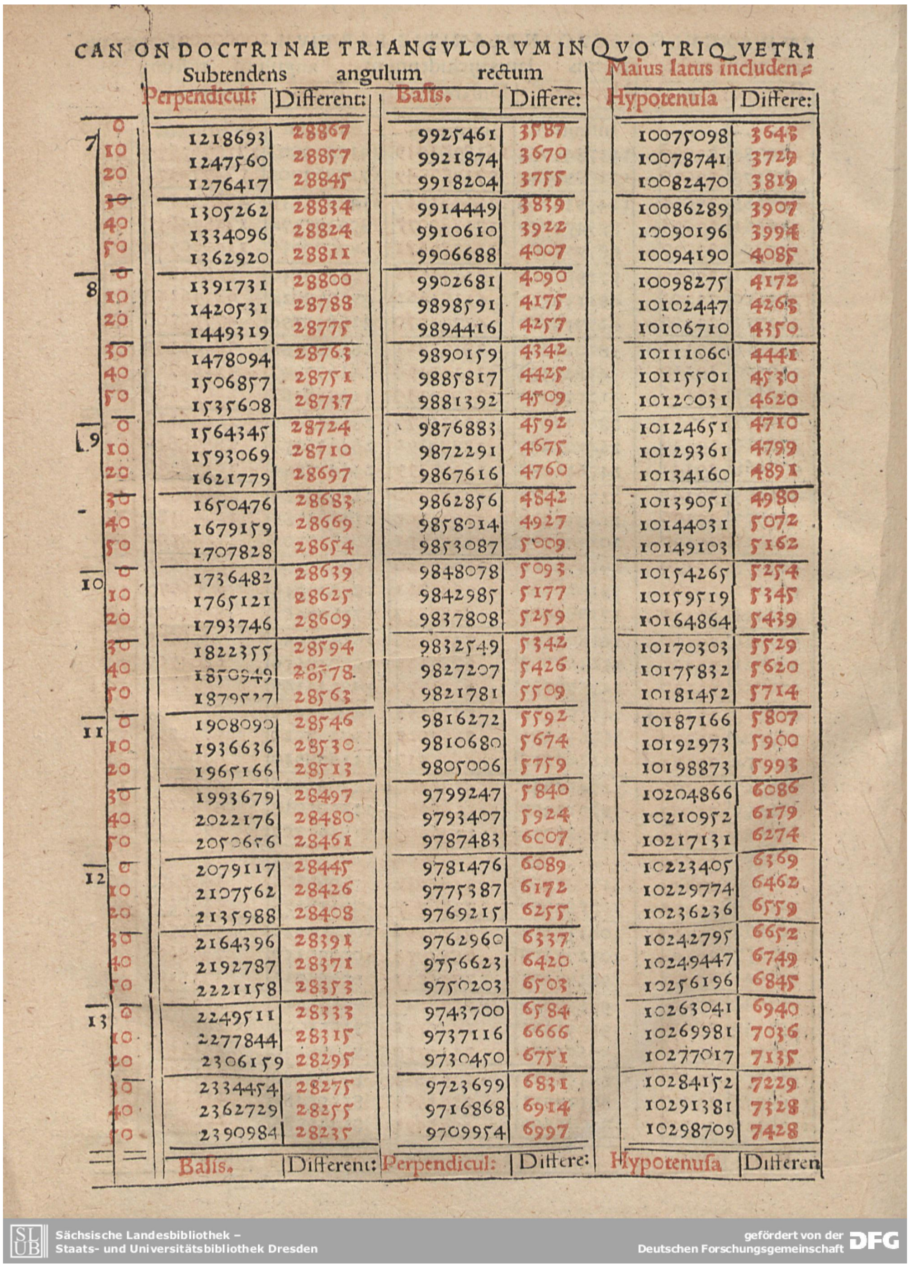
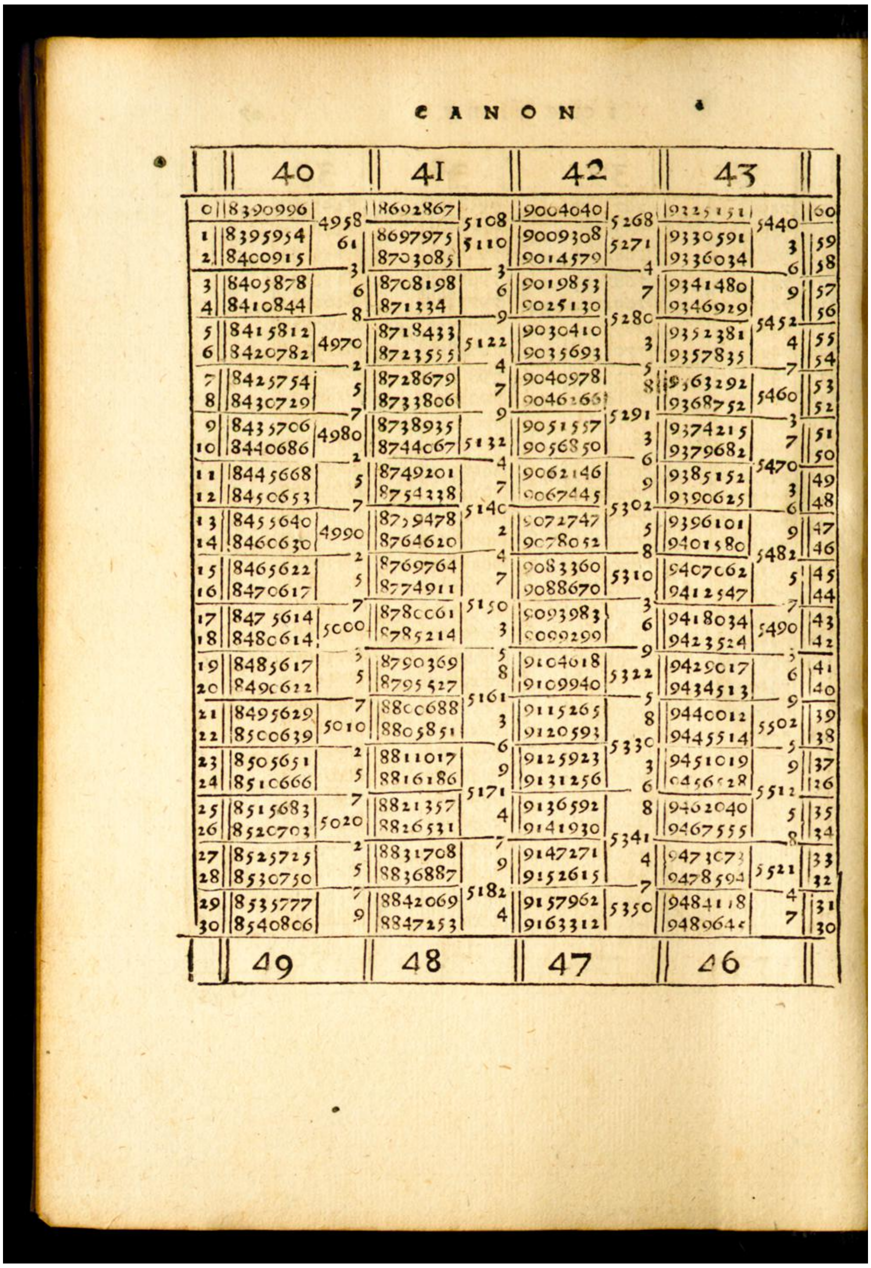
Figure 30: Excerpt of Fine's table of sines [Fine (1550)] (source: Google books).



CANON DOCTRINAE TRIANGVLORVM IN QVO TRIQVETRI

		Subtendens angulum rectum				Maius latus includens	
		Perpendicul:	Different:	Basis.	Differe:	Hypotenusa	Differe:
7	0	1218693	28867	9925461	3587	10075098	3643
	10	1247560	28857	9921874	3670	10078741	3729
	20	1276417	28845	9918204	3755	10082470	3819
	30	1305262	28834	9914449	3839	10086289	3907
	40	1334096	28824	9910610	3922	10090196	3994
	50	1362920	28811	9906688	4007	10094190	4085
8	0	1391731	28800	9902681	4090	10098275	4172
	10	1420531	28788	9898591	4175	10102447	4263
	20	1449319	28775	9894416	4257	10106710	4350
	30	1478094	28763	9890159	4342	10111060	4441
	40	1506857	28751	9885817	4425	10115501	4530
	50	1535608	28737	9881392	4509	10120031	4620
9	0	1564345	28724	9876883	4592	10124651	4710
	10	1593069	28710	9872291	4675	10129361	4799
	20	1621779	28697	9867616	4760	10134160	4891
	30	1650476	28683	9862856	4842	10139051	4980
	40	1679159	28669	9858014	4927	10144031	5072
	50	1707828	28654	9853087	5009	10149103	5162
10	0	1736482	28639	9848078	5093	10154265	5254
	10	1765121	28625	9842985	5177	10159519	5345
	20	1793746	28609	9837808	5259	10164864	5439
	30	1822355	28594	9832549	5342	10170303	5529
	40	1850949	28578	9827207	5426	10175832	5620
	50	1879527	28563	9821781	5509	10181452	5714
11	0	1908090	28546	9816272	5592	10187166	5807
	10	1936636	28530	9810680	5674	10192973	5900
	20	1965166	28513	9805006	5759	10198873	5993
	30	1993679	28497	9799247	5840	10204866	6086
	40	2022176	28480	9793407	5924	10210952	6179
	50	2050656	28461	9787483	6007	10217131	6274
12	0	2079117	28445	9781476	6089	10223405	6369
	10	2107562	28426	9775387	6172	10229774	6462
	20	2135988	28408	9769215	6255	10236236	6559
	30	2164396	28391	9762960	6337	10242795	6652
	40	2192787	28371	9756623	6420	10249447	6749
	50	2221158	28353	9750203	6503	10256196	6845
13	0	2249511	28333	9743700	6584	10263041	6940
	10	2277844	28315	9737116	6666	10269981	7036
	20	2306159	28295	9730450	6751	10277017	7135
	30	2334454	28275	9723699	6831	10284152	7229
	40	2362729	28255	9716868	6914	10291381	7328
	50	2390984	28235	9709954	6997	10298709	7428
		Basis.	Different:	Perpendicul:	Differe:	Hypotenusa	Differen

Figure 31: Excerpt of Rheticus's table of the six trigonometric functions [Rheticus (1551)] (source: Dresden).



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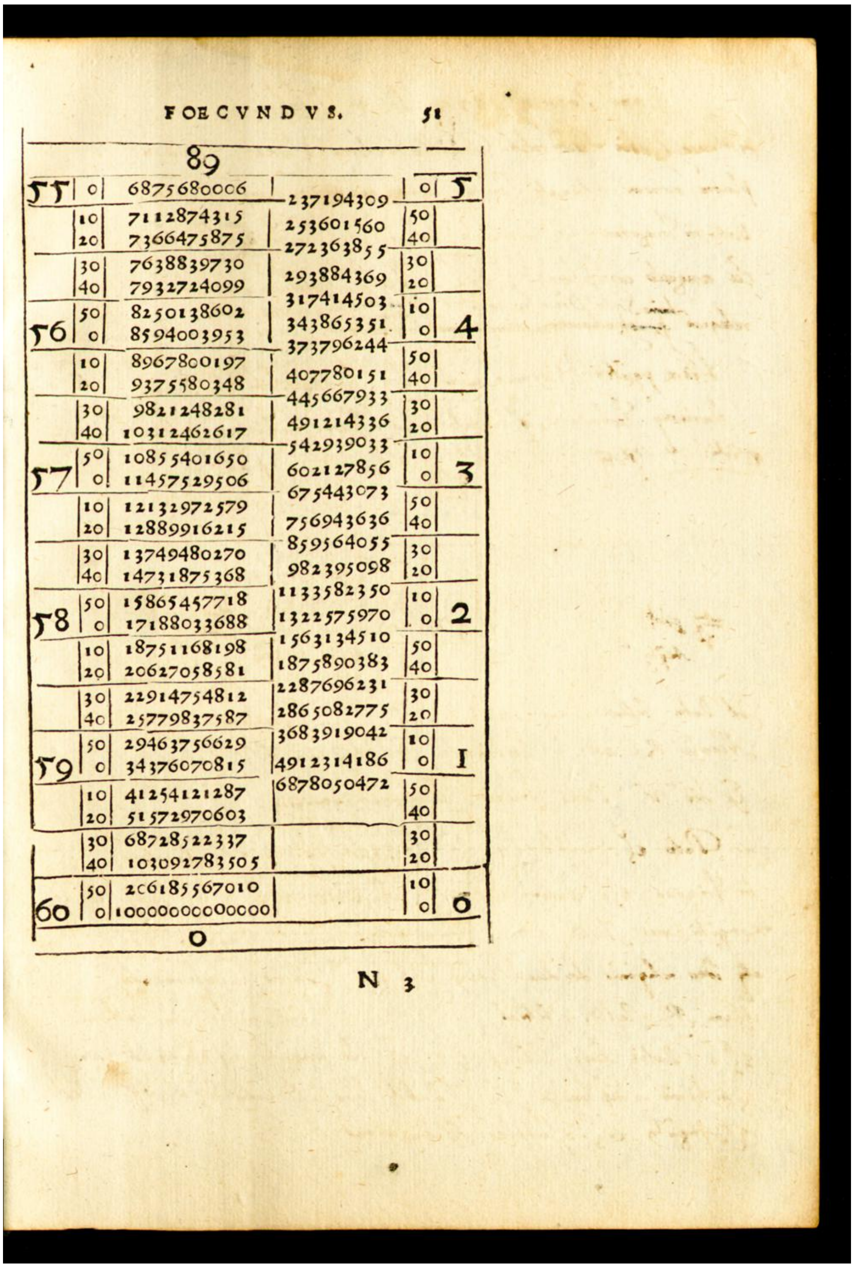
	40		41		42		43		
0	8390996	4958	8692867	5108	9004040	5268	9325151	5440	60
1	8395954	61	8697975	5110	9009308	5271	9330591	3	59
2	8400915	3	8703085	3	9014579	4	9336034	6	58
3	8405878	6	8708198	6	9019853	7	9341480	9	57
4	8410844	8	871334	9	9025130	5280	9346929	5452	56
5	8415812	4970	8718433	5122	9030410		9352381	4	55
6	8420782	2	8723555	4	9035693	3	9357835	7	54
7	8425754	5	8728679	7	9040978	5	9363292		53
8	8430729	7	8733806	9	9046266	8	9368752	5460	52
9	8435706	4980	8738935		9051557	5291	9374215	3	51
10	8440686	2	8744067	5132	9056850	3	9379682	7	50
11	8445668	5	8749201	4	9062146	6	9385152	5470	49
12	8450653	7	8754338	7	9067445	9	9390625	3	48
13	8455640	4990	8759478	5140	9072747	5302	9396101	6	47
14	8460630	2	8764620	2	9078052	5	9401580	9	46
15	8465622	5	8769764	4	9083360	8	9407062	5482	45
16	8470617	7	8774911	7	9088670	5310	9412547	5	44
17	8475614	5000	8780061	5150	9093983	3	9418034	7	43
18	8480614	3	8785214	3	9099299	6	9423524	5490	42
19	8485617	5	8790369	5	9104618	9	9429017	3	41
20	8490622	7	8795527	8	9109940	5322	9434513	6	40
21	8495629	5010	8800688	5161	9115265	5	9440012	9	39
22	8500639	2	8805851	3	9120593	8	9445514	5502	38
23	8505651	5	8811017	6	9125923	5330	9451019	5	37
24	8510666	7	8816186	9	9131256	3	9456528	9	36
25	8515683	5020	8821357	5171	9136592	6	9462040	5512	35
26	8520703	2	8826531	4	9141930	8	9467555	5	34
27	8525725	5	8831708	7	9147271	5341	9473073	8	33
28	8530750	7	8836887	9	9152615	4	9478594	5521	32
29	8535777	9	8842069	5182	9157962	7	9484118	4	31
30	8540806		8847253	4	9163312	5350	9489646	7	30
	49		48		47		46		

Figure 32: An excerpt from Reinhold's table of tangents [Reinhold (1554)] (source: e-rara).



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		89			
55	0	6875680006	237194309	0	5
	10	7112874315	253601560	50	
	20	7366475875	272363855	40	
	30	7638839730	293884369	30	
	40	7932724099		20	
	50	8250138602	317414503	10	
56	0	8594003953	343865351	0	4
	10	8967800197	373796244	50	
	20	9375580348	407780151	40	
	30	9821248281	445667933	30	
	40	10312462617	491214336	20	
	50	10855401650	542939033	10	
57	0	11457529506	602127856	0	3
	10	12132972579	675443073	50	
	20	12889916215	756943636	40	
	30	13749480270	859564055	30	
	40	14731875368	982395098	20	
	50	15865457718	1133582350	10	
58	0	17188033688	1322575970	0	2
	10	18751168198	1563134510	50	
	20	20627058581	1875890383	40	
	30	22914754812	2287696231	30	
	40	25779837587	2865082775	20	
	50	29463756629	3683919042	10	
59	0	34376070815	4912314186	0	I
	10	41254121287	6878050472	50	
	20	51572970603		40	
	30	68728522337		30	
	40	103092783505		20	
	50	206185567010		10	
60	0	100000000000000		0	0
		0			

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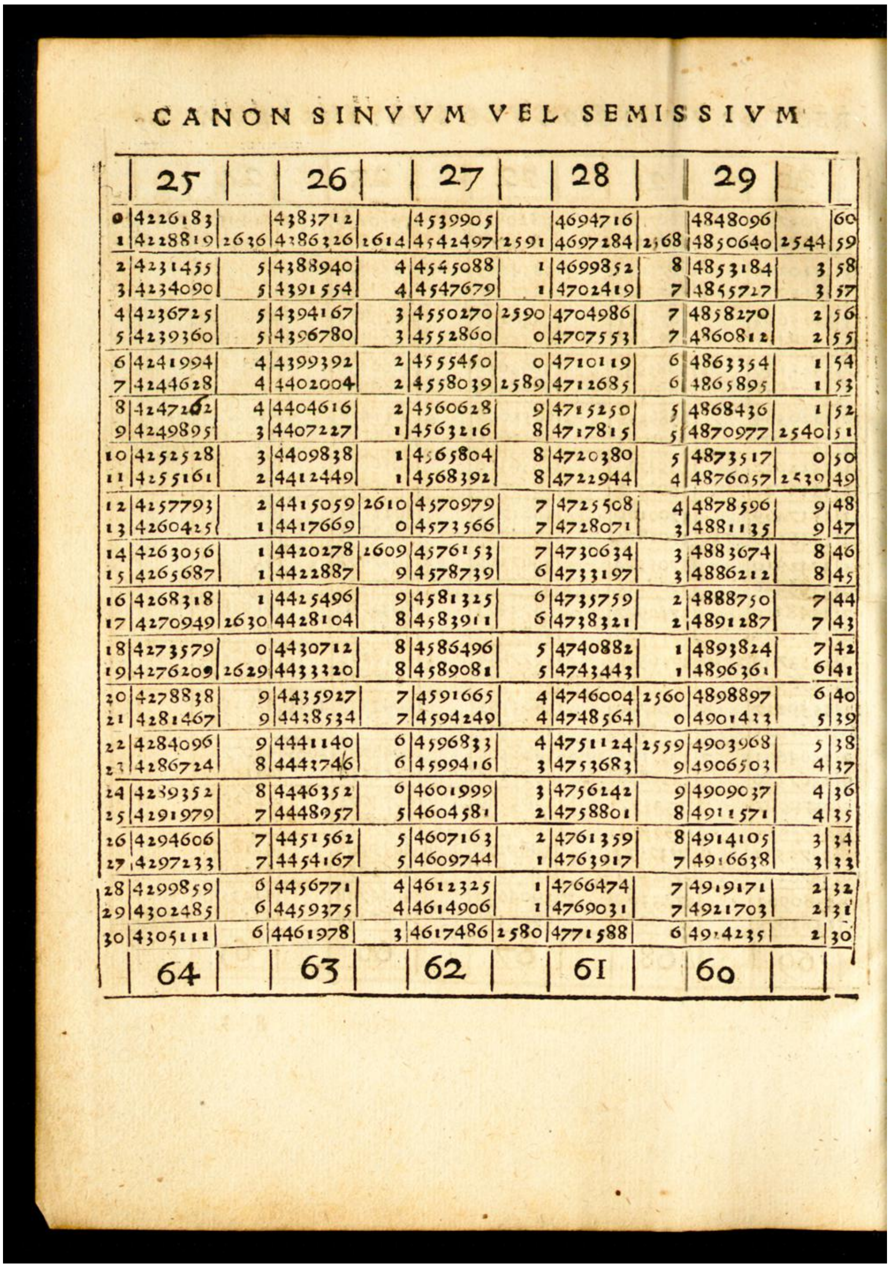
Figure 33: The end of Reinhold's table of tangents, with values every 10 seconds [Reinhold (1554)] (source: e-rara).



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	25		26		27		28		29		
0	4226183		4383712		4539905		4694716		4848096		60
1	4228819	2636	4386326	2614	4542497	2591	4697284	2568	4850640	2544	59
2	4231455	5	4388940	4	4545088	1	4699852	8	4853184	3	58
3	4234090	5	4391554	4	4547679	1	4702419	7	4855727	3	57
4	4236725	5	4394167	3	4550270	2590	4704986	7	4858270	2	56
5	4239360	5	4396780	3	4552860	0	4707553	7	4860812	2	55
6	4241994	4	4399392	2	4555450	0	4710119	6	4863354	1	54
7	4244628	4	4402004	2	4558039	2589	4712685	6	4865895	1	53
8	4247262	4	4404616	2	4560628	9	4715250	5	4868436	1	52
9	4249895	3	4407227	1	4563216	8	4717815	5	4870977	2540	51
10	4252528	3	4409838	1	4565804	8	4720380	5	4873517	0	50
11	4255161	2	4412449	1	4568392	8	4722944	4	4876057	2539	49
12	4257793	2	4415059	2610	4570979	7	4725508	4	4878596	9	48
13	4260425	1	4417669	0	4573566	7	4728071	3	4881135	9	47
14	4263056	1	4420278	2609	4576153	7	4730634	3	4883674	8	46
15	4265687	1	4422887	9	4578739	6	4733197	3	4886212	8	45
16	4268318	1	4425496	9	4581325	6	4735759	2	4888750	7	44
17	4270949	2630	4428104	8	4583911	6	4738321	2	4891287	7	43
18	4273579	0	4430712	8	4586496	5	4740882	1	4893824	7	42
19	4276209	2629	4433320	8	4589081	5	4743443	1	4896361	6	41
20	4278838	9	4435927	7	4591665	4	4746004	2560	4898897	6	40
21	4281467	9	4438534	7	4594249	4	4748564	0	4901433	5	39
22	4284096	9	4441140	6	4596833	4	4751124	2559	4903968	5	38
23	4286724	8	4443746	6	4599416	3	4753683	9	4906503	4	37
24	4289352	8	4446352	6	4601999	3	4756242	9	4909037	4	36
25	4291979	7	4448957	5	4604581	2	4758801	8	4911571	4	35
26	4294606	7	4451562	5	4607163	2	4761359	8	4914105	3	34
27	4297233	7	4454167	5	4609744	1	4763917	7	4916638	3	33
28	4299859	6	4456771	4	4612325	1	4766474	7	4919171	2	32
29	4302485	6	4459375	4	4614906	1	4769031	7	4921703	2	31
30	4305111	6	4461978	3	4617486	2580	4771588	6	4924235	2	30
	64		63		62		61		60		

Figure 34: An excerpt from Reinhold's table of sines [Reinhold (1554)] (source: e-rara).

	0	1	2	3	4	5	6	7	8	9
m.	Sinus.	Sinus.	Sinus.	Sinus.	Sinus.	Sinus.	Sinus.	Sinus.	Sinus.	Sinus.
0	00	1745	3489	5233	6975	8715	10452	12186	13917	15643
1	29	1774	3519	5262	7004	8744	10481	12215	13946	15672
2	58	1803	3548	5291	7033	8773	10510	12244	13974	15700
3	87	1832	3577	5320	7062	8802	10639	12273	14003	15729
4	116	1861	3606	5349	7091	8831	10568	12302	14032	15758
5	145	1890	3635	5378	7120	8860	10597	12331	14061	15787
6	174	1919	3664	5407	7149	8889	10626	12360	14090	15815
7	203	1948	3693	5436	7178	8918	10655	12389	14118	15844
8	232	1977	3722	5465	7207	8947	10684	12417	14147	15873
9	261	2007	3751	5495	7236	8976	10713	12446	14176	15901
10	290	2036	3780	5524	7265	9005	10742	12475	14205	15930
11	319	2065	3809	5553	7294	9034	10771	12504	14234	15959
12	349	2094	3838	5582	7323	9063	10799	12533	14262	15988
13	378	2123	3867	5611	7352	9092	10828	12562	14291	16016
14	407	2152	3896	5640	7381	9121	10857	12591	14320	16045
15	436	2181	3925	5669	7410	9150	10886	12619	14349	16074
16	465	2210	3955	5698	7439	9179	10915	12648	14378	16102
17	494	2239	3984	5727	7468	9208	10944	12677	14406	16131
18	523	2268	4013	5756	7497	9237	10973	12706	14435	16160
19	552	2297	4042	5785	7526	9266	11002	12735	14464	16189
20	581	2326	4071	5814	7555	9294	11031	12764	14493	16217
21	610	2355	4100	5843	7584	9323	11060	12793	14521	16246
22	639	2385	4129	5872	7613	9352	11089	12821	14550	16275
23	669	2414	4158	5901	7642	9381	11117	12850	14579	16303
24	698	2443	4187	5930	7671	9410	11146	12879	14608	16332
25	727	2472	4216	5959	7700	9439	11175	12908	14637	16361
26	756	2501	4245	5988	7729	9468	11204	12937	14665	16379
27	785	2530	4274	6017	7758	9497	11233	12966	14694	16418
28	814	2559	4303	6046	7787	9526	11262	12994	14723	16447
29	843	2588	4332	6075	7816	9555	11291	13023	14752	16476
30	873	2617	4361	6104	7845	9584	11320	13052	14780	16504
31	901	2646	4391	6133	7874	9613	11349	13081	14809	16533
32	930	2675	4420	6162	7903	9642	11378	13110	14838	16562
33	959	2704	4449	6191	7932	9671	11407	13139	14867	16590
34	989	2734	4478	6220	7961	9700	11435	13167	14896	16619
35	1018	2763	4507	6250	7990	9729	11464	13196	14924	16648
36	1047	2792	4536	6279	8019	9758	11493	13225	14953	16676
37	1076	2821	4565	6308	8048	9787	11522	13254	14982	16705
38	1105	2850	4594	6337	8077	9816	11551	13283	15011	16734
39	1134	2879	4622	6366	8106	9845	11580	13312	15039	16762
40	1163	2908	4652	6395	8135	9874	11609	13340	15068	16791
41	1192	2937	4681	6424	8164	9903	11638	13369	15097	16820
42	1221	2966	4710	6453	8193	9931	11667	13398	15126	16848
43	1250	2995	4739	6482	8222	9960	11695	13427	15154	16877
44	1279	3024	4768	6511	8251	9989	11724	13456	15183	16906
45	1308	3053	4797	6540	8280	10018	11753	13485	15212	16934
46	1338	3082	4826	6569	8309	10047	11782	13513	15241	16963
47	1367	3112	4855	6598	8338	10076	11811	13542	15269	16992
48	1396	3141	4884	6627	8367	10105	11840	13571	15298	17020
49	1425	3170	4914	6656	8396	10134	11869	13600	15327	17049
50	1454	3199	4943	6685	8425	10163	11898	13629	15350	17078
51	1483	3228	4972	6714	8454	10192	11927	13658	15384	17106
52	1512	3257	5001	6743	8483	10221	11955	13686	15413	17135
53	1541	3286	5030	6772	8512	10250	11984	13715	15442	17164
54	1570	3315	5059	6801	8531	10279	12013	13744	15471	17192
55	1599	3344	5088	6830	8570	10308	12042	13773	15499	17221
56	1628	3373	5117	6859	8599	10337	12071	13802	15528	17250
57	1657	3402	5146	6888	8628	10366	12100	13830	15557	17278
58	1687	3431	5175	6917	8657	10394	12129	13859	15585	17307
59	1716	3460	5204	6946	8686	10423	12158	13888	15614	17336
60	1745	3489	5233	6975	8715	10452	12186	13917	15643	17364

Figure 35: An excerpt of Bassantin's table of sines [Bassantin (1557)].

Tabula ſecunda per L. G. ſupputata.

Altitu. poli. Gr	Multiplicandus	Gr.	Multiplicandus.
1	1745	26	43837
2	3489	27	45399
3	5233	28	46947
4	6975	29	48480
5	8716	30	50000
6	10453	31	51504
7	12187	32	52992
8	13917	33	54464
9	15643	34	55919
10	17365	35	57357
11	19080	36	58779
12	20791	37	60180
13	22495	38	61565
14	24192	39	62932
15	25882	40	64279
16	27563	41	65605
17	29237	42	66912
18	30902	43	68199
19	32557	44	69465
20	34202	45	70710
21	35837	46	71934
22	37460	47	73135
23	39072	48	74314
24	40674	49	75431
25	42262	50	76604
Bore	Numerus Multiplican	Altitu. poli bor	Numerus Multiplicād.

In hac tabella ſecunda ſemidiameter que ſinus eſt totus. preſupponitur. 100000. partium.

Figure 36: An excerpt of Gaurico's sine table with the heading *tabula fecunda* [Gaurico (1557)].

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Tabella ſecunda per Campanum Nouarienſem iamdiu ſupputata.

Multiplicandus		Multiplicandus	
Gr.	Numerus	Gr.	Numerus.
0	0000	26	48772
1	1745	27	50952
2	3492	28	53170
3	5240	29	55432
4	6992	30	57724
5	8748	31	60086
6	10521	32	62486
7	12278	33	64940
8	14053	34	67452
9	15238	35	70022
10	17633	36	72854
11	19439	37	75356
12	21256	38	78129
13	23087	39	80978
14	24932	40	83909
15	26794	41	86929
16	28674	42	93040
17	30573	43	94787
18	32492	44	96571
19	34433	45	100000
20	36396	46	103552
21	38387	47	107236
22	40402	48	111602
23	42448	49	115037
24	44522	50	119197
25	46631		

Figure 37: Gaurico's table of tangents [Gaurico (1557)].

Arcus G.	Arcus M.	Sinus Partes	Arcus G.	Arcus M.	Sinus Partes	Arcus G.	Arcus M.	Sinus Partes
			5	0	8716	10	0	17365
0	10	290		10	9005		10	17650
	20	581		20	9295		20	17937
	30	873		30	9584		30	18223
	40	1163		40	9874		40	18509
0	50	1454	5	50	10163	10	50	18795
1	0	1745	6	0	10453	11	0	19080
	10	2036		10	10742		10	19366
	20	2327		20	11031		20	19651
	30	2617		30	11320		30	19937
	40	2908		40	11609		40	20222
1	50	3199	6	50	11897	11	50	20505
2	0	3489	7	0	12187	12	0	20791
	10	3780		10	12475		10	21075
	20	4071		20	12764		20	21359
	30	4362		30	13052		30	21644
	40	4652		40	13340		40	21927
2	50	4943	7	50	13629	12	50	22211
3	0	5233	8	0	13917	13	0	22495
	10	5523		10	14205		10	22778
	20	5814		20	14493		20	23061
	30	6104		30	14781		30	23344
	40	6395		40	15068		40	23627
3	50	6685	8	50	15355	13	50	23909
4	0	6975	9	0	15643	14	0	24192
	10	7265		10	15930		10	24474
	20	7555		20	16217		20	24756
	30	7846		30	16504		30	25038
	40	8136		40	16794		40	25319
4	50	8425	9	50	17078	14	50	25600

Figure 38: An excerpt of Gaurico's sine table [Gaurico (1557)].

TABELLA SINVS RECTI. 65

g	Sinus.	Dfa	6^{ma}	g	Sinus.	Dfa	6^{ma}
1	1745	1745	29. 5	46	71934	1223.	20. 23
2	3490	1745	29. 5	47	73135	1201.	20. 1
3	5234	1744	29. 4	48	74315	1180.	19. 40
4	6976	1742	29. 2	49	75471	1156.	19. 16
5	8716	1740	29. 0	50	76604	1133.	18. 53
6	10453	1737	28. 57	51	77715	1111.	18. 31
7	12187	1734	28. 54	52	78801	1086	18. 6
8	13917	1730	28. 50	53	79864	1063	17. 43
9	15643	1726	28. 46	54	80902	1038	17. 18
10	17365	1722	28. 42	55	81915	1013	16. 53
11	19081	1716	28. 36	56	82904	989	16. 29
12	20791	1710	28. 30	57	83867	963	16. 3
13	22495	1704	28. 24	58	84805	938	15. 38
14	24192	1697	28. 17	59	85717	912	15. 21
15	25882	1690	28. 10	60	86603	886	14. 46
16	27564	1682	28. 2	61	87462	859	14. 19
17	29237	1673	27. 53	62	88295	833	13. 53
18	30902	1665	27. 45	63	89101	806	13. 26
19	32557	1655	27. 35	64	89879	778	12. 58
20	34202	1645	27. 25	65	90631	752	12. 32
21	35837	1635	27. 15	66	91355	724	12. 4
22	37461	1624	27. 4	67	92051	696	11. 36
23	39073	1612	26. 52	68	92718	667	11. 7
24	40674	1601	26. 41	69	93358	640	10. 40
25	42262	1588	26. 28	70	93969	611	10 11
26	43837	1575	26. 15	71	94552	583	9. 43
27	45399	1562	26. 2	72	95106	554	9. 14
28	46947	1548	25. 48	73	95631	525	8. 45
29	48481	1534	25. 34	74	96126	495	8. 15
30	50000	1519	25. 19	75	96593	467	7. 47
31	51504	1504	25. 4	76	97030	437	7. 17
32	52992	1488	24. 48	77	97437	407	6. 47
33	54464	1472	24. 32	78	97815	378	6. 18
34	55919	1455	24. 15	79	98163	348	5. 48
35	57358	1439	23. 59	80	98481	318	5. 18
36	58779	1421	23. 41	81	98769	288	4. 48
37	60182	1403	23. 23	82	99027	258	4. 18
38	61566	1384	23. 4	83	99255	228	3. 48
39	62932	1366	22. 46	84	99452	197	3. 17
40	64279	1347	22. 27	85	99620	168	2. 48
41	65606	1327	22. 7	86	99756	139	2. 16
42	66913	1307	21. 47	87	99863	107	1. 47
43	68200	1287	21. 27	88	99939	76	1. 6
44	69466	1266	21. 6	89	99985	46	0. 46
45	70711	1245	20. 45	90	100000	15	0. 15

Figure 39: Maurolico's table of sines [Maurolico (1558)] (source: Österreichische Nationalbibliothek).

TABELLA FOECUNDA.

g	Vmbra	Dfa	6^{ma}	g	Vmbra	Dfa	6^{ma}
1	1745.	1745	29. 5	46	103551	3551.	59. 11
2	3492	1747	29. 7	47	107236	3685.	61. 25
3	5241	1749	29. 9	48	111062	3826.	63. 46
4	6992	1751	29. 11	49	115037	3975.	66. 15
5	8748	1756	29. 16	50	119197	4160.	69. 20
6	10510	1762	29. 22	51	123491	4294	71. 34
7	12278	1768	29. 28	52	127994	4503	75 5
8	14053	1775	29. 35	53	132704	4710	78. 30
9	15838	1785	29. 45	54	137639	4935	82. 15
10	17633	1795	29. 55	55	142813	5174	86. 14
11	19439	1806	30. 6	56	148253	5440	90. 40
12	21256	1817	30. 17	57	153987	5734	95. 34
13	23087	1831	30. 31	58	160033	6046	100.46
14	24932	1845	30. 45	59	166429	6396	106.36
15	26794	1862	31. 2	60	173205	6776	112.56
16	28674	1880	31. 20	61	180405	7200	120. 0
17	30573	1899	31. 39	62	188073	7668	127.48
18	32492	1919	31. 59	63	196261	8188	136.28
19	34433	1941	32. 21	64	205030	8769	146. 9
20	36396	1963	32. 43	65	214451	9421	157. 1
21	38387	1991	33. 1,1	66	224603	10152	169. 12
22	40402	2015	33. 35	67	235585	10982	183. 2
23	42448	2046	34. 6	68	247509	11924	198.44
24	44522	2074	34. 34	69	260509	13000	216. 40
25	46631	2109	35. 9	70	274747	14238	237. 18
26	48772	2141	35. 41	71	290421	15674	261. 14
27	50952	2180	36. 20	72	307768	17347	289. 7
28	53170	2218	36. 58	73	327084	19316	321. 56
29	55432	2262	37. 42	74	348742	21658	360. 58
30	57735	2303	38. 23	75	373205	24463	407. 43
31	60086	2351	39. 11	76	401078	27873	464. 33
32	62486	2400	40. 0	77	433148	32070	534. 30
33	64940	2454	40. 54	78	470453	37305	621. 45
34	67452	2512	41. 51	79	514455	44002	733. 22
35	70022	2570	42. 50	80	567128	52673	877. 53
36	72654	2632	43. 52	81	631375	64247	1070. 47
37	75356	2702	45. 2	82	711537	80162	1336. 2
38	78129	2773	46. 13	83	814435	102898	1714. 18
39	80978	2849	47. 29	84	951436	137001	2283. 21
40	83909	2931	48. 51	85	1143005	191569	3192. 49
41	86929	3020	50. 20	86	1430067	287062	4784. 22
42	90040	3111	51. 51	87	1908113	478046	7967. 26
43	63252	3212	53. 52	88	2863625	955512	15925. 12
44	96571	3319	55. 19	89	5728995	2865370	47756. 10
45	100000	3429	57. 9	90	Infinitum.	Infinitum.	Infinitum.

89. 15. . . . . . 7638998.
89. 30. . . . . 11458872.
89. 45. . . 22918163.
89. 55. . . 68754439.
89. 59. . 343772546.

Figure 40: Maurolico's table of tangents [Maurolico (1558)] (source: Österreichische Nationalbibliothek).

TABELLA BENEFICA 66

g	Radius	Dfa	60m	g	Radius	Dfa	60m
1	100015	.. 15	0. 15	46	143955	.2534	42. 14
2	100061	.. 46	0. 46	47	146628	.2673	44. 33
3	100137	.. 76	1. 16	48	149448	.2820	47. 0
4	100244	.. 107	2. 47	49	152425	.2977	49. 37
5	100382	.. 138	2. 18	50	155572	.3147	52. 27
6	100551	.. 169	2. 49	51	158902	.3330	55. 30
7	100751	.. 200	3. 20	52	162427	.3525	58. 45
8	100983	.. 232	3. 52	53	166165	.3738	62. 18
9	101246	.. 264	4. 24	54	170131	.3966	66. 6
10	101543	.. 297	4. 57	55	174344	.4213	70. 13
11	101871	.. 328	5. 28	56	178830	.4486	74. 46
12	102234	.. 363	6. 3	57	183608	.4778	79. 38
13	102630	.. 396	6. 30	58	188708	.5100	85. 0
14	103061	.. 431	7. 11	59	194160	.5452	90. 52
15	103528	.. 467	7. 47	60	200000	.5840	97. 20
16	104030	.. 502	8. 22	61	206267	.6267	104. 27
17	104569	.. 539	8. 59	62	213006	.6739	112. 19
18	105146	.. 577	9. 37	63	220269	.7263	121. 3
19	105762	.. 616	10. 16	64	228117	.7848	130. 48
20	106418	.. 656	10. 56	65	236620	.8503	141. 43
21	107115	.. 697	11. 37	66	245859	.9239	153. 59
22	107854	.. 739	12. 19	67	255930	10071	167. 51
23	108636	.. 782	13. 2	68	266947	11017	183. 37
24	109464	.. 828	13. 48	69	279043	12096	201. 36
25	110338	.. 874	14. 34	70	292380	13337	222. 17
26	111260	.. 922	15. 22	71	307155	14775	246. 15
27	112233	.. 973	16. 13	72	323607	16452	274. 12
28	113257	. 1024	17. 4	73	342030	18423	307. 3
29	114335	. 1078	17. 58	74	362796	20766	346. 6
30	115470	. 1135	18. 55	75	386370	23574	392. 54
31	116664	. 1194	19. 54	76	413357	26987	449. 47
32	117918	. 1254	20. 54	77	444541	31184	519. 44
33	119236	. 1318	21. 58	78	480973	36432	607. 12
34	120621	. 1385	23. 5	79	524084	43111	718. 31
35	122078	. 1457	24. 17	80	575877	51793	863. 13
36	123606	. 1528	25. 28	81	639245	63368	1056. 8
37	125214	. 1608	26. 48	82	718530	79285	1321. 25
38	126902	. 1688	28. 8	83	820552	102022	1700. 22
39	128676	. 1774	29. 34	84	956677	136125	2268. 45
40	130541	. 1865	31. 5	85	1147371	190694	3178. 14
41	132501	. 1960	32. 40	86	1433558	286187	4769. 47
42	134563	. 2062	34. 22	87	1910732	477174	7952. 54
43	136733	. 2170	39. 10	88	2865371	954639	15910. 39
44	139016	. 2283	38. 3	89	5729868	2864497	47741. 37
45	141421	. 2405	40. 5	90	Infinitum.	Infinitum.	Infinitum.

0. 0 100000
89. 15 . . . 7639653
89. 30 . . . 11459309
89. 45 . . . 22918381
89. 55 . . . 68754512
89. 59 . . . 343772560.

Figure 41: Maurolico's table of secants [Maurolico (1558)] (source: Österreichische Nationalbibliothek).

Tabula Sinuum ad 10000000. particulas computata.

G / m	0 Sinus	portio uni9 2 10	1 Sinus	portio uni9 2 10	2 Sinus	portio uni9 2 10	3 Sinus	portio uni9 2 10	4 Sinus	portio uni9 2 10
0	0		174524	48 5	348995	48 4	523360	48 4	697565	48 4
1	2909	48 5	177433		351902		526265		700467	
2	5818		180341		354809		529170		703369	
3	8727		183250		357716		532075		706270	
4	11636		186158		360623		534980		709172	
5	14544		189066		363530		537884		712073	
6	17453		191975		366437		540789		714975	
7	20362		194883		369344		543694		717876	
8	23271		197792		372251		546598		720777	
9	26180		200700		375158		549503		723678	
10	29088		203608		378064		552407		726579	
11	31997		206517		380971		555312		729480	
12	34906		209425		383878		558216		732381	
13	37815		212333		386785		561120		735282	48 3
14	40724		215241		389692		564024		738183	
15	43632		218149		392598		566928		741084	
16	46541		221057		395505		569832		743985	
17	49450		223965		398412		572736		746886	
18	52359		226873		401318		575640		749787	
19	55268		229781		404225		578544		752688	
20	58177		232689		407131		581448		755588	
21	61086		235597		410038		584352		758489	
22	63995		238505		412944		587256		761389	
23	66904		241413		415851		590160		764290	
24	69813		244321		418757		593064		767180	
25	72721		247229		421663		595967		770090	
26	75630		250137		424570		598871		772991	
27	78539		253045		427476		601775		775891	
28	81448		255953		430382		604678		778791	
29	84357		258861		433288		607582		781691	
30	87265		261769		436194		610485		784591	

P

Figure 42: Eisenmenger's table of sines [Eisenmenger (1562)].

TABVLA FOECVNDA.

G	Numerus	G	Numerus	G	Numerus
0	00000	31	60086	61	180402
1	1745	32	62486	62	188075
2	3492	33	64940	63	196263
3	5240	34	67452	64	205034
4	6992	35	70022	65	214450
5	8748	36	72654	66	224697
6	10511	37	75356	67	235583
7	12278	38	78129	68	247513
8	14053	39	80978	69	260511
9	15838	40	83909	70	274753
10	17633	41	86929	71	290422
11	19439	42	90040	72	307767
12	21256	43	93254	73	327088
13	23087	44	96571	74	348748
14	24932	45	100000	75	373211
15	26794	46	103551	76	401089
16	28674	47	107236	77	433148
17	30573	48	111062	78	470453
18	32492	49	115037	79	514438
19	34433	50	119177	80	567118
20	36396	51	123491	81	631377
21	38387	52	127994	82	711569
22	40402	53	132704	83	814456
23	42448	54	137639	84	951387
24	44522	55	142813	85	1143131
25	46631	56	148253	86	1430203
26	48772	57	153987	87	1908217
27	50952	58	160035	88	2863563
28	53170	59	166429	89	5729796
29	55432	60	173207	90	Infinitum
30	57734				

Figure 43: Excerpt of Schreckenfuchs's table of tangents [Schreckenfuchs (1569)].

Arcus.		Sinus.	Arcus.		Sinus.	Arcus.		Sinus.	Arcus.		Sinus.
G.	M.	par.	G.	M.	par.	G.	M.	par.	G.	M.	par.
0	15	261	11	30	11962	23	0	23443	34	15	33768
0	30	523	11	5	12218	23	15	23684	34	30	33984
0	45	785	12	0	12474	23	30	23934	34	45	34199
1	0	1047	12	15	12730	23	45	24164	35	0	34414
1	15	1308	12	30	12086	24	0	24404	35	15	34628
1	30	1570	12	45	13241	24	15	24643	35	30	34842
1	45	1832	13	0	13497	24	30	24881	35	45	35054
2	0	2093	13	15	13752	24	45	25119	36	0	35267
2	15	2355	13	30	14006	25	0	25357	36	15	35478
2	30	2617	14	0	14515	25	15	25594	36	30	35689
2	45	2878	14	15	14769	25	30	25830	36	45	35899
3	0	3140	14	30	15022	25	45	26066	37	0	36189
3	15	3401	14	45	15276	26	0	26302	37	15	36317
3	30	3652	15	0	15529	26	15	26537	37	30	36525
3	45	3924	15	15	15781	26	30	26771	37	45	36733
4	0	4185	15	30	16034	26	45	27005	38	0	36939
4	15	4446	15	45	16286	27	0	27239	38	15	37145
4	30	4707	16	0	16538	27	15	27472	38	30	37350
4	45	4968	16	15	16789	27	30	27704	38	45	37555
5	0	5229	16	30	17040	27	45	27936	39	0	37759
5	15	5490	16	45	17291	28	0	28168	39	15	37962
5	30	5750	17	0	17541	28	15	28399	39	30	38164
5	45	6011	17	15	17792	28	30	28629	39	45	38366
6	0	6271	17	30	18042	28	45	28858	40	0	38567
6	15	6532	17	45	18291	29	0	29088	40	15	38767
6	30	6792	18	0	18541	29	15	29317	40	30	38966
6	45	7052	18	15	18789	29	30	29545	40	45	39165
7	0	7312	18	30	19038	29	45	29742	41	0	39363
7	15	7571	18	45	19286	30	0	30000	41	15	39560
7	30	7831	19	0	19534	30	15	30226	41	30	39757
7	45	8091	19	15	19781	30	30	30452	41	45	39952
8	0	8350	19	30	20028	30	45	30677	42	0	40147
8	15	8609	19	45	20275	31	0	30902	42	15	40342
8	30	8868	20	0	20521	31	15	31126	42	30	40535
8	45	9127	20	15	20767	31	30	31349	42	45	40728
9	0	9386	20	30	21012	31	45	31572	43	0	40919
9	15	9644	20	45	21257	32	0	31795	43	15	41110
9	30	9902	21	0	21502	32	15	32016	43	30	41301
9	45	10160	21	15	21746	32	30	32237	43	45	41449
10	0	10418	21	30	21990	32	45	32458	44	0	41679
10	15	10676	21	45	22233	33	0	32678	44	15	41867
10	30	10934	22	0	22476	33	15	32897	49	30	42024
10	45	11191	22	15	22718	33	30	33116	44	45	42240
11	0	11448	22	30	22961	33	45	33334	45	0	42426
11	15	11075	22	45	23202	34	0	33551	45	15	42611

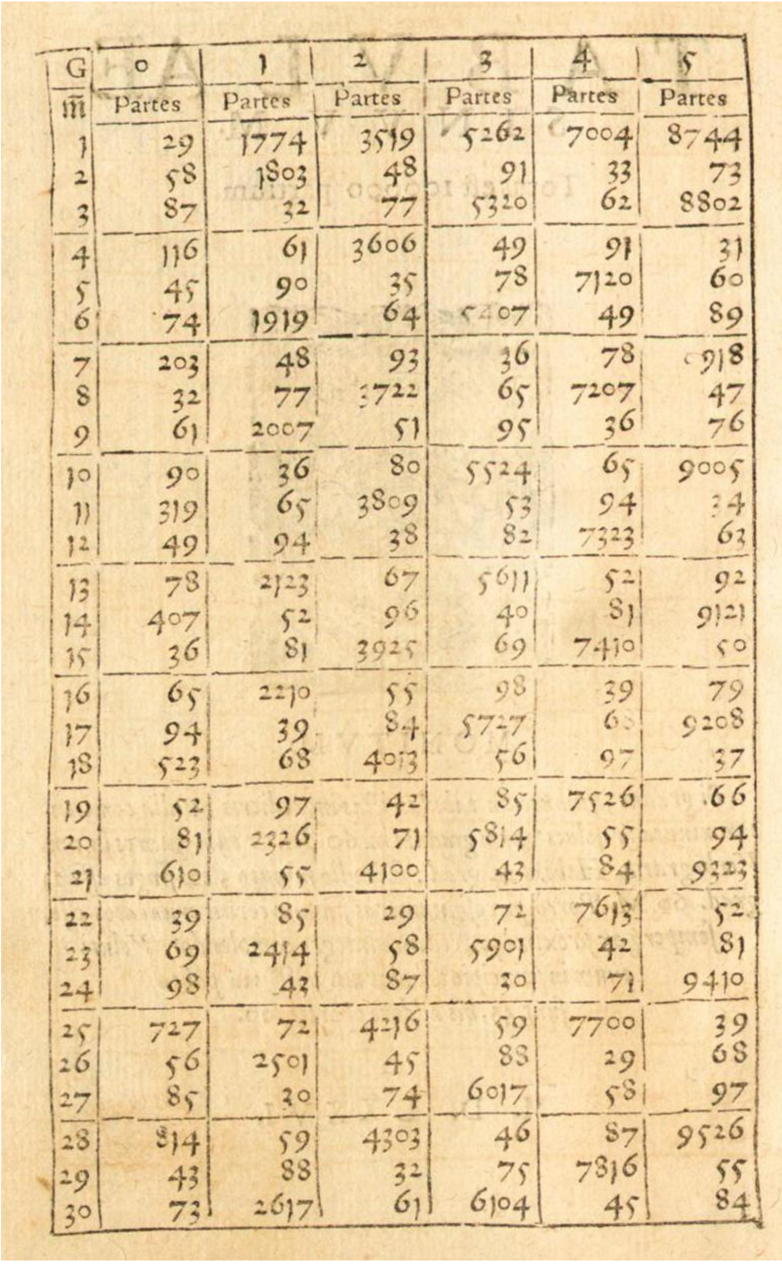
Figure 44: Excerpt of Schreckenfuchs's first table of sines [Schreckenfuchs (1569)].

Tabula sinuum rectorum.

G.	M.	par.	M.	Sa.	la.
0	15	0	15	42	30
0	30	0	31	25	0
6	45	0	47	7	30
1	0	1	2	50	0
1	15	1	18	32	0
1	30	1	34	14	0
1	45	1	49	56	0
2	0	2	5	38	0
2	15	2	21	20	0
2	30	2	37	2	0
2	45	2	52	43	30
3	0	3	8	24	30
3	15	3	24	5	30
3	30	3	39	46	30
3	45	3	55	27	0
4	0	4	11	7	30
4	15	4	26	47	30
4	45	4	58	27	30
5	0	5	13	46	30
5	15	5	29	24	0
5	30	5	45	2	30
5	45	6	0	40	30
6	0	6	16	18	0
6	15	6	31	55	0
6	30	6	47	32	0
6	45	7	3	8	0
7	0	7	18	43	30
7	15	7	34	19	0
7	30	7	49	53	30
7	45	8	5	28	0
8	0	8	21	1	30
8	15	8	36	34	30
8	30	8	52	7	0
8	45	9	7	38	30

G.	M.	par.	M.	Sa.	la.
9	0	9	23	9	30
9	15	9	38	40	30
9	30	9	54	10	30
9	45	10	9	39	30
10	0	10	25	8	0
10	15	10	40	36	0
10	30	10	56	3	0
10	45	11	11	29	0
11	0	11	26	54	30
11	15	11	42	19	30
11	30	11	57	43	30
11	45	12	13	6	30
12	0	12	28	29	0
12	15	12	43	50	30
12	30	12	59	11	0
12	45	13	14	30	30
15	0	13	29	49	0
13	15	13	45	7	0
13	30	14	0	24	0
13	45	14	15	40	0
14	0	14	30	55	0
14	15	14	46	9	0
14	30	15	1	22	0
14	45	15	16	34	0
15	0	15	31	45	0
15	15	15	46	55	0
15	30	16	2	4	0
15	45	16	17	11	0
16	0	16	32	17	30
16	15	16	47	23	0
16	30	17	2	27	30
16	45	17	17	30	30
17	0	17	32	32	30
17	15	17	47	33	0

Figure 45: Excerpt of Schreckenfuchs's second table of sines [Schreckenfuchs (1569)].



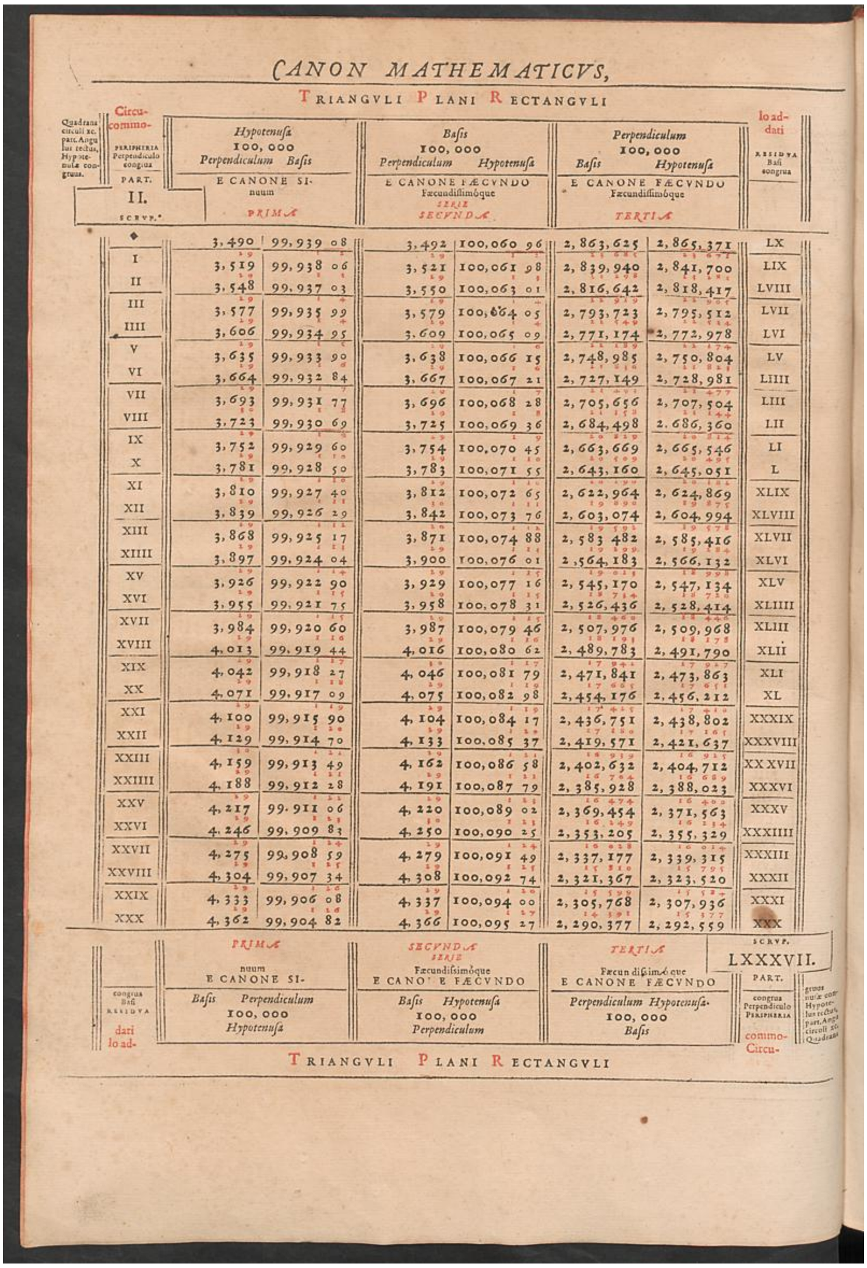
G	0	1	2	3	4	5
m̄	Partes	Partes	Partes	Partes	Partes	Partes
1	29	1774	3519	5262	7004	8744
2	58	1803	48	91	33	73
3	87	32	77	5320	62	8802
4	116	61	3606	49	91	31
5	45	90	35	78	7120	60
6	74	1919	64	5407	49	89
7	203	48	93	36	78	918
8	32	77	3722	65	7207	47
9	61	2007	51	95	36	76
10	90	36	80	5524	65	9005
11	319	65	3809	53	94	34
12	49	94	38	82	7323	63
13	78	2123	67	5611	52	92
14	407	52	96	40	81	9121
15	36	81	3925	69	7410	50
16	65	2210	55	98	39	79
17	94	39	84	5727	68	9208
18	523	68	4013	56	97	37
19	52	97	42	85	7526	66
20	81	2326	71	5814	55	94
21	610	55	4100	43	84	9323
22	39	85	29	72	7613	52
23	69	2414	58	5901	42	81
24	98	43	87	30	71	9410
25	727	72	4216	59	7700	39
26	56	2501	45	88	29	68
27	85	30	74	6017	58	97
28	814	59	4303	46	87	9526
29	43	88	32	75	7816	55
30	73	2617	61	6104	45	84

Figure 46: Excerpt of Witekind's table of sines [Witekind (1576)].

174 *Tabula sinuum rectorum siue semichordarum minutim extensa.*

	0	1	2	3	4
m.	Sinus.	Sinus.	Sinus.	Sinus.	Sinus.
0	00	1745	3489	5233	6975
1	29	1774	3519	5262	7004
2	58	1803	3548	5291	7033
3	87	1832	3577	5320	7062
4	116	1861	3606	5349	7091
5	145	1890	3635	5378	7120
6	174	1919	3664	5407	7149
7	203	1948	3693	5436	7178
8	232	1977	3722	5465	7207
9	261	2007	3751	5405	7236
10	290	2036	3780	5584	7265
11	319	2065	3809	5553	7294
12	349	2094	3838	5582	7323
13	783	2123	3867	5611	7352
14	407	2152	3896	5640	7381
15	436	2181	3925	5669	7410
16	465	2210	3955	5698	7439
17	494	2239	3984	5727	7468
18	523	2268	4013	5756	7497
19	552	2297	4042	5785	7526
20	581	2326	4071	5814	7555
21	610	2355	4100	5843	7584
22	639	2385	4129	5872	7613
23	669	2414	4158	5901	7642
24	698	2443	4187	5930	7671
25	727	2472	4216	5959	7700
26	756	2501	4245	5988	7729
27	785	2530	4274	5017	7758
28	814	2559	4303	6046	7787
29	843	2588	4332	6075	7816
30	872	2617	4361	6104	7845

Figure 47: Excerpt of Peucer's table of sines [Peucer (1579)].



CANON MATHEMATICVS,

TRIANGVLI PLANI RECTANGVLI

Circumcommodo PERIPHERIA Perpendiculo congrua PART. II. SCRVP.	Hypotenusa 100,000 — Perpendiculum (E CANONE SINUUM PRIMAE)	Basis	Basis 100,000 — Perpendiculum (E CANONE FAECVNDO Faecundissimoque SERIE SECVNDAE)	Hypotenusa	Perpendiculum 100,000 — Basis (E CANONE FAECVNDO Faecundissimoque TERTIAE)	Hypotenusa	lo addati RESIDVA Basi congrua
◆	3,490	99,939 08	3,492	100,060 96	2,863,625	2,865,371	LX
I	3,519	99,938 06	3,521	100,061 98	2,839,940	2,841,700	LIX
II	3,548	99,937 03	3,550	100,063 01	2,816,642	2,818,417	LVIII
III	3,577	99,935 99	3,579	100,064 05	2,793,723	2,795,512	LVII
IIII	3,606	99,934 95	3,609	100,065 09	2,771,174	2,772,978	LVI
V	3,635	99,933 90	3,638	100,066 15	2,748,985	2,750,804	LV
VI	3,664	99,932 84	3,667	100,067 21	2,727,149	2,728,981	LIIII
VII	3,693	99,931 77	3,696	100,068 28	2,705,656	2,707,504	LIII
VIII	3,723	99,930 69	3,725	100,069 36	2,684,498	2.686,360	LII
IX	3,752	99,929 60	3,754	100,070 45	2,663,669	2,665,546	LI
X	3,781	99,928 50	3,783	100,071 55	2,643,160	2,645,051	L
XI	3,810	99,927 40	3,812	100,072 65	2,622,964	2,624,869	XLIX
XII	3,839	99,926 29	3,842	100,073 76	2,603,074	2,604,994	XLVIII
XIII	3,868	99,925 17	3,871	100,074 88	2,583 482	2,585,416	XLVII
XIIII	3,897	99,924 04	3,900	100,076 01	2,564,183	2,566,132	XLVI
XV	3,926	99,922 90	3,929	100,077 16	2,545,170	2,547,134	XLV
XVI	3,955	99,921 75	3,958	100,078 31	2,526,436	2,528,414	XLIIII
XVII	3,984	99,920 60	3,987	100,079 46	2,507,976	2,509,968	XLIII
XVIII	4,013	99,919 44	4,016	100,080 62	2,489,783	2,491,790	XLII
XIX	4,042	99,918 27	4,046	100,081 79	2,471,841	2,473,863	XLI
XX	4,071	99,917 09	4,075	100,082 98	2,454,176	2,456,212	XL
XXI	4,100	99,915 90	4,104	100,084 17	2,436,751	2,438,802	XXXIX
XXII	4,129	99,914 70	4,133	100,085 37	2,419,571	2,421,637	XXXVIII
XXIII	4,159	99,913 49	4,162	100,086 58	2,402,632	2,404,712	XX XVII
XXIIII	4,188	99,912 28	4,191	100,087 79	2,385,928	2,388,023	XXXVI
XXV	4,217	99.911 06	4,220	100,089 02	2,369,454	2,371,563	XXXV
XXVI	4,246	99,909 83	4,250	100,090 25	2,353,205	2,355,329	XXXIIII
XXVII	4,275	99,908 59	4,279	100,091 49	2,337,177	2,339,315	XXXIII
XXVIII	4,304	99,907 34	4,308	100,092 74	2,321,367	2,323,520	XXXII
XXIX	4,333	99,906 08	4,337	100,094 00	2,305,768	2,307,936	XXXI
XXX	4,362	99,904 82	4,366	100,095 27	2,290,377	2,292,559	XXX
congrua Basi RESIDVA dari lo ad-	Basis (PRIMAE E CANONE SINUUM)	Perpendiculum — 100,000 Hypotenusa	Basis (SECVNDAE SERIE E CANONE FAECVNDO Faecundissimoque)	Hypotenusa — 100,000 Perpendiculum	Perpendiculum (TERTIAE E CANONE FAECVNDO Faecundissimoque)	Hypotenusa — 100,000 Basis	SCRVP. LXXXVII. PART. congrua Perpendiculo PERIPHERIA commodo Circu-

TRIANGVLI PLANI RECTANGVLI

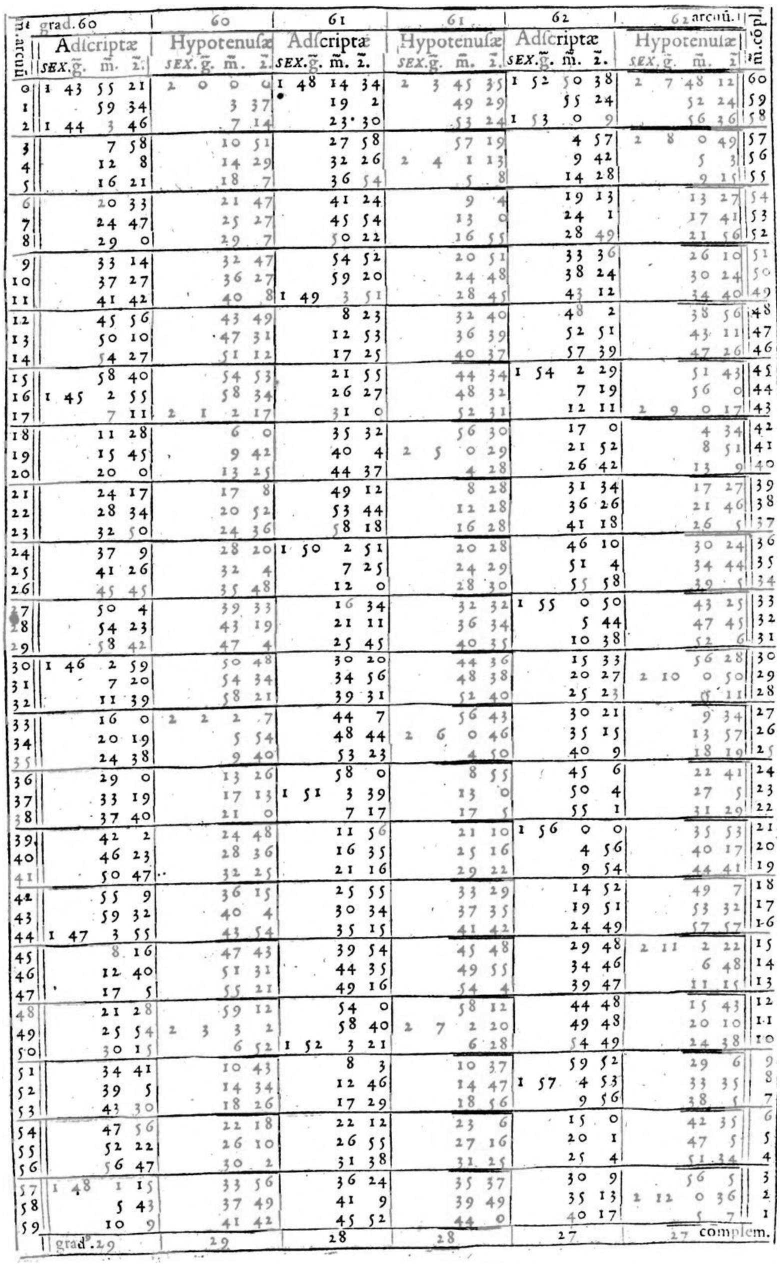
Figure 48: Excerpt of Viète's *Canon mathematicus* [Viète (1579)] (source: e-rara).

CANON SINVVM. 27

m. arcus	grad. 18 Sinus. g.	m.	2.	71 Sinus. g.	m.	2.	19 Sinus. g.	m.	2.	70 Sinus. g.	m.	2.	20 Sinus. g.	m.	2.	69 arcuū. Sinus. g.	m.	2.	m. arcus
0	18	32	28	57	3	48	19	32	3	56	43	52	20	31	16	56	22	54	60
1		33	27		3	29		33	2		43	31		32	15		22	32	59
2		34	27		3	9		34	1		43	11		33	14		22	10	58
3		35	27		2	50		35	1		42	50		34	13		21	49	57
4		36	27		2	30		36	0		42	30		35	12		21	27	56
5		37	26		2	11		37	0		42	9		36	11		21	6	55
6		38	26		1	51		37	59		41	48		37	10		20	44	54
7		39	26		1	31		38	58		41	28		38	9		20	22	53
8		40	25		1	12		39	58		41	7		39	8		20	1	52
9		41	25		0	52		40	57		40	47		40	7		19	39	51
10		42	25		0	33		41	56		40	26		41	6		19	17	50
11		43	25		0	13		42	56		40	6		42	5		18	56	49
12		44	24	56	59	54		43	55		39	45		43	4		18	34	48
13		45	24		59	34		44	54		39	24		44	3		18	13	47
14		46	24		59	15		45	54		39	4		45	2		17	51	46
15		47	23		58	55		46	53		38	43		46	1		17	29	45
16		48	23		58	35		47	52		38	22		47	0		17	8	44
17		49	23		58	15		48	52		38	2		47	59		16	46	43
18		50	22		57	56		49	51		37	41		48	58		16	24	42
19		51	22		57	36		50	50		37	20		49	57		16	2	41
20		52	22		57	16		51	50		36	59		50	56		15	40	40
21		53	21		56	56		52	49		36	38		51	55		15	18	39
22		54	21		56	36		53	48		36	17		52	54		14	56	38
23		55	20		56	17		54	47		35	57		53	53		14	34	37
24		56	20		55	57		55	47		35	36		54	51		14	13	36
25		57	20		55	37		56	46		35	15		55	50		13	51	35
26		58	19		55	17		57	45		34	54		56	49		13	29	34
27		59	19		54	57		58	44		34	33		57	48		13	7	33
28	19	0	19		54	38		59	44		34	12		58	47		12	45	32
29		1	18		54	18	20	0	43		33	51		59	46		12	23	31
30		2	18		53	58		1	42		33	31	21	0	45		12	1	30
31		3	17		53	38		2	41		33	9		1	44		11	39	29
32		4	17		53	18		3	40		32	48		2	42		11	17	28
33		5	16		52	58		4	40		32	27		3	41		10	55	27
34		6	16		52	38		5	39		32	6		4	40		10	33	26
35		7	15		52	18		6	38		31	45		5	39		10	11	25
36		8	15		51	58		7	37		31	24		6	38		9	48	24
37		9	15		51	38		8	37		31	3		7	36		9	26	23
38		10	14		51	17		9	36		30	42		8	35		9	4	22
39		11	14		50	57		10	35		30	21		9	34		8	42	21
40		12	13		50	37		11	34		30	0		10	33		8	20	20
41		13	13		50	17		12	33		29	38		11	32		7	58	19
42		14	12		49	57		13	32		29	17		12	30		7	36	18
43		15	12		49	37		14	32		28	50		13	29		7	13	17
44		16	11		49	17		15	31		28	35		14	28		6	51	16
45		17	11		48	57		16	30		28	14		15	27		6	29	15
46		18	10		48	37		17	29		27	53		16	26		6	7	14
47		19	10		48	16		18	28		27	31		17	24		5	44	13
48		20	9		47	56		19	27		27	10		18	23		5	22	12
49		21	9		47	36		20	26		26	49		19	22		5	0	11
50		22	8		47	15		21	26		26	27		20	20		4	37	10
51		23	8		46	55		22	25		26	6		21	19		4	15	9
52		24	7		46	35		23	24		25	45		22	18		3	52	8
53		25	7		46	14		24	23		25	23		23	16		3	30	7
54		26	6		45	54		25	22		25	2		24	15		3	8	6
55		27	5		45	34		26	21		24	40		25	14		2	45	5
56		28	5		45	13		27	20		24	19		26	13		2	23	4
57		29	4		44	53		28	19		23	58		27	11		2	1	3
58		30	4		44	33		29	18		23	36		28	10		1	38	2
59		31	3		44	12		30	17		23	15		29	8		1	16	1

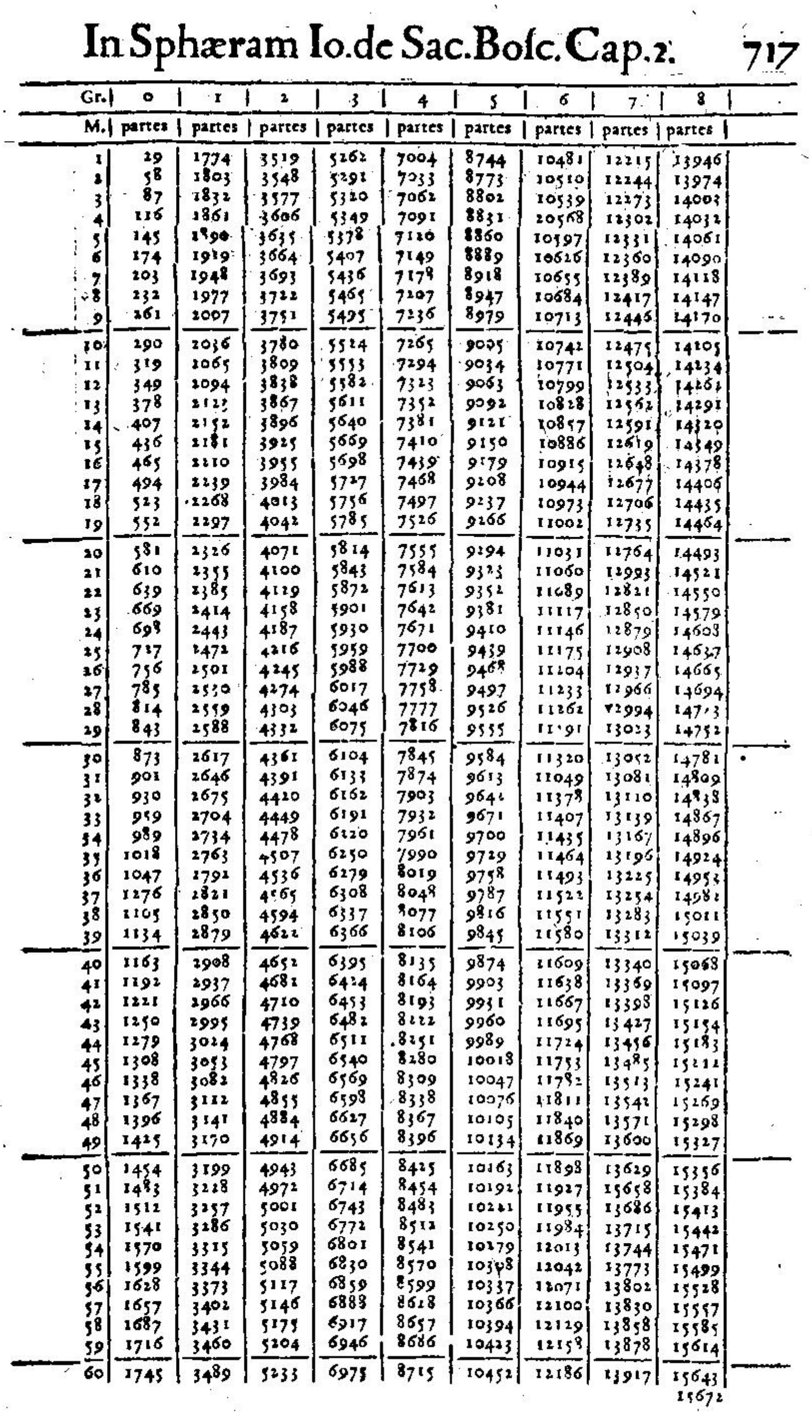
c iiij

Figure 49: Excerpt of Bressieu's table of sines [Bressieu (1581)] (source: Google Books).



m. arcus	grad. 60 Adscriptæ SEX. g. m. 2.	60 Hypotenusæ SEX. g. m. 2.	61 Adscriptæ SEX. g. m. 2.	61 Hypotenusæ SEX. g. m. 2.	62 Adscriptæ SEX. g. m. 2.	62 arcus. Hypotenusæ SEX. g. m. 2.	m. compl.
0	1 43 55 21	2 0 0 0	1 48 14 34	2 3 45 35	1 52 50 38	2 7 48 12	60
1	59 34	3 37	19 2	49 29	55 24	52 24	59
2	1 44 3 46	7 14	23 30	53 24	1 53 0 9	56 36	58
3	7 58	10 51	27 58	57 19	4 57	2 8 0 49	57
4	12 8	14 29	32 26	2 4 1 13	9 42	5 3	56
5	16 21	18 7	36 54	5 8	14 28	9 15	55
6	20 33	21 47	41 24	9 4	19 13	13 27	54
7	24 47	25 27	45 54	13 0	24 1	17 41	53
8	29 0	29 7	50 22	16 55	28 49	21 56	52
9	33 14	32 47	54 52	20 51	33 36	26 10	51
10	37 27	36 27	59 20	24 48	38 24	30 24	50
11	41 42	40 8	1 49 3 51	28 45	43 12	34 40	49
12	45 56	43 49	8 23	32 40	48 2	38 56	48
13	50 10	47 31	12 53	36 39	52 51	43 11	47
14	54 27	51 12	17 25	40 37	57 39	47 26	46
15	58 40	54 53	21 55	44 34	1 54 2 29	51 43	45
16	1 45 2 55	58 34	26 27	48 32	7 19	56 0	44
17	7 11	2 1 2 17	31 0	52 31	12 11	2 9 0 17	43
18	11 28	6 0	35 32	56 30	17 0	4 34	42
19	15 45	9 42	40 4	2 5 0 29	21 52	8 51	41
20	20 0	13 25	44 37	4 28	26 42	13 9	40
21	24 17	17 8	49 12	8 28	31 34	17 27	39
22	28 34	20 52	53 44	12 28	36 26	21 46	38
23	32 50	24 36	58 18	16 28	41 18	26 5	37
24	37 9	28 20	1 50 2 51	20 28	46 10	30 24	36
25	41 26	32 4	7 25	24 29	51 4	34 44	35
26	45 45	35 48	12 0	28 30	55 58	39 5	34
27	50 4	39 33	16 34	32 32	1 55 0 50	43 25	33
28	54 23	43 19	21 11	36 34	5 44	47 45	32
29	58 42	47 4	25 45	40 35	10 38	52 6	31
30	1 46 2 59	50 48	30 20	44 36	15 33	56 28	30
31	7 20	54 34	34 56	48 38	20 27	2 10 0 50	29
32	11 39	58 21	39 31	52 40	25 23	5 11	28
33	16 0	2 2 2 7	44 7	56 43	30 21	9 34	27
34	20 19	5 54	48 44	2 6 0 46	35 15	13 57	26
35	24 38	9 40	53 23	4 50	40 9	18 19	25
36	29 0	13 26	58 0	8 55	45 6	22 41	24
37	33 19	17 13	1 51 3 39	13 0	50 4	27 5	23
38	37 40	21 0	7 17	17 5	55 1	31 29	22
39	42 2	24 48	11 56	21 10	1 56 0 0	35 53	21
40	46 23	28 36	16 35	25 16	4 56	40 17	20
41	50 47	32 25	21 16	29 22	9 54	44 41	19
42	55 9	36 15	25 55	33 29	14 52	49 7	18
43	59 32	40 4	30 34	37 35	19 51	53 32	17
44	1 47 3 55	43 54	35 15	41 42	24 49	57 57	16
45	8 16	47 43	39 54	45 48	29 48	2 11 2 22	15
46	12 40	51 31	44 35	49 55	34 46	6 48	14
47	17 5	55 21	49 16	54 4	39 47	11 15	13
48	21 28	59 12	54 0	58 12	44 48	15 43	12
49	25 54	2 3 3 2	58 40	2 7 2 20	49 48	20 10	11
50	30 15	6 52	1 52 3 21	6 28	54 49	24 38	10
51	34 41	10 43	8 3	10 37	59 52	29 6	9
52	39 5	14 34	12 46	14 47	1 57 4 53	33 35	8
53	43 30	18 26	17 29	18 56	9 56	38 5	7
54	47 56	22 18	22 12	23 6	15 0	42 35	6
55	52 22	26 10	26 55	27 16	20 1	47 5	5
56	56 47	30 2	31 38	31 25	25 4	51 34	4
57	1 48 1 15	33 56	36 24	35 37	30 9	56 5	3
58	5 43	37 49	41 9	39 49	35 13	2 12 0 36	2
59	10 9	41 42	45 52	44 0	40 17	5 7	1
	grad. 29	29	28	28	27	27 complem.	

Figure 50: Excerpt of Bressieu's table of tangents (odd columns) and secants (even columns) [Bressieu (1581)] (source: Google Books). Note that the faded parts are artefacts of the way Google Books stores images.



In Sphæram Io.de Sac.Bofc.Cap.2. 717

Gr.	0	1	2	3	4	5	6	7	8
M.	partes	partes	partes	partes	partes	partes	partes	partes	partes
1	29	1774	3519	5262	7004	8744	10481	12215	13946
2	58	1803	3548	5291	7033	8773	10510	12244	13974
3	87	1832	3577	5320	7062	8802	10539	12273	14003
4	116	1861	3606	5349	7091	8831	20568	12302	14032
5	145	1890	3635	5378	7120	8860	10597	12331	14061
6	174	1919	3664	5407	7149	8889	10626	12360	14090
7	203	1948	3693	5436	7178	8918	10655	12389	14118
8	232	1977	3722	5465	7207	8947	10684	12417	14147
9	261	2007	3751	5495	7236	8979	10713	12446	14170
10	290	2036	3780	5524	7265	9005	10742	12475	14105
11	319	2065	3809	5553	7294	9034	10771	12504	14234
12	349	2094	3838	5582	7323	9063	10799	12533	14262
13	378	2123	3867	5611	7352	9092	10828	12562	14291
14	407	2152	3896	5640	7381	9121	10857	12591	14320
15	436	2181	3925	5669	7410	9150	10886	12619	14349
16	465	2210	3955	5698	7439	9179	10915	12648	14378
17	494	2239	3984	5727	7468	9208	10944	12677	14406
18	523	2268	4013	5756	7497	9237	10973	12706	14435
19	552	2297	4042	5785	7526	9266	11002	12735	14464
20	581	2326	4071	5814	7555	9294	11031	12764	14493
21	610	2355	4100	5843	7584	9323	11060	12993	14521
22	639	2385	4129	5872	7613	9352	11689	12821	14550
23	669	2414	4158	5901	7642	9381	11117	12850	14579
24	698	2443	4187	5930	7671	9410	11146	12879	14608
25	727	2472	4216	5959	7700	9439	11175	12908	14637
26	756	2501	4245	5988	7729	9468	11204	12937	14665
27	785	2530	4274	6017	7758	9497	11233	12966	14694
28	814	2559	4303	6046	7777	9526	11262	12994	[illegible]
29	843	2588	4332	6075	7816	9555	[illegible]	13023	14752
30	873	2617	4361	6104	7845	9584	11320	13052	14781
31	901	2646	4391	6133	7874	9613	11049	13081	14809
32	930	2675	4420	6162	7903	9642	11378	13110	14838
33	959	2704	4449	6191	7932	9671	11407	13139	14867
34	989	2734	4478	6120	7961	9700	11435	13167	14896
35	1018	2763	4507	6250	7990	9729	11464	13196	14924
36	1047	2792	4536	6279	8019	9758	11493	13225	14953
37	1276	2821	4565	6308	8048	9787	11522	13254	14982
38	1105	2850	4594	6337	8077	9816	11551	13283	15011
39	1134	2879	4622	6366	8106	9845	11580	13312	15039
40	1163	2908	4652	6395	8135	9874	11609	13340	15068
41	1192	2937	4681	6424	8164	9903	11638	13369	15097
42	1221	2966	4710	6453	8193	9931	11667	13398	15126
43	1250	2995	4739	6482	8222	9960	11695	13427	15154
44	1279	3024	4768	6511	8251	9989	11724	13456	15183
45	1308	3053	4797	6540	8280	10018	11753	13485	15212
46	1338	3082	4826	6569	8309	10047	11782	13513	15241
47	1367	3112	4855	6598	8338	10076	11811	13542	15269
48	1396	3141	4884	6627	8367	10105	11840	13571	15298
49	1425	3170	4914	6656	8396	10134	11869	13600	15327
50	1454	3199	4943	6685	8425	10163	11898	13629	15356
51	1483	3228	4972	6714	8454	10192	11927	13658	15384
52	1512	3257	5001	6743	8483	10221	11955	13686	15413
53	1541	3286	5030	6772	8512	10250	11984	13715	15442
54	1570	3315	5059	6801	8541	10279	12013	13744	15471
55	1599	3344	5088	6830	8570	10308	12042	13773	15499
56	1628	3373	5117	6859	8599	10337	12071	13802	15528
57	1657	3402	5146	6888	8628	10366	12100	13830	15557
58	1687	3431	5175	6917	8657	10394	12129	13858	15585
59	1716	3460	5204	6946	8686	10423	12158	13878	15614
60	1745	3489	5233	6975	8715	10452	12186	13917	15643
									15672

Figure 51: An excerpt from Giuntini's table of sines [Giuntini (1581)] (source: Google books).

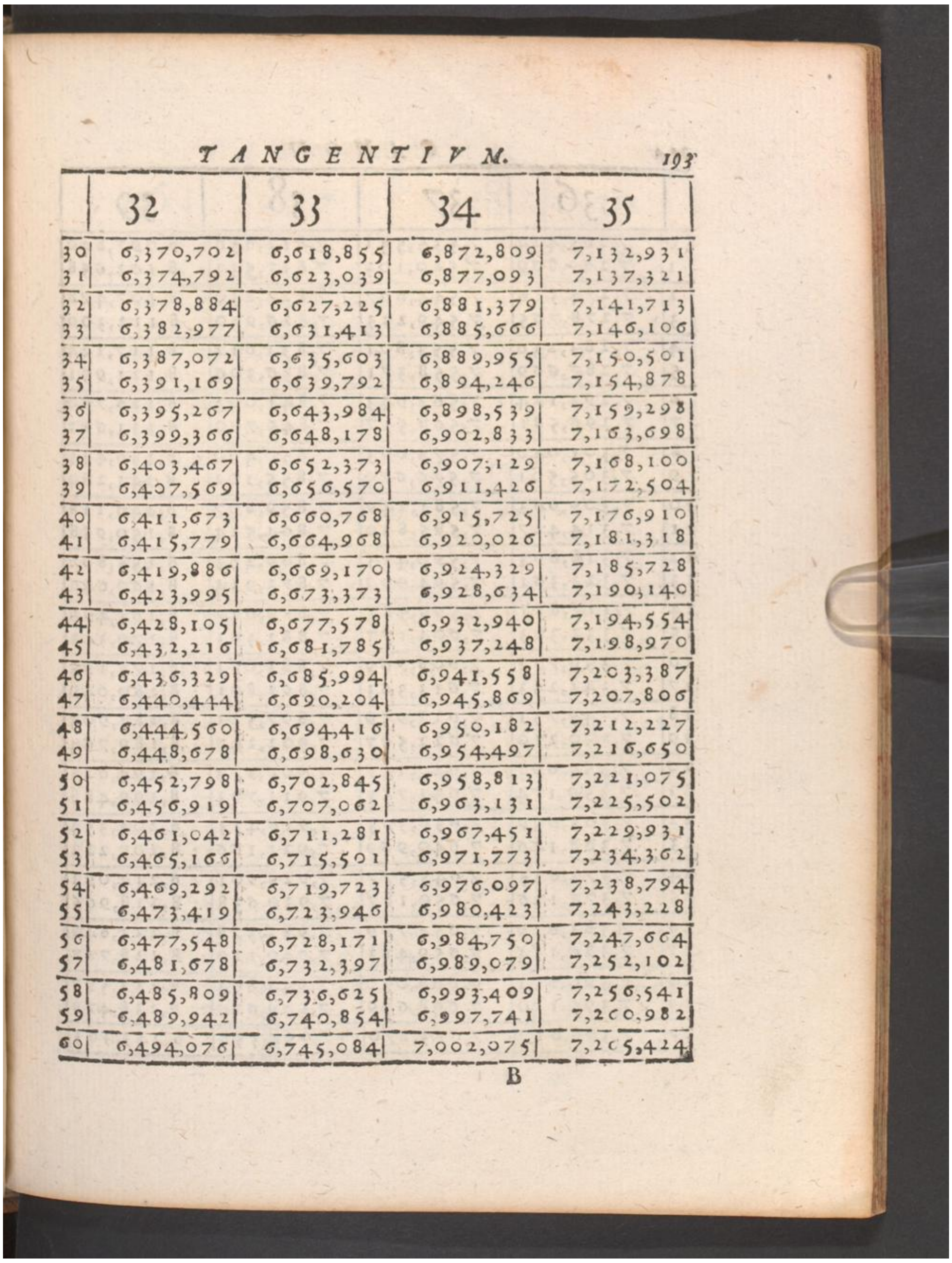
Pars Prima. 23

Sinuum rectorutabula.m

G	80	81	82	33	84	85	86	87	88	89
M	Sinus	Sinus	Sinus	Sinus	Sinus	Sinus	Sinus	Sinus	Sinus	Sinus
30	98628	98901	99144	99357	99539	99691	99813	99904	99964	99996
31	98633	98905	99148	99360	99542	99594	99815	99906	99965	99996
32	98638	98910	99152	99363	99545	99696	99817	99907	99966	99996
33	98642	98914	99155	99367	99547	99698	99818	99908	99967	99996
34	98647	98918	99159	99370	99550	99700	99820	99909	99967	99997
35	98652	98922	99163	99373	99553	99703	96822	99911	99968	99997
36	98657	98927	99167	99376	99556	99705	99823	99912	99969	99997
37	98661	98931	99170	99380	99558	99707	99825	99913	99970	99997
38	98666	98935	99174	99383	99561	99709	99827	99914	99970	99997
39	98671	98939	99178	99386	99564	99711	99829	99915	99971	99998
40	98676	98944	99182	99389	99567	99714	99830	99917	99972	99998
41	98681	98948	99185	99392	99569	99716	99832	99918	99972	99998
42	98685	98952	99189	99396	99572	99718	99834	99919	99973	99998
43	98690	98956	99193	99399	99575	99720	99835	99920	99974	99998
44	98694	98960	99196	99402	99577	99722	99837	99921	99974	99998
45	98699	98965	99200	99405	99580	99725	99839	99922	99975	99999
46	98704	98969	99204	99408	99583	99727	99840	99924	99975	99999
47	98708	98973	99207	99411	99585	99729	99842	99925	99976	99999
48	98713	98977	99211	99415	99588	99731	99844	99926	99977	99999
49	98718	98981	99215	99418	99591	99733	99845	99927	99978	99999
50	98722	98985	99218	99421	99593	99735	99847	99928	99978	99999
51	98727	98990	99222	99424	99596	99737	99848	99929	99979	99999
52	98732	98994	99225	99427	99598	99739	99850	99930	99980	99999
53	98736	98998	99229	99430	99601	99742	99852	99931	99981	99999
54	98741	99002	99233	99433	99604	99744	99853	99932	99981	99999
55	98745	99006	99236	99436	99606	99746	99855	99933	99982	99999
56	98750	99010	99240	99439	99609	99748	99856	99934	99982	99999
57	98755	99014	99244	99443	99611	99750	99858	99935	99983	99999
58	98759	99018	99247	99446	99614	99752	99859	99937	99983	99999
59	98764	99022	99251	9944[illegible]	99616	99754	99861	99938	99984	100000
60	98768	99025	99254	99452	99619	99756	99862	99939	99984	100000

Sinuum

Figure 52: An excerpt from Padovani's table of sines [Giuntini (1581)] (source: Google books).

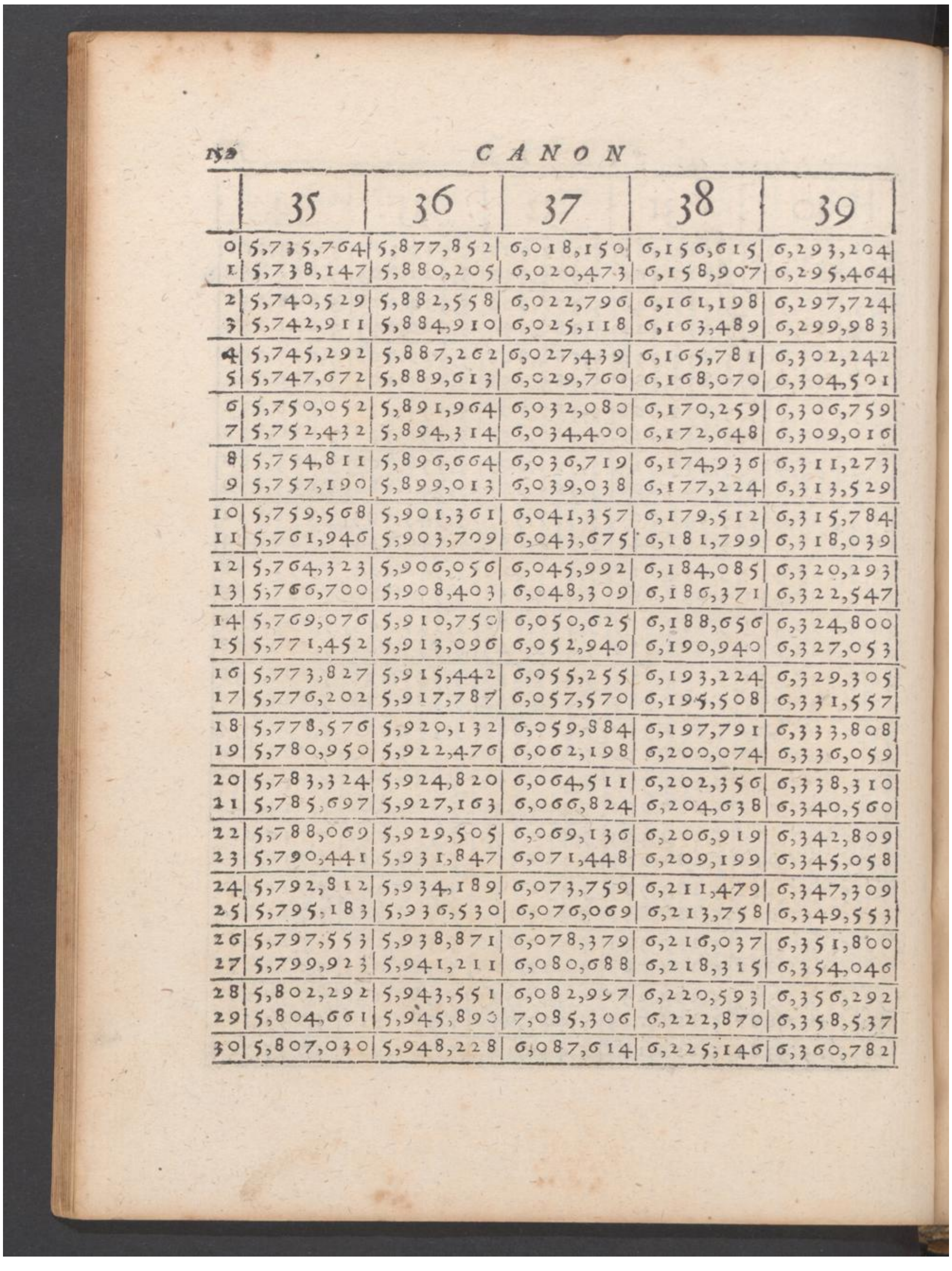


TANGENTIVM. 193

	32	33	34	35
30	6,370,702	6,618,855	6,872,809	7,132,931
31	6,374,792	6,623,039	6,877,093	7,137,321
32	6,378,884	6,627,225	6,881,379	7,141,713
33	6,382,977	6,631,413	6,885,666	7,146,106
34	6,387,072	6,635,603	6,889,955	7,150,501
35	6,391,169	6,639,792	6,894,246	7,154,878
36	6,395,267	6,643,984	6,898,539	7,159,298
37	6,399,366	6,648,178	6,902,833	7,163,698
38	6,403,467	6,652,373	6,907,129	7,168,100
39	6,407,569	6,656,570	6,911,426	7,172,504
40	6,411,673	6,660,768	6,915,725	7,176,910
41	6,415,779	6,664,968	6,920,026	7,181,318
42	6,419,886	6,669,170	6,924,329	7,185,728
43	6,423,995	6,673,373	6,928,634	7,190,140
44	6,428,105	6,677,578	6,932,940	7,194,554
45	6,432,216	6,681,785	6,937,248	7,198,970
46	6,436,329	6,685,994	6,941,558	7,203,387
47	6,440,444	6,690,204	6,945,869	7,207,806
48	6,444,560	6,694,416	6,950,182	7,212,227
49	6,448,678	6,698,630	6,954,497	7,216,650
50	6,452,798	6,702,845	6,958,813	7,221,075
51	6,456,919	6,707,062	6,963,131	7,225,502
52	6,461,042	6,711,281	6,967,451	7,229,931
53	6,465,166	6,715,501	6,971,773	7,234,362
54	6,469,292	6,719,723	6,976,097	7,238,794
55	6,473,419	6,723,946	6,980,423	7,243,228
56	6,477,548	6,728,171	6,984,750	7,247,664
57	6,481,678	6,732,397	6,989,079	7,252,102
58	6,485,809	6,736,625	6,993,409	7,256,541
59	6,489,942	6,740,854	6,997,741	7,260,982
60	6,494,076	6,745,084	7,002,075	7,265,424

B

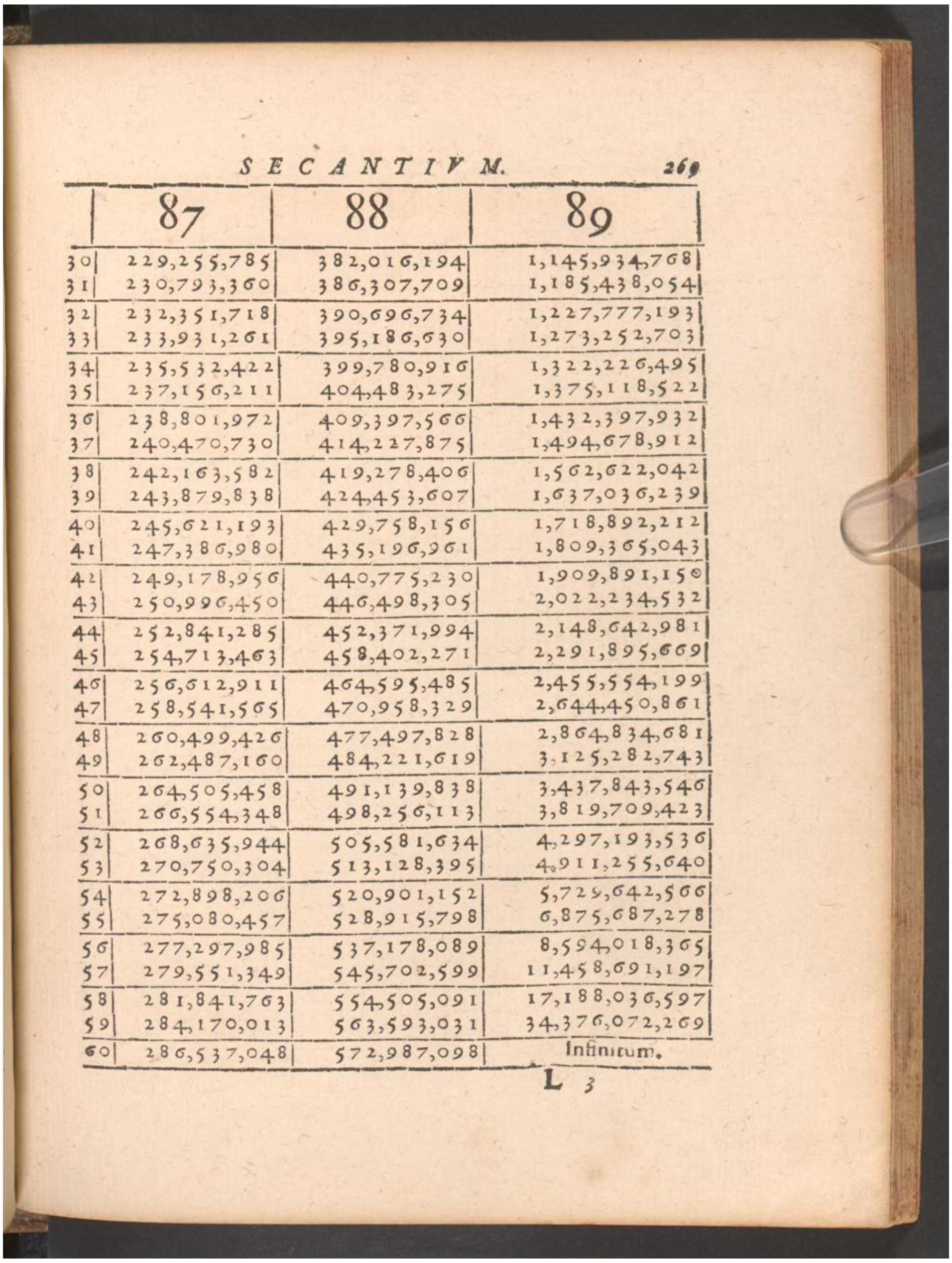
Figure 53: An excerpt from Fincke's table of tangents [Fincke (1583)] (source: e-rara).



152 *CANON*

	35	36	37	38	39
0	5,735,764	5,877,852	6,018,150	6,156,615	6,293,204
1	5,738,147	5,880,205	6,020,473	6,158,907	6,295,464
2	5,740,529	5,882,558	6,022,796	6,161,198	6,297,724
3	5,742,911	5,884,910	6,025,118	6,163,489	6,299,983
4	5,745,292	5,887,262	6,027,439	6,165,781	6,302,242
5	5,747,672	5,889,613	6,029,760	6,168,070	6,304,501
6	5,750,052	5,891,964	6,032,080	6,170,259	6,306,759
7	5,752,432	5,894,314	6,034,400	6,172,648	6,309,016
8	5,754,811	5,896,664	6,036,719	6,174,936	6,311,273
9	5,757,190	5,899,013	6,039,038	6,177,224	6,313,529
10	5,759,568	5,901,361	6,041,357	6,179,512	6,315,784
11	5,761,946	5,903,709	6,043,675	6,181,799	6,318,039
12	5,764,323	5,906,056	6,045,992	6,184,085	6,320,293
13	5,766,700	5,908,403	6,048,309	6,186,371	6,322,547
14	5,769,076	5,910,750	6,050,625	6,188,656	6,324,800
15	5,771,452	5,913,096	6,052,940	6,190,940	6,327,053
16	5,773,827	5,915,442	6,055,255	6,193,224	6,329,305
17	5,776,202	5,917,787	6,057,570	6,195,508	6,331,557
18	5,778,576	5,920,132	6,059,884	6,197,791	6,333,808
19	5,780,950	5,922,476	6,062,198	6,200,074	6,336,059
20	5,783,324	5,924,820	6,064,511	6,202,356	6,338,310
21	5,785,697	5,927,163	6,066,824	6,204,638	6,340,560
22	5,788,069	5,929,505	6,069,136	6,206,919	6,342,809
23	5,790,441	5,931,847	6,071,448	6,209,199	6,345,058
24	5,792,812	5,934,189	6,073,759	6,211,479	6,347,309
25	5,795,183	5,936,530	6,076,069	6,213,758	6,349,553
26	5,797,553	5,938,871	6,078,379	6,216,037	6,351,800
27	5,799,923	5,941,211	6,080,688	6,218,315	6,354,046
28	5,802,292	5,943,551	6,082,997	6,220,593	6,356,292
29	5,804,661	5,945,893	7,085,306	6,222,870	6,358,537
30	5,807,030	5,948,228	6,087,614	6,225,146	6,360,782

Figure 54: An excerpt from Fincke's table of sines [Fincke (1583)] (source: e-rara).


SECANTIVM. 269

	87	88	89
30	229,255,785	382,016,194	1,145,934,768
31	230,793,360	386,307,709	1,185,438,054
32	232,351,718	390,696,734	1,227,777,193
33	233,931,261	395,186,630	1,273,252,703
34	235,532,422	399,780,916	1,322,226,495
35	237,156,211	404,483,275	1,375,118,522
36	238,801,972	409,397,566	1,432,397,932
37	240,470,730	414,227,875	1,494,678,912
38	242,163,582	419,278,406	1,562,622,042
39	243,879,838	424,453,607	1,637,036,239
40	245,621,193	429,758,156	1,718,892,212
41	247,386,980	435,196,961	1,809,365,043
42	249,178,956	440,775,230	1,909,891,150
43	250,996,450	446,498,305	2,022,234,532
44	252,841,285	452,371,994	2,148,642,981
45	254,713,463	458,402,271	2,291,895,669
46	256,612,911	464,595,485	2,455,554,199
47	258,541,565	470,958,329	2,644,450,861
48	260,499,426	477,497,828	2,864,834,681
49	262,487,160	484,221,619	3,125,282,743
50	264,505,458	491,139,838	3,437,843,546
51	266,554,348	498,256,113	3,819,709,423
52	268,635,944	505,581,634	4,297,193,536
53	270,750,304	513,128,395	4,911,255,640
54	272,898,206	520,901,152	5,729,642,566
55	275,080,457	528,915,798	6,875,687,278
56	277,297,985	537,178,089	8,594,018,365
57	279,551,349	545,702,599	11,458,691,197
58	281,841,763	554,505,091	17,188,036,597
59	284,170,013	563,593,031	34,376,072,269
60	286,537,048	572,987,098	Infinitum.

L 3

Figure 55: An excerpt from Fincke's table of secants [Fincke (1583)] (source: e-rara).

132 TABVLA

Gradus Quadrantis pro sinubus

Minuta Graduum Quadrantis pro sinubus rectis arcuum eiusdem Quadrantis.

	0	1	2	3	4	
0	0000	174524	348995	523360	697565	60
1	2909	177433	351902	526265	700467	59
2	5818	180341	354809	529170	703369	58
3	8727	183250	357716	532075	706270	57
4	11636	186158	360623	534980	709172	56
5	14544	189066	363530	537884	712073	55
6	17453	191975	366437	540789	714975	54
7	20362	194883	369344	543694	717876	53
8	23271	197792	372251	546598	720777	52
9	26180	200700	375158	549503	723678	51
10	29088	203608	378064	552407	726579	50
11	31997	206517	380971	555312	729480	49
12	34906	209425	383878	558216	732381	48
13	37815	212333	386785	561120	735282	47
14	40724	215241	389692	564024	738183	46
15	43632	218149	392598	566928	741084	45
16	46541	221057	395505	569832	743985	44
17	49450	223965	398412	572736	746886	43
18	52359	226873	401318	575640	749787	42
19	55268	229781	404225	578544	752688	41
20	58177	232689	407131	581448	755588	40
21	61086	235597	410038	584352	758489	39
22	63995	238505	412944	587256	761389	38
23	66904	241413	415851	590160	764290	37
24	69813	244321	418757	593064	767190	36
25	72721	247229	421663	595967	770090	35
26	75630	250137	424570	598871	772991	34
27	78539	253045	427476	601775	775891	33
28	81448	255953	430382	604678	778791	32
29	84357	258861	433288	607582	781691	31
30	87265	261769	436194	610485	784591	30
	89	88	87	86	85	

Minuta Graduũ Quadrãtis pro sinubus rectis cõplementorũ arcuũ eiusdem Quadrãtis

Gradus Quadrantis pro sinubus rectis.

Figure 56: An excerpt from Clavius's table of sines [Clavius (1586)].

102 TABVLA

Gradus Quadrantis pro tangentibus

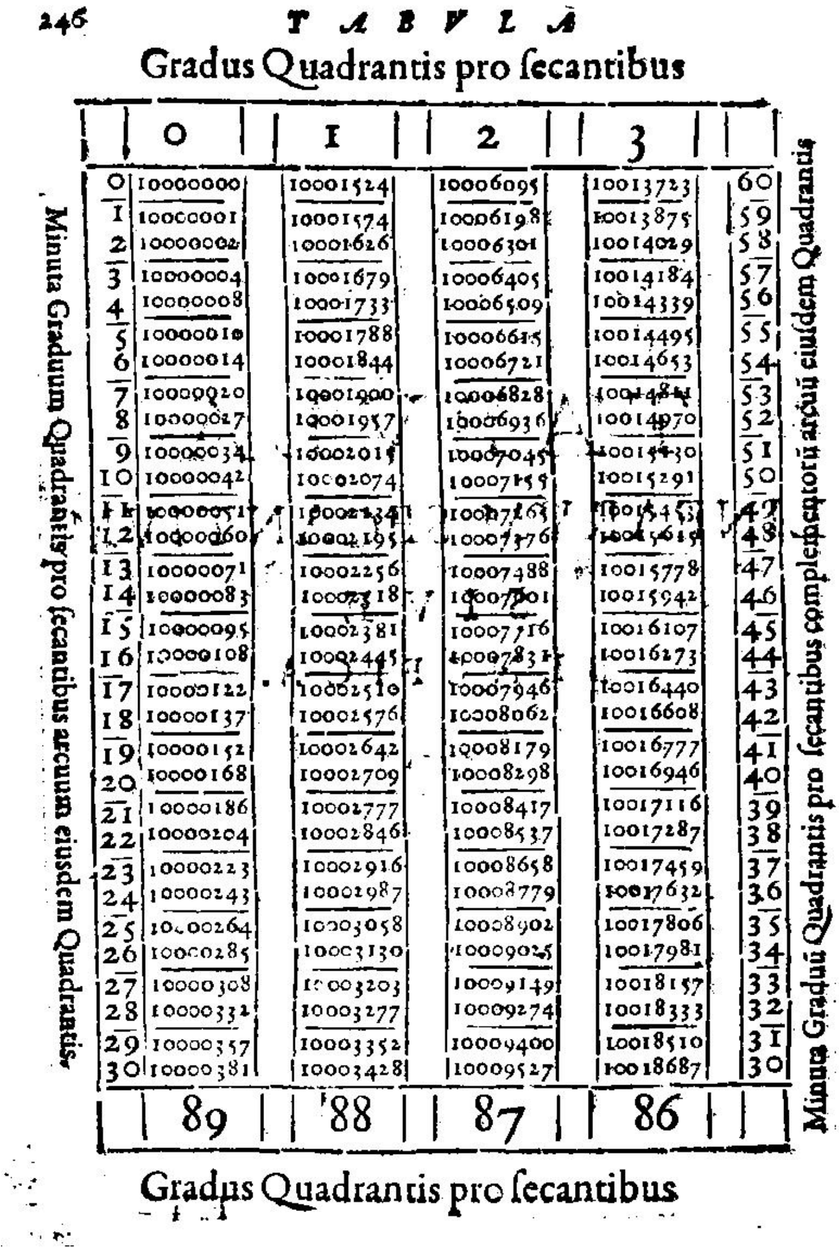
Minuta Graduum Quadrantis pro tangentibus arcuum eiusdem Quadrantis.

	0	1	2	3	4	
0	0000	174550	349207	524078	699269	60
1	2909	177459	352120	526995	702191	59
2	5818	180369	355033	529911	705116	58
3	8727	183279	357945	532828	708039	57
4	11636	18618[illegible]	360858	535745	710962	56
5	14544	189100	363770	538663	713886	55
6	17452	192010	366683	541580	716809	54
7	20361	194920	369596	544498	719733	53
8	23270	197830	372508	547415	722657	52
9	26179	200740	375421	550333	725580	51
10	29088	203650	378334	553251	728504	50
11	31996	2065[illegible]1	381247	556169	731428	49
12	34905	209471	384160	559087	734353	48
13	37814	212381	387073	562005	737277	47
14	40723	215291	389987	564923	740202	46
15	43632	218201	392900	567841	743127	45
16	46541	221111	395814	570759	746052	44
17	49450	224022	398727	573678	748978	43
18	52359	226932	401641	576596	751903	42
19	55268	229842	404554	579514	754829	41
20	58177	232752	407468	582433	757754	40
21	61086	235663	410382	585352	760680	39
22	63995	238574	413295	588270	763606	38
23	66904	241485	416209	591189	766532	37
24	69813	244395	419123	594108	769459	36
25	72722	247306	422037	597028	772385	35
26	75631	250217	424951	599947	775311	34
27	78540	253128	427866	602866	778238	33
28	81450	256038	430780	605786	781164	32
29	84359	258949	433694	608705	784091	31
30	87268	261859	436609	611625	787017	30
	89	88	87	86	85	

Minuta Graduũ Quadrãtis pro tãgẽtibus cõplemẽtorũ arcuũ eiusdẽ Quadrãtis

Gradus Quadrantis pro tangentibus

Figure 57: An excerpt from Clavius's table of tangents [Clavius (1586)].



246 TABVLA

Gradus Quadrantis pro ſecantibus

Minuta Graduum Quadrantis pro ſecantibus arcuum eiuſdem Quadrantis

	0	1	2	3	
0	10000000	10001524	10006095	10013723	60
1	10000001	10001574	10006198	10013875	59
2	10000002	10001626	10006301	10014029	58
3	10000004	10001679	10006405	10014184	57
4	10000008	10001733	10006509	10014339	56
5	10000010	10001788	10006615	10014495	55
6	10000014	10001844	10006721	10014653	54
7	10000020	10001900	10006828	10014[illegible]	53
8	10000027	10001957	10006936	10014970	52
9	10000034	10002015	10007045	1001[illegible]30	51
10	10000042	10002074	10007159	10015291	50
11	10000051	10002134	10007263	10015453	49
12	10000060	10002195	10007376	10015619	48
13	10000071	10002256	10007488	10015778	47
14	10000083	10002318	10007[illegible]01	10015942	46
15	10000095	10002381	10007716	10016107	45
16	10000108	10002445	10007831	10016273	44
17	10000122	10002510	10007946	10016440	43
18	10000137	10002576	10008062	10016608	42
19	10000152	10002642	10008179	10016777	41
20	10000168	10002709	10008298	10016946	40
21	10000186	10002777	10008417	10017116	39
22	10000204	10002846	10008537	10017287	38
23	10000223	10002916	10008658	10017459	37
24	10000243	10002987	10008779	10017632	36
25	10000264	10003058	10008902	10017806	35
26	10000285	10003130	10009025	10017981	34
27	10000308	10003203	10009149	10018157	33
28	10000332	10003277	10009274	10018333	32
29	10000357	10003352	10009400	10018510	31
30	10000381	10003428	10009527	10018687	30
	89	88	87	86	

Minuta Graduū Quadrantis pro ſecantibus complementorū arcuū eiuſdem Quadrantis

Gradus Quadrantis pro ſecantibus

Figure 58: An excerpt from Clavius's table of secants [Clavius (1586)].

SECANTIVM. 291

arcuum eiuſdem Quadrantis

Minuta Graduum Quadrantis pro ſecantibus arcuum eiuſdem Quadrantis.

	87	88	89	
30	229255785	382016194	1145934768	30
31	230793360	386307709	1185438054	29
32	232351718	390696734	1227777193	28
33	233931261	395186630	1273252703	27
34	235532422	399780916	1322226495	26
35	237156211	404483275	1375118522	25
36	238801972	409397566	1432397932	24
37	240470730	414127875	1494678912	23
38	242163582	419278406	1562622042	22
39	243879838	424453607	1637036239	21
40	245621193	429758156	1718892212	20
41	247386980	435196961	1809365043	19
42	249178956	440775230	1909891150	18
43	250996450	446498305	2022234532	17
44	252841285	452371994	2148642981	16
45	254713463	458402271	2291895669	15
46	256612911	464595485	2455554199	14
47	258541565	470958319	2644450861	13
48	260499426	477497828	2864894681	12
49	262487160	484221619	3125282743	11
50	264505458	491139838	3437843546	10
51	266554348	498256113	3819709423	9
52	268635944	505581634	4297193536	8
53	270750304	513128395	4911255640	7
54	272898206	520901152	5729642566	6
55	275080457	528915798	6875687278	5
56	277297985	537178089	8594018365	4
57	279551349	545702599	11458691197	3
58	281841763	554505091	17188036598	2
59	284170013	563593031	34376072269	1
60	286537048	572987098	Infinita.	0
	2	1	0	

Minuta Graduũ Quadrantis pro ſecantibus cõplementorũ arcuũ eiuſdẽ Quadrãtis.

complementorum arcuum eiuſdem Quadrantis.

Figure 59: Another excerpt from Clavius's table of secants [Clavius (1586)].

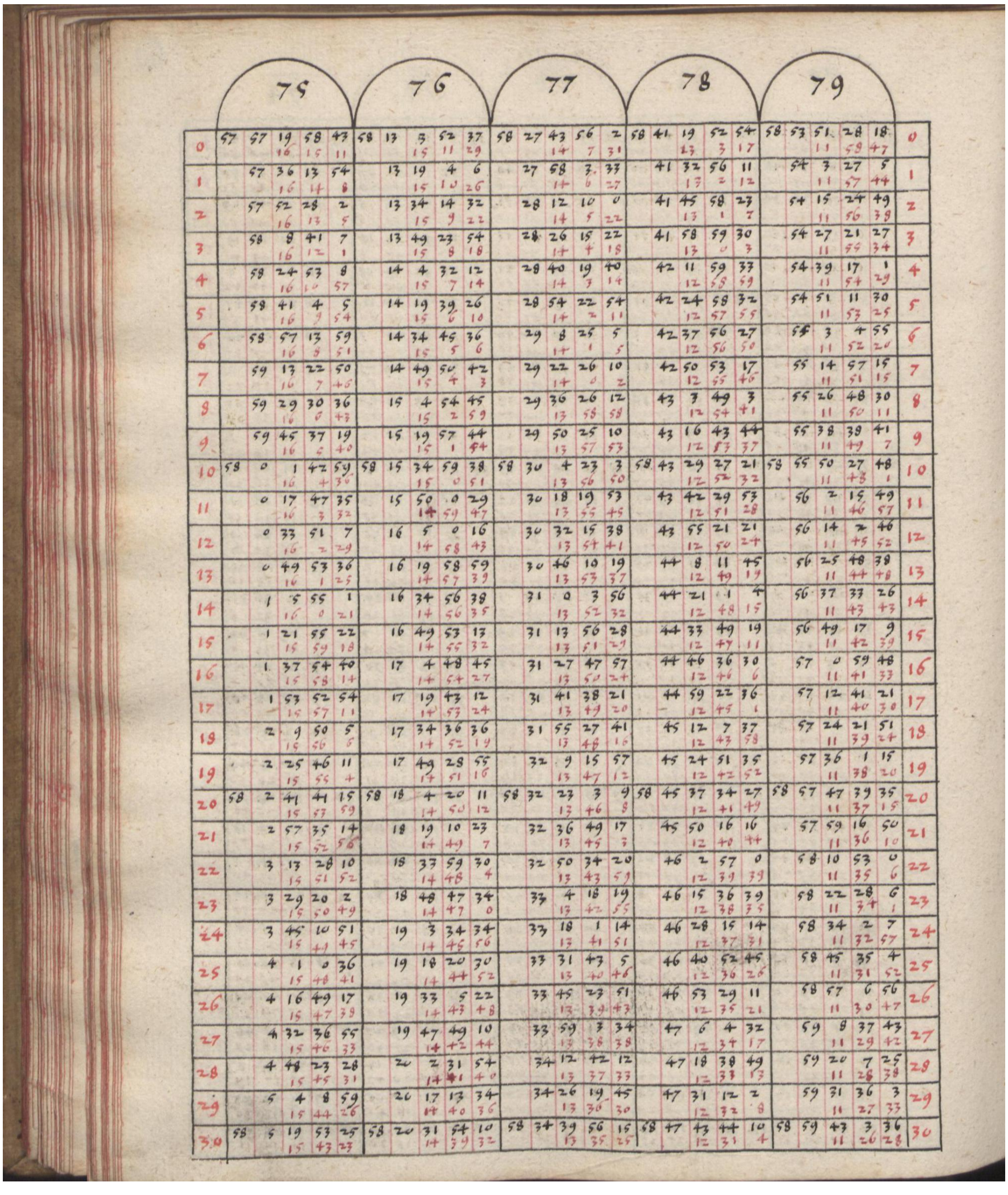


Figure 60: An excerpt of Bürgi's table of sines (1587). Bürgi's *Fundamentum Astronomiæ* manuscript is kept at the Biblioteka Uniwersytecka Wrocław, under call number IV Qu 38a. This excerpt of Bürgi's sine table was provided by Dieter Launert and is included in LOCOMAT (http://locomat.loria.fr) with permission.

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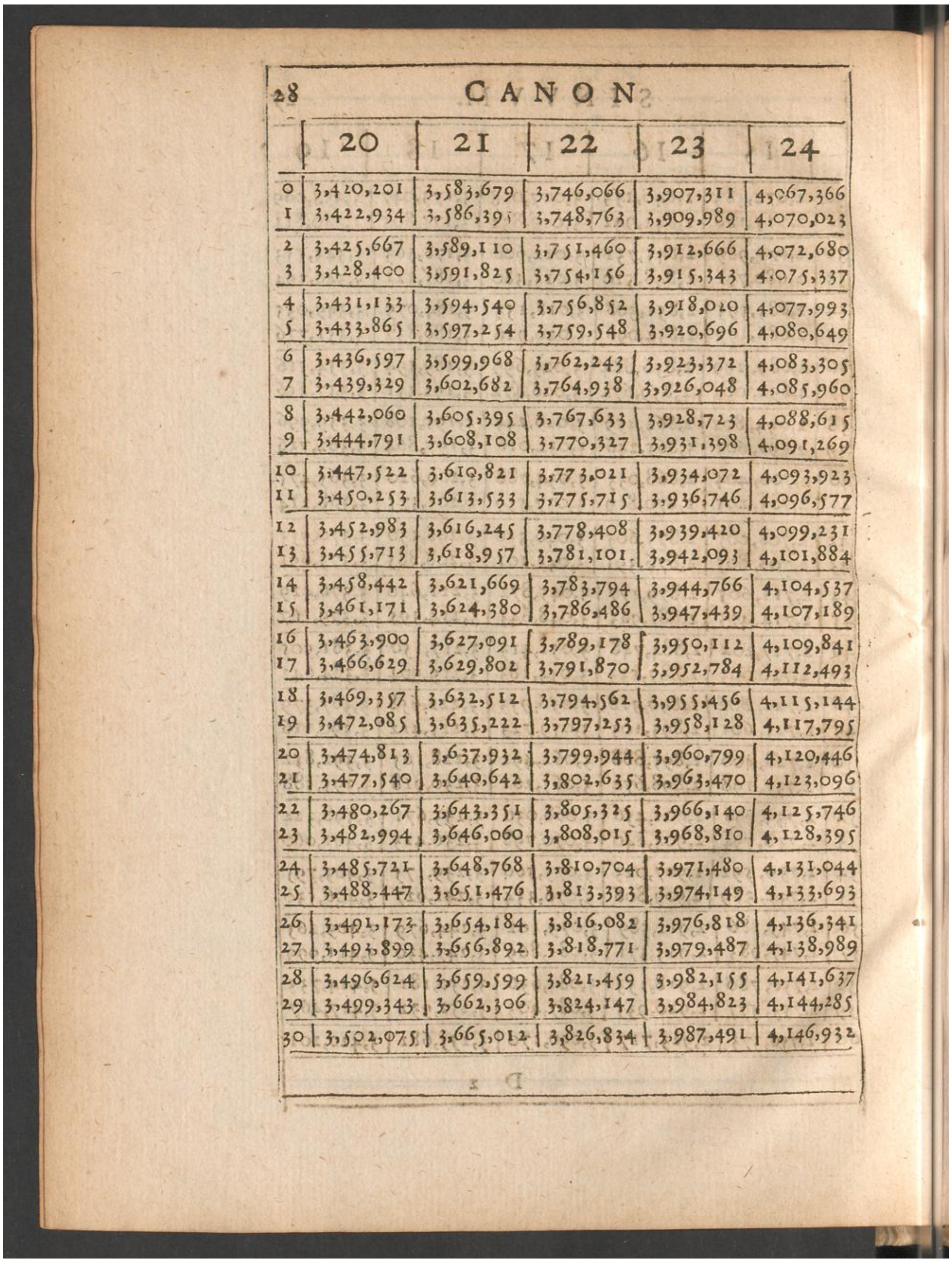
Incipit tabula.

G / M.	0	1	2	3	4	5
1	17	1064	2111	3157	4202	5246
2	34	1082	2128	3175	4220	5264
3	52	1099	2146	3192	4237	5281
4	69	1116	2163	3209	4255	5298
5	87	1134	2181	3227	4272	5316
6	104	1151	2198	3244	4289	5333
7	122	1169	2216	3262	4307	5351
8	139	1186	2233	3279	4324	5368
9	157	1204	2250	3297	4342	5385
10	174	1221	2268	3314	4359	5403
11	191	1239	2285	3331	4376	5420
12	209	1256	2203	3349	4394	5437
13	226	1274	2320	3366	4411	5455
14	244	1291	2338	3384	4429	5472
15	261	1308	2355	3401	4446	5490
16	279	1326	2373	3418	4463	5507
17	296	1343	2390	3436	4481	5524
18	314	1361	2407	3453	4498	5542
19	331	1378	2425	3471	4516	5559
20	349	1396	2442	3488	4533	5577
21	366	1413	2460	3506	4550	5594
22	383	1431	2477	3523	4568	5611
23	401	1448	2495	3540	4585	5629
24	418	1465	2512	3558	4603	5646
25	436	1483	2529	3575	4620	5663
26	453	1500	2547	3593	4637	5681
27	471	1518	2564	3610	4655	5698
28	488	1535	2582	3628	4672	5716
29	506	1553	2599	3645	4690	5733
30	523	1570	2617	3662	4707	5750

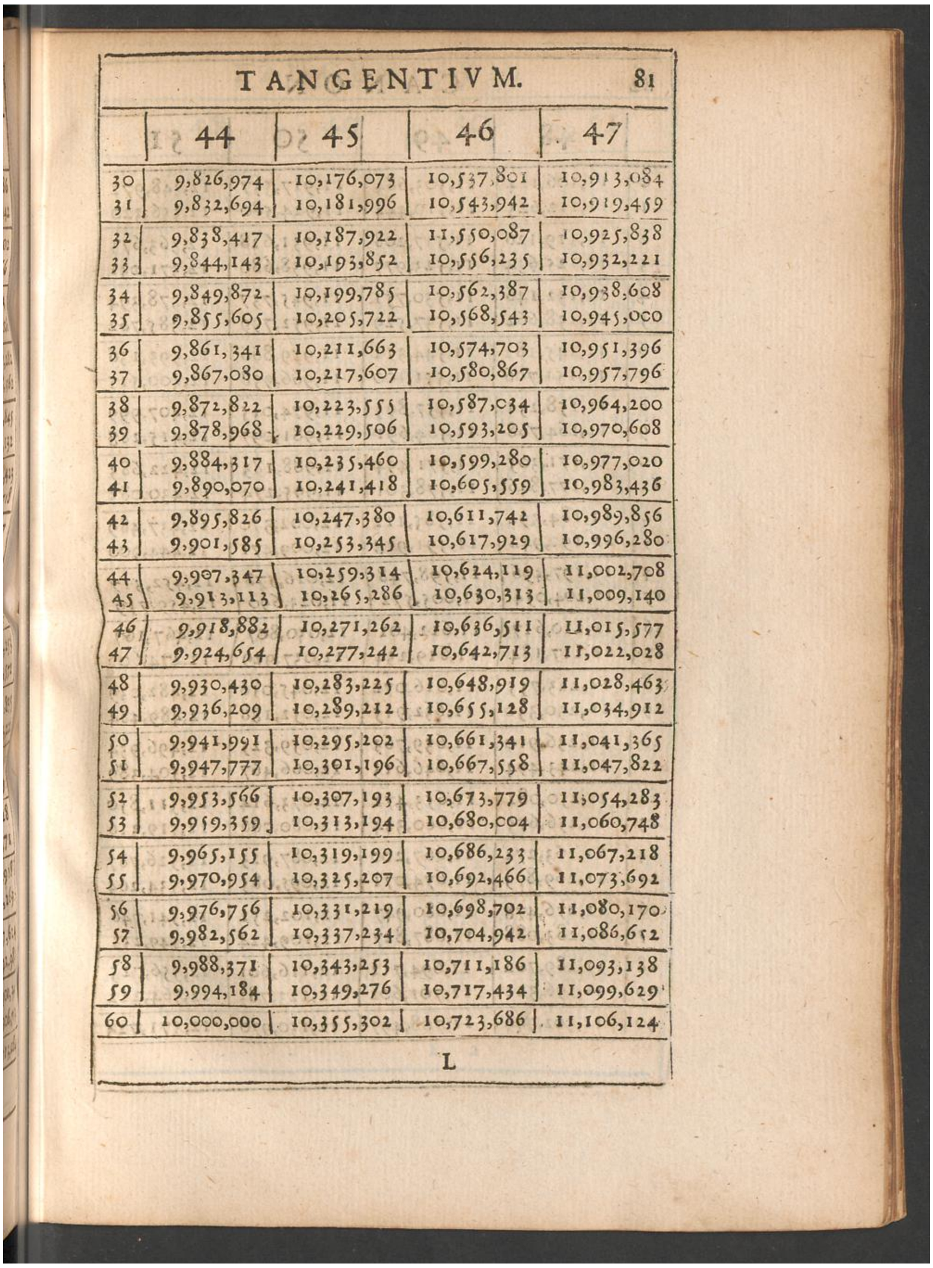
Figure 61: An excerpt from Gallucci's table of sines [Gallucci (1588)] (source: Google books).



28 CANON

	20	21	22	23	24
0	3,420,201	3,583,679	3,746,066	3,907,311	4,067,366
1	3,422,934	3,586,39[illegible]	3,748,763	3,909,989	4,070,023
2	3,425,667	3,589,110	3,751,460	3,912,666	4,072,680
3	3,428,400	3,591,825	3,754,156	3,915,343	4,075,337
4	3,431,133	3,594,540	3,756,852	3,918,020	4,077,993
5	3,433,865	3,597,254	3,759,548	3,920,696	4,080,649
6	3,436,597	3,599,968	3,762,243	3,923,372	4,083,305
7	3,439,329	3,602,682	3,764,938	3,926,048	4,085,960
8	3,442,060	3,605,395	3,767,633	3,928,723	4,088,615
9	3,444,791	3,608,108	3,770,327	3,931,398	4,091,269
10	3,447,522	3,610,821	3,773,021	3,934,072	4,093,923
11	3,450,253	3,613,533	3,775,715	3,936,746	4,096,577
12	3,452,983	3,616,245	3,778,408	3,939,420	4,099,231
13	3,455,713	3,618,957	3,781,101	3,942,093	4,101,884
14	3,458,442	3,621,669	3,783,794	3,944,766	4,104,537
15	3,461,171	3,624,380	3,786,486	3,947,439	4,107,189
16	3,463,900	3,627,091	3,789,178	3,950,112	4,109,841
17	3,466,629	3,629,802	3,791,870	3,952,784	4,112,493
18	3,469,357	3,632,512	3,794,562	3,955,456	4,115,144
19	3,472,085	3,635,222	3,797,253	3,958,128	4,117,795
20	3,474,813	3,637,932	3,799,944	3,960,799	4,120,446
21	3,477,540	3,640,642	3,802,635	3,963,470	4,123,096
22	3,480,267	3,643,351	3,805,325	3,966,140	4,125,746
23	3,482,994	3,646,060	3,808,015	3,968,810	4,128,395
24	3,485,721	3,648,768	3,810,704	3,971,480	4,131,044
25	3,488,447	3,651,476	3,813,393	3,974,149	4,133,693
26	3,491,173	3,654,184	3,816,082	3,976,818	4,136,341
27	3,493,899	3,656,892	3,818,771	3,979,487	4,138,989
28	3,496,624	3,659,599	3,821,459	3,982,155	4,141,637
29	3,499,343	3,662,306	3,824,147	3,984,823	4,144,285
30	3,502,075	3,665,012	3,826,834	3,987,491	4,146,932

Figure 62: An excerpt from Lansberge's table of sines [van Lansberge (1591)] (source: e-rara).



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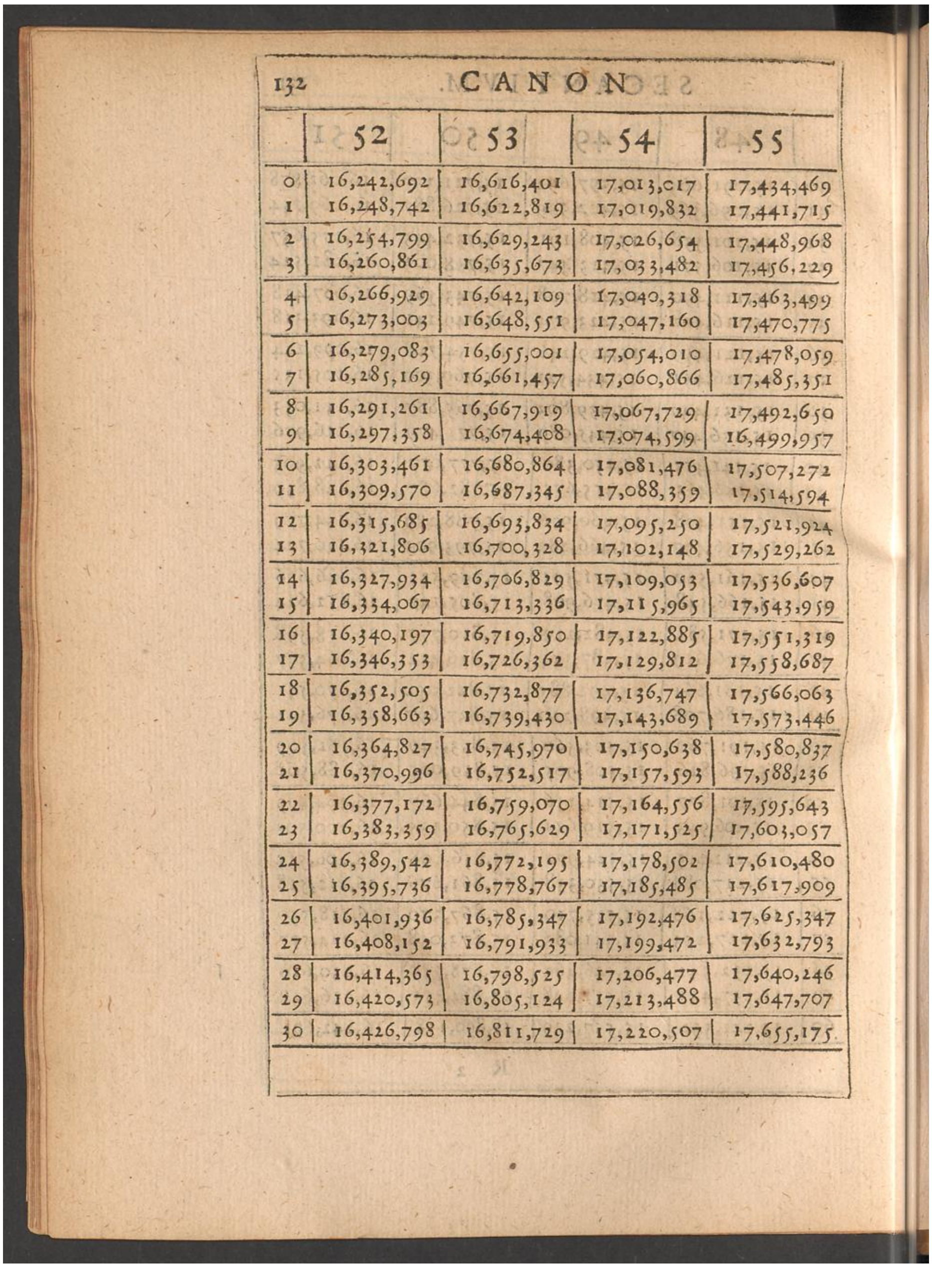
	44	45	46	47
30	9,826,974	10,176,073	10,537,801	10,913,084
31	9,832,694	10,181,996	10,543,942	10,919,459
32	9,838,417	10,187,922	11,550,087	10,925,838
33	9,844,143	10,193,852	10,556,235	10,932,221
34	9,849,872	10,199,785	10,562,387	10,938,608
35	9,855,605	10,205,722	10,568,543	10,945,000
36	9,861,341	10,211,663	10,574,703	10,951,396
37	9,867,080	10,217,607	10,580,867	10,957,796
38	9,872,822	10,223,555	10,587,034	10,964,200
39	9,878,968	10,229,506	10,593,205	10,970,608
40	9,884,317	10,235,460	10,599,280	10,977,020
41	9,890,070	10,241,418	10,605,559	10,983,436
42	9,895,826	10,247,380	10,611,742	10,989,856
43	9,901,585	10,253,345	10,617,929	10,996,280
44	9,907,347	10,259,314	10,624,119	11,002,708
45	9,913,113	10,265,286	10,630,313	11,009,140
46	9,918,882	10,271,262	10,636,511	11,015,577
47	9,924,654	10,277,242	10,642,713	11,022,028
48	9,930,430	10,283,225	10,648,919	11,028,463
49	9,936,209	10,289,212	10,655,128	11,034,912
50	9,941,991	10,295,202	10,661,341	11,041,365
51	9,947,777	10,301,196	10,667,558	11,047,822
52	9,953,566	10,307,193	10,673,779	11,054,283
53	9,959,359	10,313,194	10,680,004	11,060,748
54	9,965,155	10,319,199	10,686,233	11,067,218
55	9,970,954	10,325,207	10,692,466	11,073,692
56	9,976,756	10,331,219	10,698,702	11,080,170
57	9,982,562	10,337,234	10,704,942	11,086,652
58	9,988,371	10,343,253	10,711,186	11,093,138
59	9,994,184	10,349,276	10,717,434	11,099,629
60	10,000,000	10,355,302	10,723,686	11,106,124

L

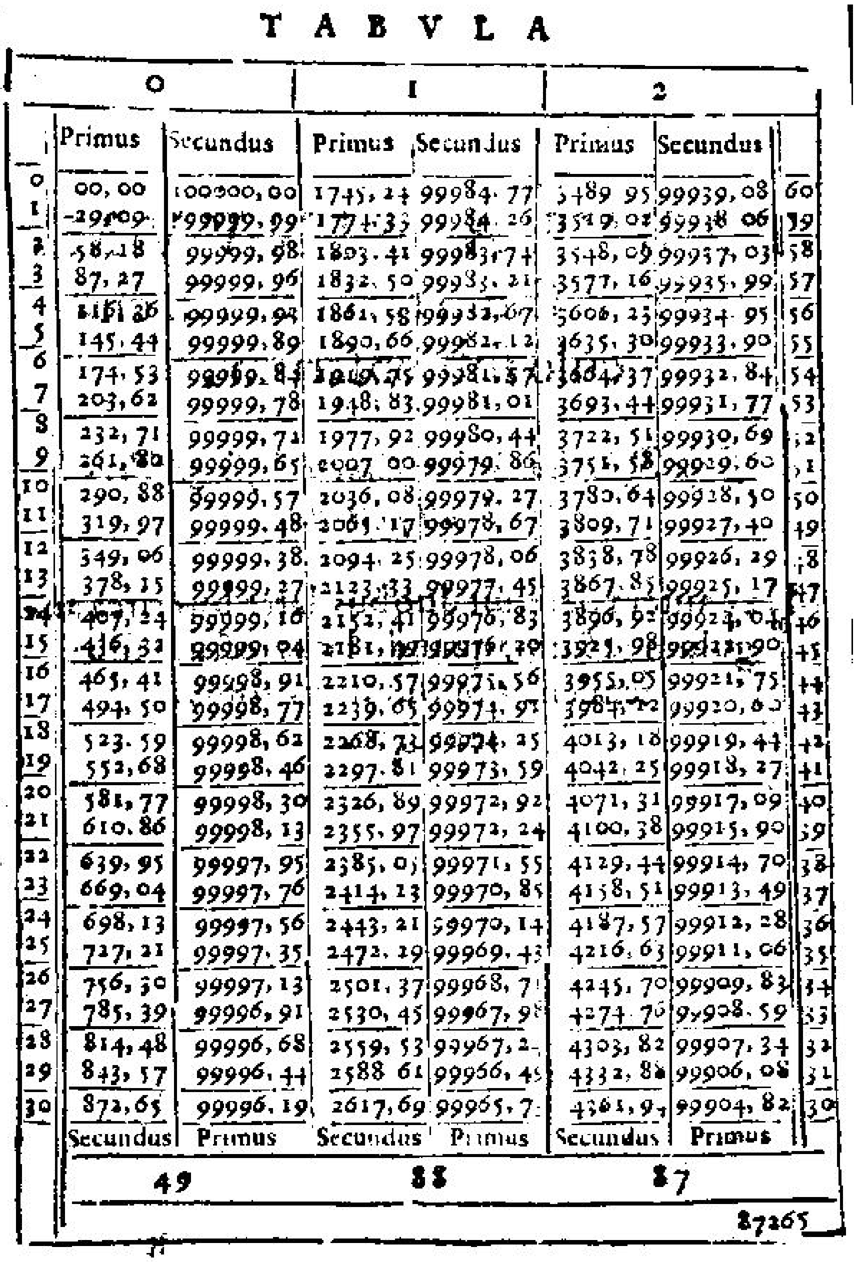
Figure 63: An excerpt from Lansberge's table of tangents [van Lansberge (1591)] (source: e-rara).



132 CANON

	52	53	54	55
0	16,242,692	16,616,401	17,013,017	17,434,469
1	16,248,742	16,622,819	17,019,832	17,441,715
2	16,254,799	16,629,243	17,026,654	17,448,968
3	16,260,861	16,635,673	17,033,482	17,456,229
4	16,266,929	16,642,109	17,040,318	17,463,499
5	16,273,003	16,648,551	17,047,160	17,470,775
6	16,279,083	16,655,001	17,054,010	17,478,059
7	16,285,169	16,661,457	17,060,866	17,485,351
8	16,291,261	16,667,919	17,067,729	17,492,650
9	16,297,358	16,674,408	17,074,599	16,499,957
10	16,303,461	16,680,864	17,081,476	17,507,272
11	16,309,570	16,687,345	17,088,359	17,514,594
12	16,315,685	16,693,834	17,095,250	17,521,924
13	16,321,806	16,700,328	17,102,148	17,529,262
14	16,327,934	16,706,829	17,109,053	17,536,607
15	16,334,067	16,713,336	17,115,965	17,543,959
16	16,340,197	16,719,850	17,122,885	17,551,319
17	16,346,353	16,726,362	17,129,812	17,558,687
18	16,352,505	16,732,877	17,136,747	17,566,063
19	16,358,663	16,739,430	17,143,689	17,573,446
20	16,364,827	16,745,970	17,150,638	17,580,837
21	16,370,996	16,752,517	17,157,593	17,588,236
22	16,377,172	16,759,070	17,164,556	17,595,643
23	16,383,359	16,765,629	17,171,525	17,603,057
24	16,389,542	16,772,195	17,178,502	17,610,480
25	16,395,736	16,778,767	17,185,485	17,617,909
26	16,401,936	16,785,347	17,192,476	17,625,347
27	16,408,152	16,791,933	17,199,472	17,632,793
28	16,414,365	16,798,525	17,206,477	17,640,246
29	16,420,573	16,805,124	17,213,488	17,647,707
30	16,426,798	16,811,729	17,220,507	17,655,175

Figure 64: An excerpt from Lansberge's table of secants [van Lansberge (1591)] (source: e-rara).

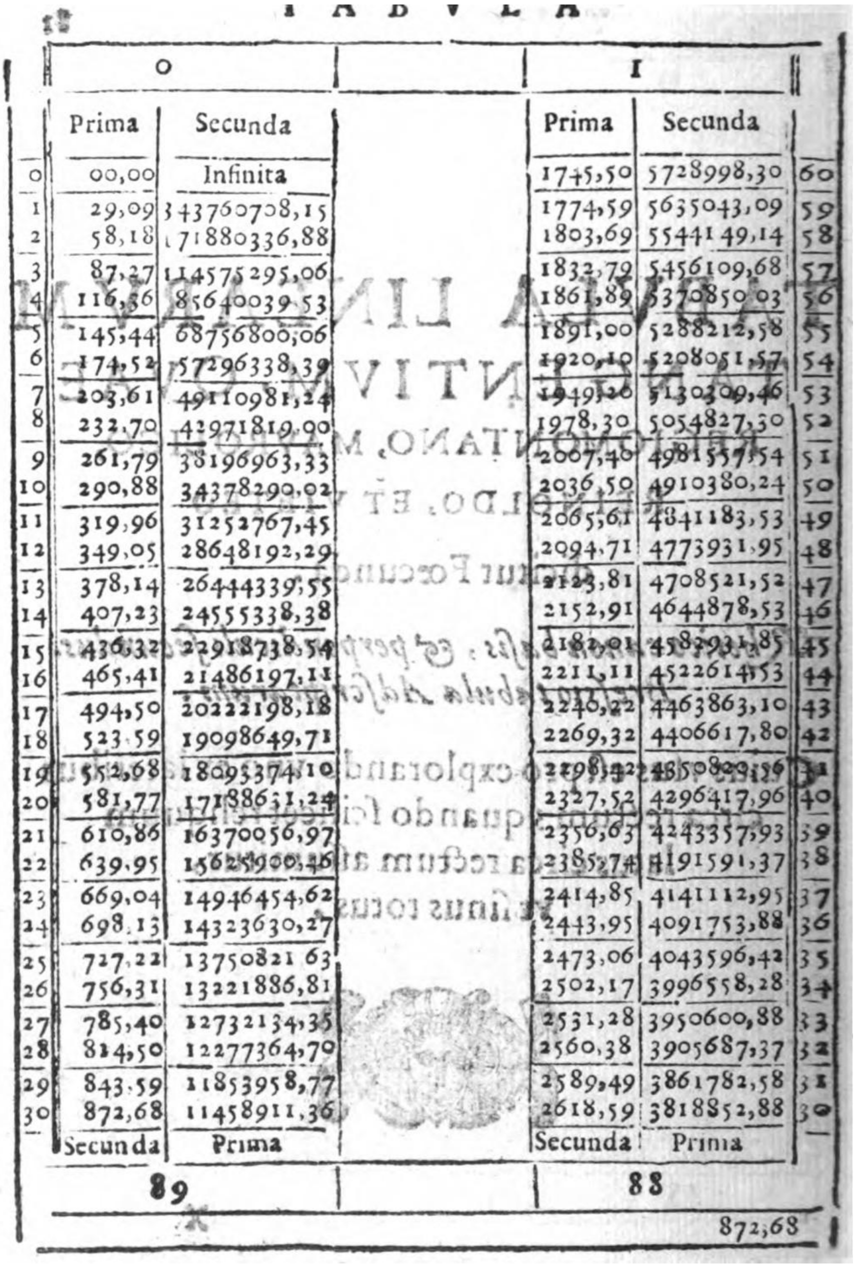


TABVLA

	0		1		2		
	Primus	Secundus	Primus	Secundus	Primus	Secundus	
0	00,00	100000,00	1745,24	99984,77	3489,95	99939,08	60
1	29,09	99999,99	1774,33	99984,26	3519,02	99938,06	59
2	58,18	99999,98	1803,41	99983,74	3548,09	99937,03	58
3	87,27	99999,96	1832,50	99983,21	3577,16	99935,99	57
4	116,36	99999,93	1861,58	99982,67	3606,23	99934,95	56
5	145,44	99999,89	1890,66	99982,12	3635,30	99933,90	55
6	174,53	99999,85	1919,75	99981,57	3664,37	99932,84	54
7	203,62	99999,78	1948,83	99981,01	3693,44	99931,77	53
8	232,71	99999,72	1977,92	99980,44	3722,51	99930,69	52
9	261,80	99999,65	2007,00	99979,86	3751,58	99929,60	51
10	290,88	99999,57	2036,08	99979,27	3780,64	99928,50	50
11	319,97	99999,48	2065,17	99978,67	3809,71	99927,40	49
12	349,06	99999,38	2094,25	99978,06	3838,78	99926,29	48
13	378,15	99999,27	2123,33	99977,45	3867,85	99925,17	47
14	407,24	99999,16	2152,41	99976,83	3896,92	99924,04	46
15	436,32	99999,04	2181,49	99976,19	3925,98	99922,90	45
16	465,41	99998,91	2210,57	99975,56	3955,05	99921,75	44
17	494,50	99998,77	2239,65	99974,91	3984,12	99920,60	43
18	523,59	99998,62	2268,73	99974,25	4013,18	99919,44	42
19	552,68	99998,46	2297,81	99973,59	4042,25	99918,27	41
20	581,77	99998,30	2326,89	99972,92	4071,31	99917,09	40
21	610,86	99998,13	2355,97	99972,24	4100,38	99915,90	39
22	639,95	99997,95	2385,05	99971,55	4129,44	99914,70	38
23	669,04	99997,76	2414,13	99970,85	4158,51	99913,49	37
24	698,13	99997,56	2443,21	99970,14	4187,57	99912,28	36
25	727,21	99997,35	2472,29	99969,43	4216,63	99911,06	35
26	756,30	99997,13	2501,37	99968,71	4245,70	99909,83	34
27	785,39	99996,91	2530,45	99967,98	4274,76	99908,59	33
28	814,48	99996,68	2559,53	99967,24	4303,82	99907,34	32
29	843,57	99996,44	2588,61	99956,45	4332,88	99906,08	31
30	872,65	99996,19	2617,69	99965,73	4361,94	99904,82	30
	Secundus	Primus	Secundus	Primus	Secundus	Primus	
	89		88		87		

87265

Figure 65: An excerpt from Magini's table of sines [Magini (1592)].



18 TABVLA

	0		I		
	Prima	Secunda	Prima	Secunda	
0	00,00	Infinita	1745,50	5728998,30	60
1	29,09	343760708,15	1774,59	5635043,09	59
2	58,18	171880336,88	1803,69	5544149,14	58
3	87,27	114575295,06	1832,79	5456109,68	57
4	116,36	85640039,53	1861,89	5370850,03	56
5	145,44	68756800,06	1891,00	5288212,58	55
6	174,52	57296338,39	1920,10	5208051,57	54
7	203,61	49110981,24	1949,20	5130309,46	53
8	232,70	42971819,90	1978,30	5054827,30	52
9	261,79	38196963,33	2007,40	4981557,54	51
10	290,88	34378290,02	2036,50	4910380,24	50
11	319,96	31252767,45	2065,61	4841183,53	49
12	349,05	28648192,29	2094,71	4773931,95	48
13	378,14	26444339,55	2123,81	4708521,52	47
14	407,23	24555338,38	2152,91	4644878,53	46
15	436,32	22918738,54	2182,01	4582931,85	45
16	465,41	21486197,11	2211,11	4522614,53	44
17	494,50	20222198,18	2240,22	4463863,10	43
18	523,59	19098649,71	2269,32	4406617,80	42
19	552,68	18093374,10	2298,42	4350829,56	41
20	581,77	17188631,24	2327,52	4296417,96	40
21	610,86	16370056,97	2356,63	4243357,93	39
22	639,95	15625900,46	2385,74	4191591,37	38
23	669,04	14946454,62	2414,85	4141112,95	37
24	698,13	14323630,27	2443,95	4091753,88	36
25	727,22	13750821,63	2473,06	4043596,42	35
26	756,31	13221886,81	2502,17	3996558,28	34
27	785,40	12732134,35	2531,28	3950600,88	33
28	814,50	12277364,70	2560,38	3905687,37	32
29	843,59	11853958,77	2589,49	3861782,58	31
30	872,68	11458911,36	2618,59	3818852,88	30
	Secunda	Prima	Secunda	Prima	
	89		88		

872,68

Figure 66: An excerpt from Magini's table of tangents [Magini (1592)].

T A B V L A

	0		I		
	Prima	Secunda	Prima	Secunda	
0	100000,00	Infinita.	100015,24	5729870,98	60
1	100000,01	343760722,69	100015,74	5635930,31	59
2	100000,02	171880365,98	100016,26	5545050,91	58
3	100000,04	114586911,97	100016,79	5457025,99	57
4	100000,08	85940183,65	100017,33	5371780,89	56
5	100000,10	68756872,73	100017,88	5289157,98	55
6	100000,14	57296425,66	100018,44	5209011,52	54
7	100000,20	49112556,40	100019,00	5131283,95	53
8	100000,27	42971935,36	100019,57	5055816,34	52
9	100000,34	38197094,23	100020,15	4982561,13	51
10	100000,42	34378435,46	100020,74	4911398,38	50
11	100000,51	31252827,43	100021,34	4842216,19	49
12	100000,60	28648346,81	100021,95	4774978,28	48
13	100000,71	26444508,61	100022,56	4709583,29	47
14	100000,83	24555541,99	100023,18	4645954,85	46
15	100000,95	22918956,69	100023,81	4584022,71	45
16	100001,08	21486429,81	100024,45	4523719,94	44
17	100001,22	20222345,32	100025,10	4464983,05	43
18	100001,37	19098911,50	100025,76	4407752,30	42
19	100001,52	18093650,43	100026,42	4351969,61	41
20	100001,68	17188922,12	100027,09	4297581,56	40
21	100001,86	16370362,39	100027,77	4244536,07	39
22	100002,04	15626220,42	100028,46	4192784,06	38
23	100002,23	14946789,12	100029,16	4142278,75	37
24	100002,43	14323979,32	100029,87	4093975,66	36
25	100002,64	13751185,22	100030,58	4044832,75	35
26	100002,85	13222264,95	100031,30	3997809,16	34
27	100003,08	12732527,03	100032,03	3951866,30	33
28	100003,32	12277771,93	100032,77	3906967,34	32
29	100003,57	11854380,54	100033,52	3863077,09	31
30	100003,81	11459347,68	100034,28	3820161,94	30
	Secunda	Prima	Secunda	Prima	
	89		88		

100003.81

Figure 67: An excerpt from Magini's table of secants [Magini (1592)].

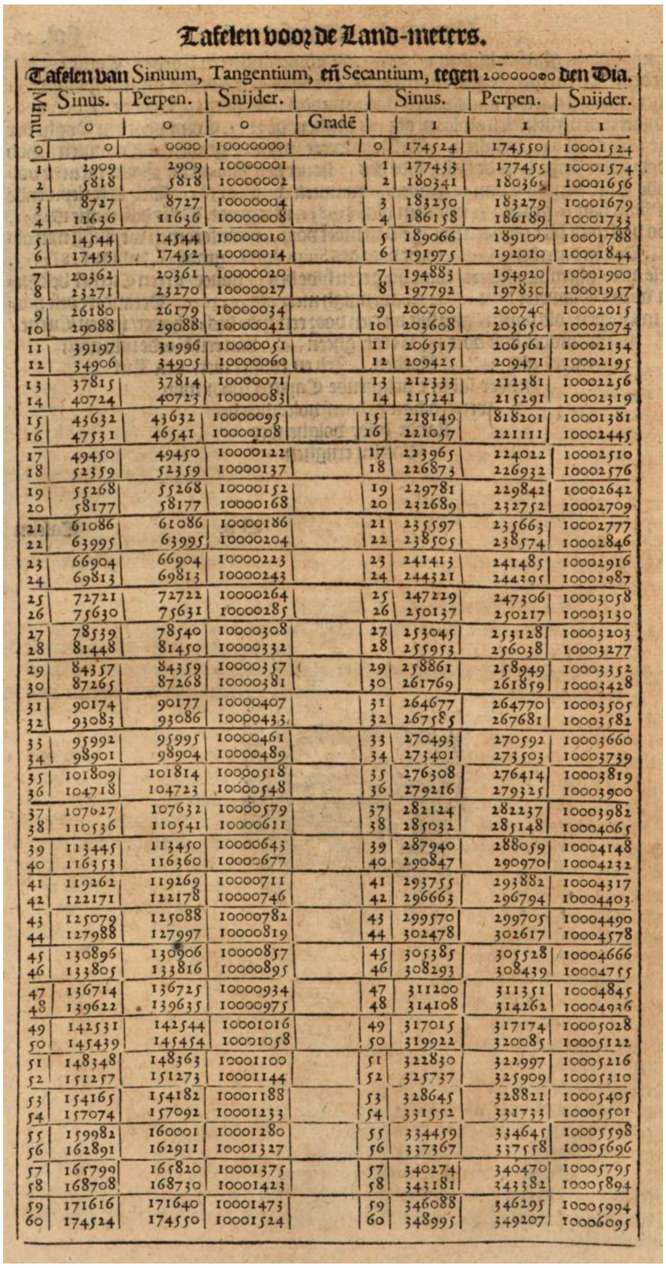
Minuta Graduum Quadrantis pro ſinubus rectis arcuum eiuſdem Quadrantis.

	0		1		2		3		4	
0	0000	48.5	174524	48.5	348995	48.4	523360	48.4	697565	48.4
1	2909		177433		351902		526265		700467	
2	5818		180341		354809		529170		703369	
3	8727		183250		357716		532075		706270	
4	11636		186158		360623		534980		709172	
5	14544		189066		363530		537884		712073	
6	17453		191975		366437		540789		714975	48.3
7	20362		194883		369344		543694		717876	
8	23271		197792		372251		546598		720777	
9	26180		200700		375158		549503		723678	
10	29088		203608		378064		552407		726579	
11	31997		206517		380971		555312		729480	
12	34906		209425		383878		558216		732381	
13	37815		212333		386785		561120		735282	
14	40724		215241		389692		564024		738183	
15	43632		218149		392598		566928		741084	
16	46541		221057		395505		569832		743985	
17	49450		223965		398412		572736		746886	
18	52359		226873		401318		575640		749787	
19	55268		229781		404225		578544		752688	
20	58177		232689		407131		581448		755588	
21	61086		235597		410038		584352		758489	
22	63995		238505		412944		587256		761389	
23	66904		241413		415851		590160		764290	
24	69813		244321		418757		593064		767190	
25	72721		247229		421663		595967		770090	
26	75630		250137		424570		598871		772991	
27	78539		253045		427476		601775		775891	
28	81448		255953		430382		604678		778791	
29	84357		258861		433288		707582		781691	
30	87265	48.5	261769	48.5	436194	48.4	610485	48.4	784591	48.3
	89		88		87		86		85	

Figure 68: An excerpt from Clavius's table of sines [Clavius (1593)].

D	0	1	2	3	4	5
M	Parts	Parts	Parts	Parts	Parts	Parts
1	29	1774	3519	5262	7004	8744
2	58	1803	48	91	33	73
3	87	32	77	5320	62	8803
4	116	61	3606	49	19	31
5	45	90	35	78	7120	60
6	74	1919	64	5407	49	89
7	203	48	93	36	78	8918
8	32	77	3722	65	7207	47
9	61	2007	51	95	36	76
10	90	36	80	5524	65	9005
11	319	65	3809	53	94	34
12	49	94	38	82	7323	63
13	78	2123	67	5611	52	92
14	407	52	96	40	81	9121
15	36	81	3925	69	7410	50
16	65	2210	55	98	39	79
17	94	39	84	5727	68	9208
18	523	68	4013	56	97	37
19	52	97	42	85	7526	66
20	81	2326	71	5814	55	94
21	610	55	4100	43	84	9325
22	39	85	29	72	7613	52
23	69	2414	58	5901	42	81
24	98	43	87	30	71	9410
25	727	72	4216	59	7700	39
26	56	2501	45	88	29	68
27	85	30	74	6017	58	97
28	814	59	4303	46	87	9526
29	43	88	32	75	7816	55
30	73	2617	61	6104	45	84

Figure 69: Excerpt of Fale's table of sines [Fale (1593)]. (source: https://archive.org/details/b30333106)

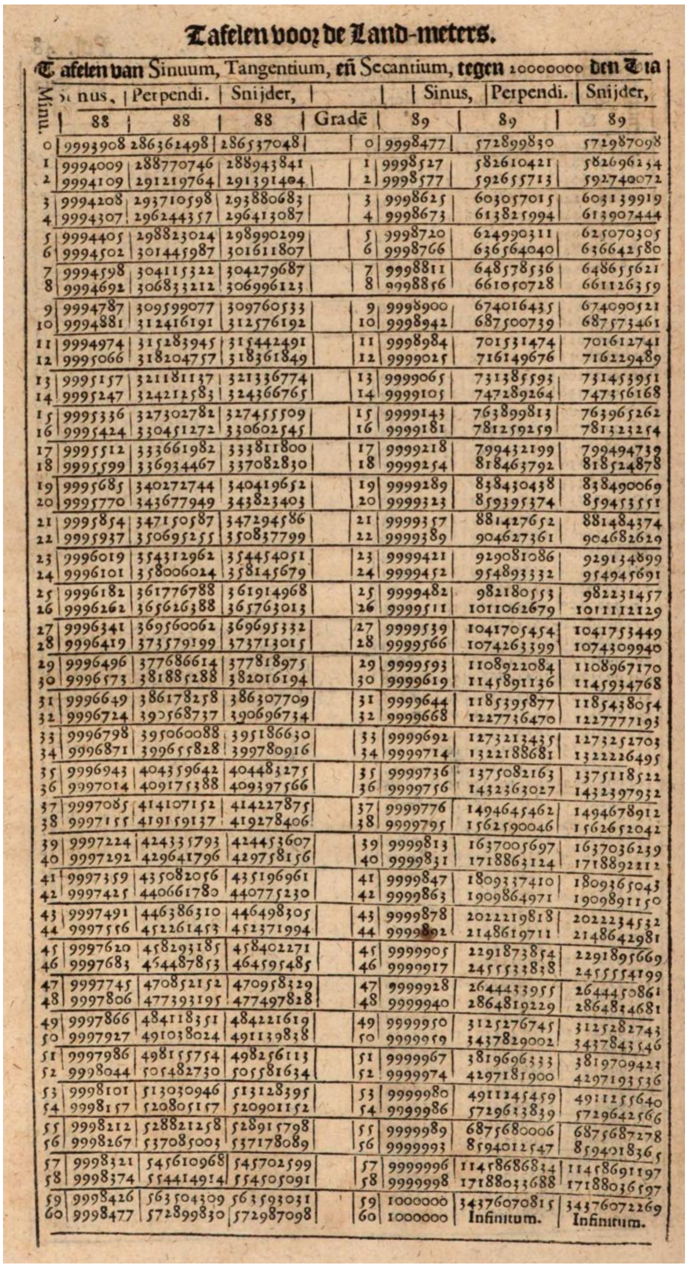


Tafelen voor de Land-meters.

Tafelen van Sinuum, Tangentium, **en** Secantium, **tegen** 10000000 **den Dia.**

Minu.	Sinus. 0	Perpen. 0	Snijder. 0	Gradē		Sinus. 1	Perpen. 1	Snijder. 1
0	0	0000	10000000		0	174524	174550	10001524
1	2909	2909	10000001		1	177433	177455	10001574
2	5818	5818	10000002		2	180341	180365	10001656
3	8727	8727	10000004		3	183250	183279	10001679
4	11636	11636	10000008		4	186158	186189	10001733
5	14544	14544	10000010		5	189066	189100	10001788
6	17453	17452	10000014		6	191975	192010	10001844
7	20362	20361	10000020		7	194883	194920	10001900
8	23271	23270	10000027		8	197792	197830	10001957
9	26180	26179	10000034		9	200700	200740	10002015
10	29088	29088	10000042		10	203608	203650	10002074
11	39197	31996	10000051		11	206517	206561	10002134
12	34906	34905	10000060		12	209425	209471	10002195
13	37815	37814	10000071		13	212333	212381	10002256
14	40724	40723	10000083		14	215241	215291	10002319
15	43632	43632	10000095		15	218149	818201	10001381
16	47531	46541	10000108		16	221057	221111	10002445
17	49450	49450	10000122		17	223965	224022	10002510
18	52359	52359	10000137		18	226873	226932	10002576
19	55268	55268	10000152		19	229781	229842	10002642
20	58177	58177	10000168		20	232689	232752	10002709
21	61086	61086	10000186		21	235597	235663	10002777
22	63995	63995	10000204		22	238505	238574	10002846
23	66904	66904	10000223		23	241413	241485	10002916
24	69813	69813	10000243		24	244321	244395	10002987
25	72721	72722	10000264		25	247229	247306	10003058
26	75630	75631	10000285		26	250137	250217	10003130
27	78539	78540	10000308		27	253045	253128	10003203
28	81448	81450	10000332		28	255953	256038	10003277
29	84357	84359	10000357		29	258861	258949	10003352
30	87265	87268	10000381		30	261769	261859	10003428
31	90174	90177	10000407		31	264677	264770	10003505
32	93083	93086	10000433		32	267585	267681	10003582
33	95992	95995	10000461		33	270493	270592	10003660
34	98901	98904	10000489		34	273401	273503	10003739
35	101809	101814	10000518		35	276308	276414	10003819
36	104718	104723	10000548		36	279216	279325	10003900
37	107627	107632	10000579		37	282124	282237	10003982
38	110536	110541	10000611		38	285032	285148	10004065
39	113445	113450	10000643		39	287940	288059	10004148
40	116353	116360	10000677		40	290847	290970	10004232
41	119262	119269	10000711		41	293755	293882	10004317
42	122171	122178	10000746		42	296663	296794	10004403
43	125079	125088	10000782		43	299570	299705	10004490
44	127988	127997	10000819		44	302478	302617	10004578
45	130896	130906	10000857		45	305385	305528	10004666
46	133805	133816	10000895		46	308293	308439	10004755
47	136714	136725	10000934		47	311200	311351	10004845
48	139622	139635	10000975		48	314108	314262	10004936
49	142531	142544	10001016		49	317015	317174	10005028
50	145439	145454	10001058		50	319922	320085	10005122
51	148348	148363	10001100		51	322830	322997	10005216
52	151257	151273	10001144		52	325737	325909	10005310
53	154165	154182	10001188		53	328645	328821	10005405
54	157074	157092	10001233		54	331552	331733	10005501
55	159982	160001	10001280		55	334459	334645	10005598
56	162891	162911	10001327		56	337367	337558	10005696
57	165799	165820	10001375		57	340274	340470	10005795
58	168708	168730	10001423		58	343181	343382	10005894
59	171616	171640	10001473		59	346088	346295	10005994
60	174524	174550	10001524		60	348995	349207	10006095

Figure 70: The first page from Ceulen's table of trigonometric functions [Ceulen (1596)].

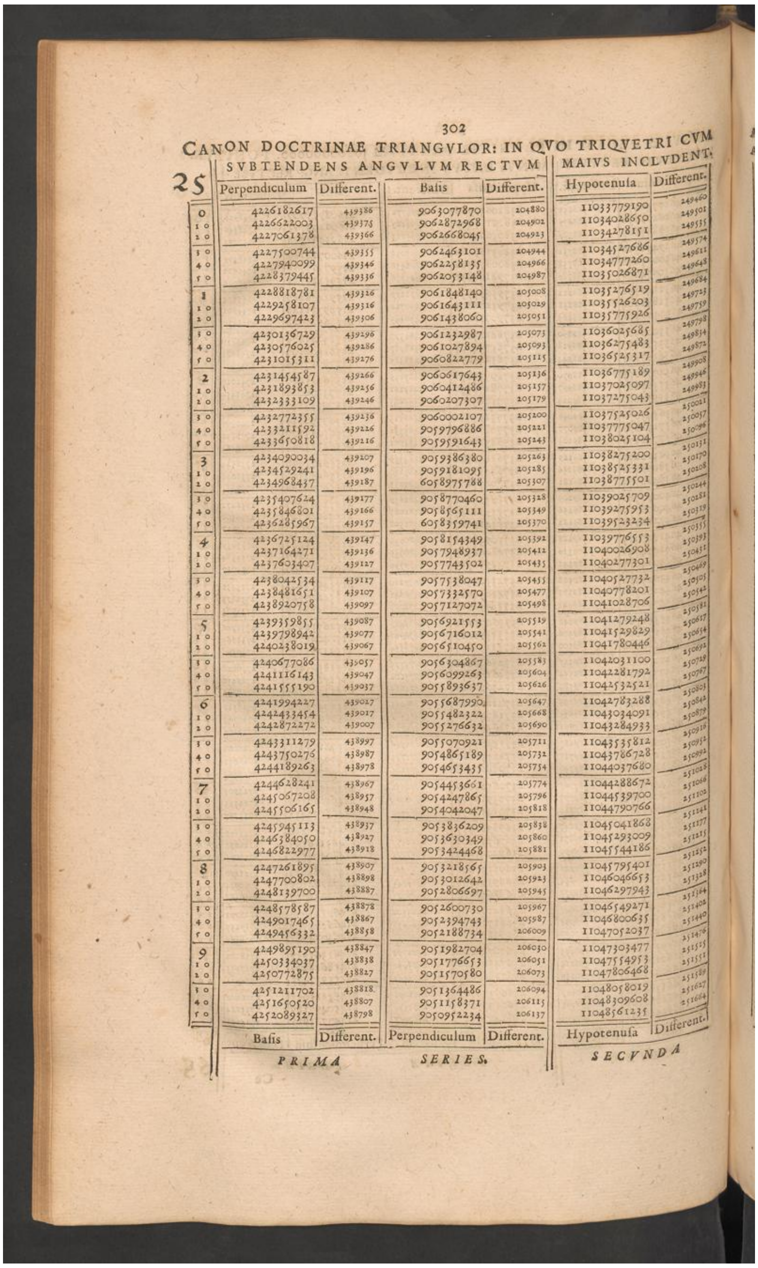


Tafelen voor de Land-meters.

Tafelen van Sinuum, Tangentium, eñ Secantium, tegen 20000000 den Tia

Minu.	Sinus, 88	Perpendi. 88	Snijder, 88	Gradē	Min.	Sinus, 89	Perpendi. 89	Snijder, 89
0	9993908	286362498	286537048		0	9998477	572899830	572987098
1	9994009	288770746	288943841		1	9998527	582610421	582696254
2	9994109	291219764	291391404		2	9998577	592655713	592740072
3	9994208	293710598	293880683		3	9998625	603057015	603139919
4	9994307	296244357	296413087		4	9998673	613825994	613907444
5	9994405	298823024	298990299		5	9998720	624990311	625070305
6	9994502	301445987	301611807		6	9998766	636564040	636642580
7	9994598	304115322	304279687		7	9998811	648578536	648655621
8	9994692	306833212	306996123		8	9998856	661050728	661126359
9	9994787	309599077	309760533		9	9998900	674016435	674090521
10	9994881	312416191	312576192		10	9998942	687500739	687573461
11	9994974	315283945	315442491		11	9998984	701531474	701612741
12	9995066	318204757	318361849		12	9999025	716149676	716229489
13	9995157	321181137	321336774		13	9999065	731385593	731453951
14	9995247	324212583	324366765		14	9999105	747289264	747356168
15	9995336	327302782	327455509		15	9999143	763899813	763965262
16	9995424	330451272	330602545		16	9999181	781259259	781323254
17	9995512	333661982	333811800		17	9999218	799432199	799494739
18	9995599	336934467	337082830		18	9999254	818463792	818524878
19	9995685	340272744	340419652		19	9999289	838430438	838490069
20	9995770	343677949	343823403		20	9999323	859395374	859453551
21	9995854	347150587	347294586		21	9999357	881427652	881484374
22	9995937	350695255	350837799		22	9999389	904627361	904682629
23	9996019	354312962	354454051		23	9999421	929081086	929134899
24	9996101	358006024	358145679		24	9999452	954893332	954945691
25	9996182	361776788	361914968		25	9999482	982180553	982231457
26	9996262	365626388	365763013		26	9999511	1011062679	1011112129
27	9996341	369560062	369695332		27	9999539	1041705454	1041753449
28	9996419	373579199	373713015		28	9999566	1074263399	1074309940
29	9996496	377686614	377818975		29	9999593	1108922084	1108967170
30	9996573	381885288	382016194		30	9999619	1145891136	1145934768
31	9996649	386178258	386307709		31	9999644	1185395877	1185438054
32	9996724	390568737	390696734		32	9999668	1227736470	1227777193
33	9996798	395060088	395186630		33	9999692	1273213435	1273252703
34	9996871	399655828	399780916		34	9999714	1322188681	1322226495
35	9996943	404359642	404483275		35	9999736	1375082163	1375118522
36	9997014	409175388	409397566		36	9999756	1432363027	1432397932
37	9997085	414107152	414227875		37	9999776	1494645462	1494678912
38	9997155	419159137	419278406		38	9999795	1562590046	1562652042
39	9997224	424335793	424453607		39	9999813	1637005697	1637036239
40	9997292	429641796	429758156		40	9999831	1718863124	1718892212
41	9997359	435082056	435196961		41	9999847	1809337410	1809365043
42	9997425	440661780	440775230		42	9999863	1909864971	1909891150
43	9997491	446386310	446498305		43	9999878	2022219818	2022234532
44	9997556	452261453	452371994		44	9999892	2148619711	2148642981
45	9997620	458293185	458402271		45	9999905	2291873854	2291895669
46	9997683	464487853	464595485		46	9999917	2455533838	2455554199
47	9997745	470852152	470958329		47	9999928	2644433955	2644450861
48	9997806	477393195	477497828		48	9999940	2864819229	2864834681
49	9997866	484118351	484221619		49	9999950	3125276745	3125282743
50	9997927	491038024	491139838		50	9999959	3437829002	3437843546
51	9997986	498155754	498256113		51	9999967	3819696333	3819709423
52	9998044	505482730	505581634		52	9999974	4297181900	4297193536
53	9998101	513030946	513128395		53	9999980	4911245459	4911255640
54	9998157	520805157	520901152		54	9999986	5729633839	5729642566
55	9998212	528821258	528915798		55	9999989	6875680006	6875687278
56	9998267	537085003	537178089		56	9999993	8594012547	8594018365
57	9998321	545610968	545702599		57	9999996	11458686834	11458691197
58	9998374	554414914	554505091		58	9999998	17188033688	17188036597
59	9998426	563504309	563593031		59	1000000	34376070815	34376072269
60	9998477	572899830	572987098		60	1000000	Infinitum.	Infinitum.

Figure 71: The last page from Ceulen's table of trigonometric functions [Ceulen (1596)].



302

CANON DOCTRINAE TRIANGVLOR: IN QVO TRIQVETRI CVM

SVBTENDENS ANGVLVM RECTVM | MAIVS INCLVDENT.

25

	Perpendiculum	Different.	Basis	Different.	Hypotenusa	Different.
0	4226182617	439386	9063077870	204880	11033779190	249460
10	4226622003	439375	9062872968	204902	11034028650	249501
20	4227061378	439366	9062668045	204923	11034278151	249535
30	4227500744	439355	9062463101	204944	11034527686	249574
40	4227940099	439346	9062258135	204966	11034777260	249611
50	4228379445	439336	9062053148	204987	11035026871	249648
1	4228818781	439326	9061848140	205008	11035276519	249684
10	4229258107	439316	9061643111	205029	11035526203	249723
20	4229697423	439306	9061438060	205051	11035775926	249759
30	4230136729	439296	9061232987	205073	11036025685	249798
40	4230576025	439286	9061027894	205093	11036275483	249834
50	4231015311	439276	9060822779	205115	11036525317	249872
2	4231454587	439266	9060617643	205136	11036775189	249908
10	4231893853	439256	9060412486	205157	11037025097	249946
20	4232333109	439246	9060207307	205179	11037275043	249983
30	4232772355	439236	9060002107	205200	11037525026	250021
40	4233211592	439226	9059796886	205221	11037775047	250057
50	4233650818	439216	9059591643	205243	11038025104	250096
3	4234090034	439207	9059386380	205263	11038275200	250131
10	4234529241	439196	9059181095	205285	11038525331	250170
20	4234968437	439187	6058975788	205307	11038775501	250208
30	4235407624	439177	9058770460	205328	11039025709	250244
40	4235846801	439166	9058565111	205349	11039275953	250281
50	4236285967	439157	6058359741	205370	11039523234	250319
4	4236725124	439147	9058154349	205392	11039776553	250355
10	4237164271	439136	9057948937	205412	11040026908	250393
20	4237603407	439127	9057743502	205435	11040277301	250431
30	4238042534	439117	9057538047	205455	11040527732	250469
40	4238481651	439107	9057332570	205477	11040778201	250505
50	4238920758	439097	9057127072	205498	11041028706	250542
5	4239359855	439087	9056921553	205519	11041279248	250581
10	4239798942	439077	9056716012	205541	11041529829	250617
20	4240238019	439067	9056510450	205562	11041780446	250654
30	4240677086	439057	9056304867	205583	11042031100	250692
40	4241116143	439047	9056099263	205604	11042281792	250729
50	4241555190	439037	9055893637	205626	11042532521	250767
6	4241994227	439027	9055687990	205647	11042783288	250803
10	4242433454	439017	9055482322	205668	11043034091	250842
20	4242872272	439007	9055276632	205690	11043284933	250879
30	4243311279	438997	9055070921	205711	11043535812	250916
40	4243750276	438987	9054865189	205732	11043786728	250952
50	4244189263	438978	9054653435	205754	11044037680	250992
7	4244628241	438967	9054453661	205774	11044288672	251028
10	4245067208	438957	9054247865	205796	11044539700	251066
20	4245506165	438948	9054042047	205818	11044790766	251102
30	4245945113	438937	9053836209	205838	11045041868	251141
40	4246384050	438927	9053630349	205860	11045293009	251177
50	4246822977	438918	9053424468	205881	11045544186	251215
8	4247261895	438907	9053218565	205903	11045795401	251252
10	4247700802	438898	9053012642	205923	11046046653	251290
20	4248139700	438887	9052806697	205945	11046297943	251328
30	4248578587	438878	9052600730	205967	11046549271	251364
40	4249017465	438867	9052394743	205987	11046800635	251402
50	4249456332	438858	9052188734	206009	11047052037	251440
9	4249895190	438847	9051982704	206030	11047303477	251476
10	4250334037	438838	9051776653	206051	11047554953	251515
20	4250772875	438827	9051570580	206073	11047806468	251551
30	4251211702	438818	9051364486	206094	11048058019	251589
40	4251650520	438807	9051158371	206115	11048309608	251627
50	4252089327	438798	9050952234	206137	11048561235	251664
	Basis	Different.	Perpendiculum	Different.	Hypotenusa	Different.

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Figure 72: Excerpt of Rheticus's *Opus palatinum* [Rheticus and Otho (1596)] (source: e-rara).

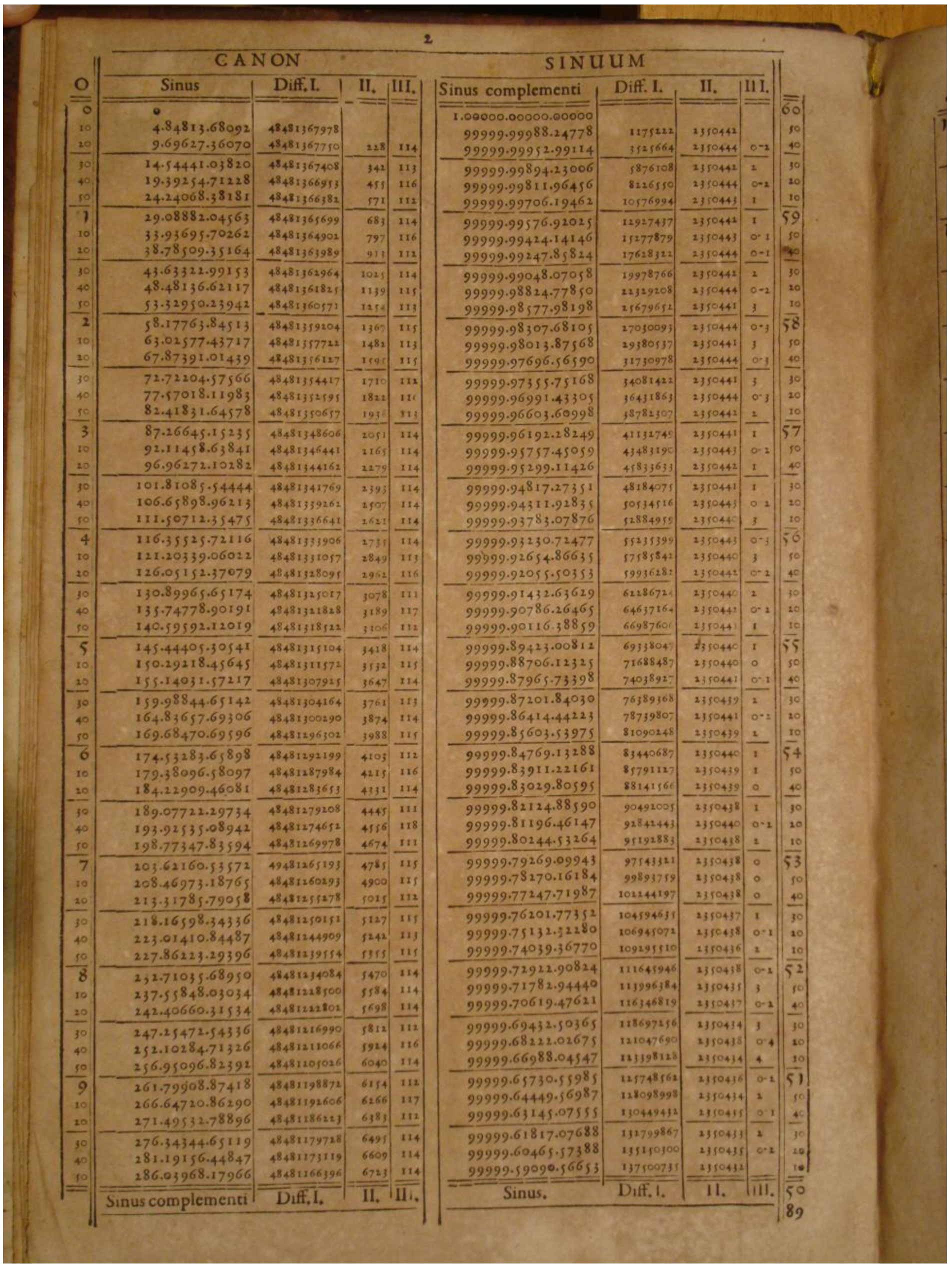

Figure 73: Excerpt of Pitiscus's *Thesaurus mathematicus* [Pitiscus (1613)] (source: École des Ponts ParisTech, Paris, photograph by the author).