A reconstruction of the tables of Rheticus' *Canon doctrinæ triangulorum* (1551)

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11 January 2011

(last version: 2 September 2021)

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Quam quisque novit artem, in hac se exerceat
Cicero

1 Trigonometric tables before Rheticus

As told by Gerhardt in his history of mathematics in Germany, Peuerbach (1423–1461) and his pupil Regiomontanus (1436–1476) [30, 43, 85] woke up the study of astronomy and built the necessary tables [33, p. 87]. Regiomontanus calculated tables of sines for every minute of arc for radiuses of 6000000 and 10000000 units.[1]

Regiomontanus had also computed a table of tangents [50], but with the smaller radius $R = 10^5$ and at intervals of 1°. There were no printed tables of secants, but an ever growing number of tables copied from those of Regiomontanus or derived from his tables.

2 Rheticus (1514–1574)

Georg Joachim was born in 1514 in Feldkirch (Austria) in what was formerly the Roman province of Raetia, from which the name Rheticus is derived. His father was a doctor.[2] After studying in Zurich from 1528 to 1532, Rheticus went to Wittenberg to complete his education. Wittenberg was then the center of the sciences. He obtained his doctorate in 1536, went to Nuremberg and Tübingen, then was called back to Wittenberg as professor of mathematics in 1536, along with Erasmus Reinhold. In 1539, he went to Frauenburg in order to assist Copernicus and pushed him to publish the details of his theory.

The first account of the Copernican system was not published by Copernicus, but in Rheticus's *Narratio prima* in 1540 [53, 66, 7, 5, 6, 8, 9, 75]. In fact, Rheticus was more zealous than Copernicus, insisting on the physical truth of the motion of the earth. Rheticus's books were condemned by the catholic church in 1550 and put on the *Index Expurgatorius*, and this condemnation applied to his works on trigonometry. As observed by De Morgan, this severity was not so remarkable, if we remember that Rheticus held the chair of mathematics at Wittenberg, where Luther had taught theology [23, p. 518].

Rheticus resumed his teaching in Wittenberg in 1541 and in 1542, Copernicus's *De lateribus et angulis triangulorum* [17, 73] was published by Rheticus in Wittenberg. This short treatise on trigonometry contains a table of sines for every minute of the quadrant and for a radius of 10^7. This table was copied from Regiomontanus's decimal table [65].

Copernicus's *De Revolutionibus* (1543), whose printing was partly overseen by Rheticus, contained the same chapter on trigonometry, but with a simpler table [16, book 1, chapter 12, ff. 15–19]. The sines were only given every $10'$ and for a radius of 100000. This table is also derived from Regiomontanus's works [36, pp. 178–179], [65].

In 1542, Rheticus went to Nuremberg and Philip Melanchton [81] then helped him to obtain a position at Leipzig University where he remained from 1542 to 1551. Rheticus

[1] For a summary of fundamental trigonometric tables printed in the 15th and 16th centuries, see my survey [65]. The present report was first written in 2011, but it has been updated and corrected in light of this survey.

[2] For extensive biographies of Rheticus, see Burmeister [13, 14], Kraai [45], Danielson [19], and Schöbi-Fink *et al*'s collections [80, 69].

had to leave Leipzig after being accused of homosexual conduct and went to Prague in 1551. He gave up his academic career and completed his medicine studies. He settled in Cracow in 1554 where he practiced medicine. Cracow was located on the same meridian as Frauenburg and Rheticus had a 15-meter obelisk erected near the city for astronomical purposes which were never completed [14, p. 9], [84]. Rheticus died during a visit in Košice, now in Slovakia, in 1574.

3 Rheticus's first canon (1551)

Rheticus published his first trigonometric canon, the *Canon doctrinæ triangulorum*, in 1551 in Leipzig [54]. This table gave the six trigonometric functions at intervals of $10'$, semi-quadrantically arranged. Each function was given to 7 places, or more exactly as integers for a radius of 10^7. This is a very rare table, and it was practically unknown when De Morgan happened to find a copy of it in the 1840s [24, 23].

Rheticus was the first to provide a uniform treatment of the six trigonometric functions. He was in particular the first to publish a table of secants.[3]

Rheticus did not consider angles in circles, but considered triangles of which one of the side was constant, and he gave the lengths of the other sides as a function of the angle at the center. The name of "sine" was not used by Rheticus, nor those of tangent and secant, the latter two having not yet been introduced.[4] The sides were called "base," "perpendicular," and "hypothenuse." The tables span 14 pages, with 7 degrees on each double page. The last double page is not totally filled, with only the degrees 42 to 44. Each double page is split in three groups of two columns. The first group (hypothenuse $= 10^7$) gives the sines and cosines, with their differences. The second group (base $= 10^7$) gives the secants and the tangents. And the third group (perpendicular $= 10^7$) gives the cosecants and the cotangents.

The heading of the tables means "Canon of the doctrine of triangles in which in a plane triangle with right angle 10^7 parts are taken for 1) the (side) subtending the right angle, 2) the greatest of the two sides including the right angle, and 3) the smallest of the two sides including the right angle" [10, p. 13].

The structure of the table was first described by De Morgan [24, 23]. A similar layout was used in Rheticus and Otho's *Opus palatinum* [55].[5]

Rheticus actually only computed two thirds of his table. The table of sines was taken from Regiomontanus [36, p. 152], but the tangents and secants were recomputed from the sines [36, p. 193]. Rheticus had therefore to compute about 1080 ratios ($90 \times 6 \times 2$).

More precisely, the sines in Rheticus's table were copied from Regiomontanus's table for $R = 10^7$ [68], [36, p. 152]. Most of the values were not changed, but some of the

[3] In 1819, Delambre, unaware of Rheticus's 1551 table, still wrote that Viète was the first to put together all these functions in 1579 [26, p. 456].

[4] It was Thomas Fincke who in 1583 first used the names "tangent" and "secant" [28, 25]. See also [35, p. 42].

[5] De Morgan considers that the semi-quadrantal arrangement of the tables finds its roots in Rheticus's three cases of triangles. In the first case, the greatest side of the triangle is the cosine, and the smallest side is the sine. The divisions would not have been the same if Rheticus had given the sines for the whole quadrant. Rheticus's arrangement led to consider the duality of sine and cosine, tangent and cotangent, and secant and cosecant [23, pp. 523–524].

typos were corrected, for instance the cosine of $38°40'$ whose value was still incorrect (as $\operatorname{Sin} 51°20'$) in Rheticus's 1542 table.

But Rheticus made new computations for the tangents and the secants using these sines. According to Glowatzki and Göttsche [36, p. 185], Rheticus merely computed the ratios for the tangents, but things are actually a bit more complicated.

First, it appears that the secants and cosecants were computed by dividing 1 (or rather 10^{14}) by the values of the cosines or sines, and truncating the results. This can readily be observed on the secants of $17°20'$, $29°$, $29°50'$, $43°$, etc., and practically every ratio whose decimal part is greater than 0.5. This is also true for the cosecants, an example being $35°30'$.

But the tangents and cotangents are another story. I don't know exactly how Rheticus computed these values, but a close examination of Rheticus's values reveals that the tangents are more accurate than the cotangents and consequently one cannot have been computed from the other. They must have been computed differently. The tangents may have been computed by dividing the sines by the cosines, but this cannot have been the case for the cotangents.[6]

It appears that the values of the cotangents are close to those obtained when computing

$$\operatorname{Cot} x = \sqrt{\operatorname{Csc}^2 x - R^2}$$

but they are not totally identical. The agreement is however much better than that obtained by merely dividing the values of the cosines by the sines of Regiomontanus, and it may even be a little better if $\operatorname{Csc}^2 x$ is rounded to seven or eight significant digits. This hypothesis may need to be tested further, but it parallels a suggestion by van Brummelen and Byrne for the computation of secants by Maurolico [77, 46], although I argue in my survey [65] that their suggestion is in fact not applicable to Maurolico's computations. However, in my survey I also suggest that Fincke used a similar procedure to compute his secants in 1583.

The 1551 canon has been reprinted in 1565 in Basel [36, p. 185].[7]

Rheticus's work remains based on Regiomontanus's tables, and although he was the first to construct a table giving all six triangle ratios, he did not compute the cosecants and cotangents sufficiently accurately for small angles, and does not yet seem to have understood that more accurate sines were needed. He has no problem giving cosecants and cotangents to 10 figures, when the sine is only given to 5 figures. This understanding of the need for more accurate sines only came later, and even the *Opus palatinum* [55] published in 1596 is still marred by this problem which will only fully be solved by Pitiscus in the early 17th century [48].

4 Rheticus's heirs and influence

After Rheticus's 1551 table, other authors such as Reinhold in 1554 [52], [36, pp. 152–153], or Fincke in 1583 [28], still continued to give simpler tables based on Regiomontanus's

[6] It will be interesting to see to what conclusions came [49] who seems to have conducted a similar investigation, but whose result is not yet published at the time I am writing this.

[7] De Morgan wrote earlier that it had been reprinted in 1580 [23, p. 521].

tables, but some authors were directly influenced by Rheticus. The first of them was François Viète.

4.1 Viète's *Canon mathematicus* (1579)

Inspired by Rheticus's table [41], François Viète (1540–1603) constructed a new table, which he called the *Canon mathematicus* [78]. The printing of the table was started in 1571 but it was only completed in 1579 [74, p. 205].

The *Canon mathematicus* contained a typographically sophisticated table of the six trigonometric functions for every minute of the quadrant and with a radius of 100000, with sometimes one or two additional figures (figure 1) [41] [26, pp. 455–456]. This was the first published canon giving the trigonometric functions every minute, but on the other hand it gave them to less places than Rheticus's first table. Viète took the sines from Regiomontanus, except for some parts which have been recomputed [36, pp. 154–155], [65]. The tangents and secants were recomputed, not from Regiomontanus's values as suggested by Glowatzki and Göttsche [36, pp. 189, 196], but using Viète's own values, either in his table of sines, or from more accurate values, for instance for $\operatorname{Sin} 1'$ [65].

4.2 Otho's *Opus palatinum* (1596)

After the publication of the 1551 canon, Rheticus continued to work on a more extensive project, where the six trigonometric functions would be given every $10''$ and for a radius of 10^{10}. This work was only completed after Rheticus's death by Lucius Valentinus Otho and published in 1596 [55, 57]. It was further corrected by Pitiscus, in particular in his *Thesaurus mathematicus* published in 1613 [48].

5 Structure of the tables and recomputation

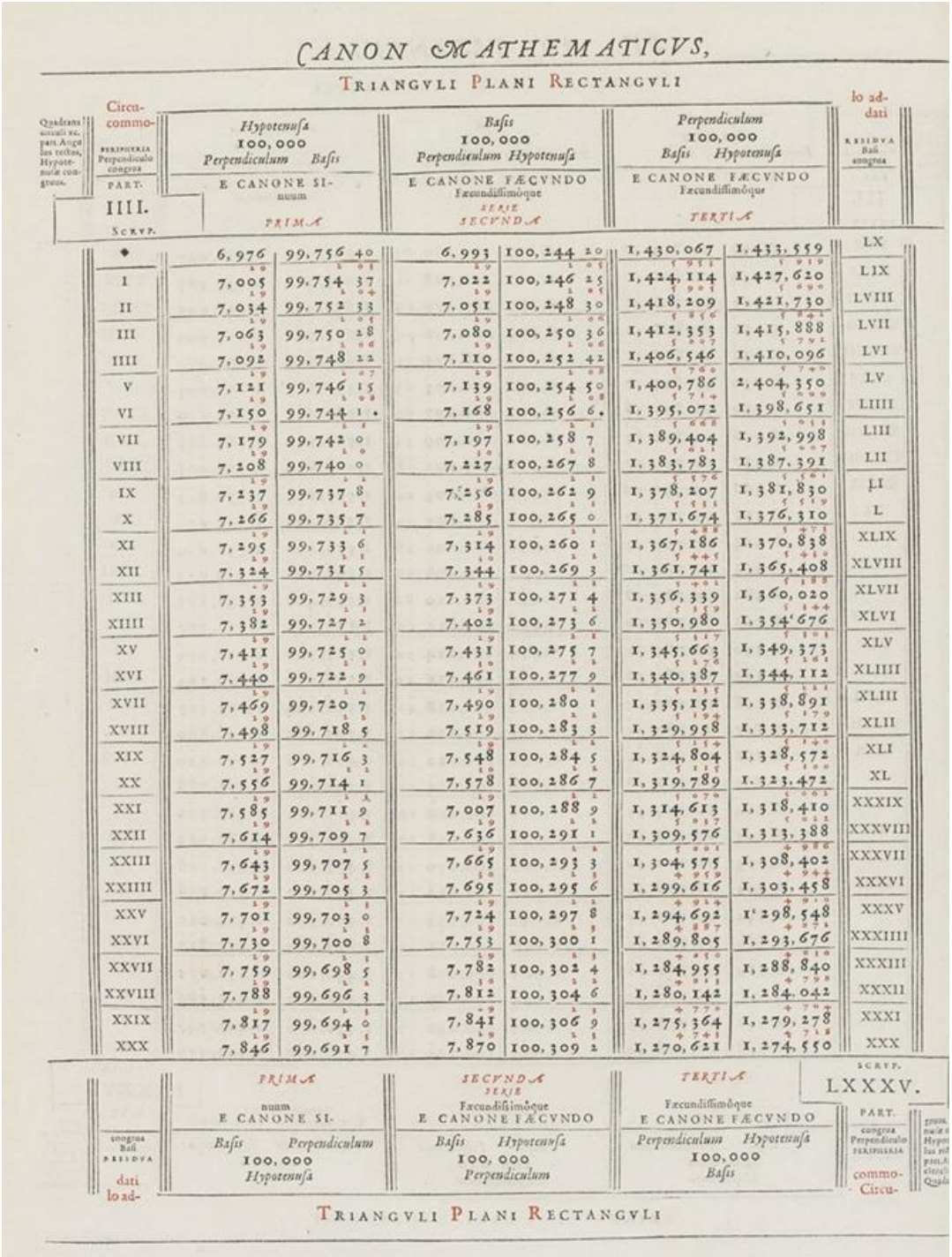
In addition to the tables described above, Rheticus's book contained a cover page, an introductory poem, and six pages of explanations which are not reproduced here. The original tables used two colors, and there are some overlaps between the red and black figures. Viète's tables also use two colors but their layout is much more successful.

Some values were omitted and I have added them in my reconstruction. The sine, secant, and tangent values for 0°, as well as the first difference of the tangents, were given as "....." or ".....," and the value of the last cosecant of the table was merely left blank.

Rheticus's tables were recomputed using the GNU `mpfr` multiple-precision floating-point library developed at INRIA [32], and give the exact values. The comparison of my table and Rheticus's will therefore immediately show where Rheticus's table contains errors.

Acknowledgements

It is my pleasure to acknowledge the help of Michela Malpangotto, who hinted to an earlier shortcoming in this study.



CANON MATHEMATICVS,

TRIANGVLI PLANI RECTANGVLI

	Hypotenusa 100,000 / Perpendiculum / Basi — E CANONE SINVUM — PRIMA		Basis 100,000 / Perpendiculum / Hypotenusa — E CANONE FÆCVNDO Fœcundissimôque — SERIE SECVNDA		Perpendiculum 100,000 / Basi / Hypotenusa — E CANONE FÆCVNDO Fœcundissimôque — TERTIA		
♦	6,976	99,756 40	6,993	100,244 20	1,430,067	1,433,559	LX
I	7,005	99,754 37	7,022	100,246 25	1,424,114	1,427,620	LIX
II	7,034	99,752 33	7,051	100,248 30	1,418,209	1,421,730	LVIII
III	7,063	99,750 28	7,080	100,250 36	1,412,353	1,415,888	LVII
IIII	7,092	99,748 22	7,110	100,252 42	1,406,546	1,410,096	LVI
V	7,121	99,746 15	7,139	100,254 50	1,400,786	2,404,350	LV
VI	7,150	99,744 1.	7,168	100,256 6.	1,395,072	1,398,651	LIIII
VII	7,179	99,742 0	7,197	100,258 7	1,389,404	1,392,998	LIII
VIII	7,208	99,740 0	7,227	100,267 8	1,383,783	1,387,391	LII
IX	7,237	99,737 8	7,256	100,262 9	1,378,207	1,381,830	LI
X	7,266	99,735 7	7,285	100,265 0	1,371,674	1,376,310	L
XI	7,295	99,733 6	7,314	100,260 1	1,367,186	1,370,838	XLIX
XII	7,324	99,731 5	7,344	100,269 3	1,361,741	1,365,408	XLVIII
XIII	7,353	99,729 3	7,373	100,271 4	1,356,339	1,360,020	XLVII
XIIII	7,382	99,727 2	7,402	100,273 6	1,350,980	1,354,676	XLVI
XV	7,411	99,725 0	7,431	100,275 7	1,345,663	1,349,373	XLV
XVI	7,440	99,722 9	7,461	100,277 9	1,340,387	1,344,112	XLIIII
XVII	7,469	99,720 7	7,490	100,280 1	1,335,152	1,338,891	XLIII
XVIII	7,498	99,718 5	7,519	100,283 3	1,329,958	1,333,712	XLII
XIX	7,527	99,716 3	7,548	100,284 5	1,324,804	1,328,572	XLI
XX	7,556	99,714 1	7,578	100,286 7	1,319,789	1,323,472	XL
XXI	7,585	99,711 9	7,007	100,288 9	1,314,613	1,318,410	XXXIX
XXII	7,614	99,709 7	7,636	100,291 1	1,309,576	1,313,388	XXXVIII
XXIII	7,643	99,707 5	7,665	100,293 3	1,304,575	1,308,402	XXXVII
XXIIII	7,672	99,705 3	7,695	100,295 6	1,299,616	1,303,458	XXXVI
XXV	7,701	99,703 0	7,724	100,297 8	1,294,692	1,298,548	XXXV
XXVI	7,730	99,700 8	7,753	100,300 1	1,289,805	1,293,676	XXXIIII
XXVII	7,759	99,698 5	7,782	100,302 4	1,284,955	1,288,840	XXXIII
XXVIII	7,788	99,696 3	7,812	100,304 6	1,280,142	1,284,042	XXXII
XXIX	7,817	99,694 0	7,841	100,306 9	1,275,364	1,279,278	XXXI
XXX	7,846	99,691 7	7,870	100,309 2	1,270,621	1,274,550	XXX

PRIMA — E CANONE SINVUM	SECVNDA SERIE — E CANONE FÆCVNDO Fœcundissimôque	TERTIA — E CANONE FÆCVNDO Fœcundissimôque	SCRVP. LXXXV. PART.
Basis / Perpendiculum 100,000 / Hypotenusa	Basis / Hypotenusa 100,000 / Perpendiculum	Perpendiculum / Hypotenusa 100,000 / Basi	

TRIANGVLI PLANI RECTANGVLI

Figure 1: A page from Viète's *Canon mathematicus* (1579). This page gives (from left to right) the sines, cosines, tangents, secants, cotangents, and cosecants for angles from 4° to 4°30′ and from 85°30′ to 86°. (source: `http://www.christies.com`, book auctioned on June 17, 2008 in New York)

References

The following list covers the most important references[8] related to Rheticus's tables. Not all items of this list are mentioned in the text, and the sources which have not been seen are marked so. I have added notes about the contents of the articles in certain cases.

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[8] **Note on the titles of the works:** Original titles come with many idiosyncrasies and features (line splitting, size, fonts, etc.) which can often not be reproduced in a list of references. It has therefore seemed pointless to capitalize works according to conventions which not only have no relation with the original work, but also do not restore the title entirely. In the following list of references, most title words (except in German) will therefore be left uncapitalized. The names of the authors have also been homogenized and initials expanded, as much as possible.

The reader should keep in mind that this list is not meant as a facsimile of the original works. The original style information could no doubt have been added as a note, but I have not done it here.

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CANON DOCTRINÆ TRIANGVLORVM IN QVO TRIQVETRI

		Subtendens angulum rectum				Maius latus includen-	
		Perpendicu:	Differen:	Basis	Differen:	Hypotenusa	Differen:
0	0	0	29089	10000000	42	10000000	42
	10	29089	29088	9999958	127	10000042	127
	20	58177	29088	9999831	212	10000169	212
	30	87265	29088	9999619	296	10000381	296
	40	116353	29086	9999323	381	10000677	381
	50	145439	29085	9998942	465	10001058	465
1	0	174524	29084	9998477	550	10001523	550
	10	203608	29082	9997927	635	10002073	635
	20	232690	29079	9997292	719	10002708	720
	30	261769	29078	9996573	803	10003428	804
	40	290847	29075	9995770	889	10004232	889
	50	319922	29073	9994881	973	10005121	974
2	0	348995	29070	9993908	1057	10006095	1059
	10	378065	29066	9992851	1142	10007154	1144
	20	407131	29063	9991709	1227	10008298	1229
	30	436194	29059	9990482	1311	10009527	1314
	40	465253	29055	9989171	1396	10010841	1398
	50	494308	29052	9987775	1480	10012239	1484
3	0	523360	29046	9986295	1564	10013723	1570
	10	552406	29042	9984731	1649	10015293	1654
	20	581448	29037	9983082	1734	10016947	1740
	30	610485	29032	9981348	1818	10018687	1825
	40	639517	29027	9979530	1903	10020512	1911
	50	668544	29021	9977627	1986	10022423	1996
4	0	697565	29015	9975641	2072	10024419	2082
	10	726580	29009	9973569	2156	10026501	2168
	20	755589	29002	9971413	2240	10028669	2253
	30	784591	28996	9969173	2324	10030922	2339
	40	813587	28989	9966849	2409	10033261	2426
	50	842576	28981	9964440	2493	10035687	2511
5	0	871557	28975	9961947	2577	10038198	2598
	10	900532	28967	9959370	2662	10040796	2684
	20	929499	28959	9956708	2746	10043480	2771
	30	958458	28950	9953962	2830	10046251	2857
	40	987408	28943	9951132	2915	10049108	2944
	50	1016351	28934	9948217	2998	10052052	3031
6	0	1045285	28925	9945219	3083	10055083	3117
	10	1074210	28916	9942136	3167	10058200	3205
	20	1103126	28906	9938969	3250	10061405	3292
	30	1132032	28897	9935719	3335	10064697	3380
	40	1160929	28887	9932384	3419	10068077	3467
	50	1189816	28877	9928965	3503	10071544	3554
		Basis:	Differen:	Perpendiculū	Differen:	Hypotenusa	Differen:

CVM ANGVLO RECTO IN PLANITIE PARTIVM 10000000 PONITVR

tiū angulū rect:		Minus latus includentium angulum rectum.					
Perpend:	Differen:	Hypotenusa	Differen:	Basis.	Differen:		
0	29089	Infinitum	Infinitum	Infinitum	Infinitum	0	90
29089	29089	3437751619	1718868537	3437737075	1718883082	50	
58178	29091	1718883082	572952947	1718853993	572967492	40	
87269	29092	1145930135	286474049	1145886501	286488594	30	
116361	29093	859456086	171882491	859397907	171897035	20	
145454	29097	687573595	114586710	687500872	114601256	10	
174551	29099	572986885	81846265	572899616	81860810	0	89
203650	29103	491140620	61383486	491038806	61398033	50	
232753	29106	429757134	47741634	429640773	47756180	40	
261859	29111	382015500	38192337	381884593	38206884	30	
290970	29116	343823163	31247393	343677709	31261942	20	
320086	29122	312575770	26038687	312415767	26053234	10	
349208	29127	286537083	22031987	286362533	22046537	0	88
378335	29134	264505096	18883868	264315996	18898418	50	
407469	29140	245621228	16365372	245417578	16379923	40	
436609	29148	229255856	14319093	229037655	14333645	30	
465757	29156	214936763	12633923	214704010	12648475	20	
494913	29165	202302840	11229614	202055535	11244168	10	
524078	29173	191073226	10047038	190811367	10061593	0	87
553251	29183	181026188	9041848	180749774	9056405	50	
582434	29192	171984340	8180258	171693369	8194814	40	
611626	29203	163804082	7436155	163498555	7450714	30	
640829	29214	156367927	6789111	156047841	6803671	20	
670043	29225	149578816	6222946	149244170	6237507	10	
699268	29237	143355870	5724721	143006663	5739285	0	86
728505	29250	137631149	5283984	137267378	5298548	50	
757755	29262	132347165	4892217	131968830	4906783	40	
787017	29276	127454948	4542425	127062047	4556992	30	
816293	29290	122912523	4228818	122505055	4243388	20	
845583	29304	118683705	3946573	118261667	3961144	10	
874887	29319	114737132	3691640	114300523	3706213	0	85
904206	29334	111045492	3460608	110594310	3475184	50	
933540	29350	107584884	3250579	107119126	3265155	40	
962890	29367	104334305	3059081	103853971	3073660	30	
992257	29384	101275224	2883997	100780311	2898579	20	
1021641	29401	98391227	2723505	97881732	2738087	10	
1051042	29420	95667722	2576023	95143645	2590610	0	84
1080462	29437	93091699	2440187	92553035	2454774	50	
1109899	29457	90651512	2314797	90098261	2329387	40	
1139356	29476	88336715	2198814	87768874	2213406	30	
1168832	29497	86137901	2091315	85555468	2105910	20	
1198329	29517	84046586	1991496	83449558	2006094	10	
Basis.	Differen:	Hypotenusa	Differen:	Perpendicu:	Differen:		

CANON DOCTRINÆ TRIANGVLORVM IN QVO TRIQVETRI

		Subtendens angulum rectum				Maius latus includen-	
		Perpendicu:	Differen:	Basis	Differen:	Hypotenusa	Differen:
7	0	1218693	28867	9925462	3588	10075098	3643
	10	1247560	28856	9921874	3670	10078741	3730
	20	1276416	28846	9918204	3755	10082471	3819
	30	1305262	28834	9914449	3839	10086290	3906
	40	1334096	28823	9910610	3923	10090196	3996
	50	1362919	28812	9906687	4006	10094192	4084
8	0	1391731	28800	9902681	4091	10098276	4173
	10	1420531	28788	9898590	4174	10102449	4261
	20	1449319	28775	9894416	4257	10106710	4351
	30	1478094	28763	9890159	4342	10111061	4441
	40	1506857	28750	9885817	4425	10115502	4530
	50	1535607	28738	9881392	4509	10120032	4619
9	0	1564345	28724	9876883	4592	10124651	4710
	10	1593069	28710	9872291	4676	10129361	4800
	20	1621779	28697	9867615	4759	10134161	4890
	30	1650476	28683	9862856	4843	10139051	4981
	40	1679159	28669	9858013	4926	10144032	5071
	50	1707828	28654	9853087	5009	10149103	5163
10	0	1736482	28639	9848078	5093	10154266	5254
	10	1765121	28625	9842985	5177	10159520	5345
	20	1793746	28609	9837808	5259	10164865	5438
	30	1822355	28594	9832549	5343	10170303	5529
	40	1850949	28579	9827206	5425	10175832	5621
	50	1879528	28562	9821781	5509	10181453	5714
11	0	1908090	28546	9816272	5592	10187167	5806
	10	1936636	28530	9810680	5675	10192973	5900
	20	1965166	28513	9805005	5758	10198873	5993
	30	1993679	28497	9799247	5841	10204866	6086
	40	2022176	28479	9793406	5923	10210952	6180
	50	2050655	28462	9787483	6007	10217132	6274
12	0	2079117	28444	9781476	6089	10223406	6368
	10	2107561	28427	9775387	6172	10229774	6463
	20	2135988	28408	9769215	6255	10236237	6558
	30	2164396	28390	9762960	6337	10242795	6653
	40	2192786	28372	9756623	6420	10249448	6749
	50	2221158	28353	9750203	6502	10256197	6844
13	0	2249511	28333	9743701	6585	10263041	6941
	10	2277844	28315	9737116	6667	10269982	7036
	20	2306159	28295	9730449	6750	10277018	7134
	30	2334454	28275	9723699	6832	10284152	7231
	40	2362729	28255	9716867	6914	10291383	7328
	50	2390984	28235	9709953	6996	10298711	7425
		Basis:	Differen:	Perpendiculū	Differen:	Hypotenusa	Differen:

CVM ANGVLO RECTO IN PLANITIE PARTIVM 10000000 PONITVR

tiū angulū rect:		Minus latus includentium angulum rectum.					
Perpend:	Differen:	Hypotenusa	Differen:	Basis.	Differen:		
1227846	29538	82055090	1898640	81443464	1913240	0	83
1257384	29559	80156450	1812115	79530224	1826718	50	
1286943	29582	78344335	1731359	77703506	1745965	40	
1316525	29604	76612976	1655870	75957541	1670477	30	
1346129	29628	74957106	1585197	74287064	1599809	20	
1375757	29651	73371909	1518944	72687255	1533558	10	
1405408	29676	71852965	1456745	71153697	1471362	0	82
1435084	29700	70396220	1398278	69682335	1412898	50	
1464784	29726	68997942	1343251	68269437	1357875	40	
1494510	29752	67654691	1291398	66911562	1306024	30	
1524262	29778	66363293	1242481	65605538	1257110	20	
1554040	29804	65120812	1196280	64348428	1210913	10	
1583844	29833	63924532	1152599	63137515	1167236	0	81
1613677	29860	62771933	1111259	61970279	1125898	50	
1643537	29889	61660674	1072094	60844381	1086737	40	
1673426	29918	60588580	1034955	59757644	1049602	30	
1703344	29948	59553625	999704	58708042	1014354	20	
1733292	29978	58553921	966216	57693688	980870	10	
1763270	30009	57587705	934374	56712818	949032	0	80
1793279	30040	56653331	904073	55763786	918734	50	
1823319	30071	55749258	875215	54845052	889880	40	
1853390	30105	54874043	847710	53955172	862379	30	
1883495	30137	54026333	821473	53092793	836146	20	
1913632	30171	53204860	796429	52256647	811107	10	
1943803	30205	52408431	772507	51445540	787188	0	79
1974008	30240	51635924	749640	50658352	764325	50	
2004248	30275	50886284	727767	49894027	742457	40	
2034523	30311	50158517	706830	49151570	721525	30	
2064834	30347	49451687	686780	48430045	701477	20	
2095181	30385	48764907	667564	47728568	682267	10	
2125566	30422	48097343	649137	47046301	663844	0	78
2155988	30460	47448206	631458	46382457	646170	50	
2186448	30499	46816748	614485	45736287	629202	40	
2216947	30538	46202263	598183	45107085	612904	30	
2247485	30578	45604080	582515	44494181	597241	20	
2278063	30619	45021565	567450	43896940	582181	10	
2308682	30660	44454115	552957	43314759	567693	0	77
2339342	30702	43901158	539008	42747066	553748	50	
2370044	30744	43362150	525574	42193318	540320	40	
2400788	30787	42836576	512633	41652998	527384	30	
2431575	30830	42323943	500158	41125614	514914	20	
2462405	30875	41823785	488130	40610700	502891	10	
Basis.	Differen:	Hypotenusa	Differen:	Perpendicu:	Differen:		

CANON DOCTRINÆ TRIANGVLORVM IN QVO TRIQVETRI

		Subtendens angulum rectum				Maius latus includen-	
		Perpendicu:	Differen:	Basis	Differen:	Hypotenusa	Differen:
14	0	2419219	28214	9702957	7078	10306136	7524
	10	2447433	28194	9695879	7160	10313660	7622
	20	2475627	28173	9688719	7243	10321282	7721
	30	2503800	28152	9681476	7324	10329003	7820
	40	2531952	28130	9674152	7406	10336823	7920
	50	2560082	28108	9666746	7488	10344743	8019
15	0	2588190	28087	9659258	7569	10352762	8119
	10	2616277	28065	9651689	7652	10360881	8220
	20	2644342	28042	9644037	7732	10369101	8321
	30	2672384	28019	9636305	7815	10377422	8422
	40	2700403	27997	9628490	7896	10385844	8524
	50	2728400	27974	9620594	7977	10394368	8626
16	0	2756374	27950	9612617	8059	10402994	8729
	10	2784324	27927	9604558	8140	10411723	8831
	20	2812251	27902	9596418	8221	10420554	8935
	30	2840153	27879	9588197	8302	10429489	9039
	40	2868032	27855	9579895	8383	10438528	9142
	50	2895887	27830	9571512	8464	10447670	9248
17	0	2923717	27805	9563048	8546	10456918	9352
	10	2951522	27781	9554502	8626	10466270	9458
	20	2979303	27755	9545876	8706	10475728	9563
	30	3007058	27730	9537170	8788	10485291	9670
	40	3034788	27704	9528382	8868	10494961	9777
	50	3062492	27678	9519514	8949	10504738	9884
18	0	3090170	27652	9510565	9029	10514622	9992
	10	3117822	27626	9501536	9110	10524614	10100
	20	3145448	27599	9492426	9189	10534714	10209
	30	3173047	27572	9483237	9271	10544923	10318
	40	3200619	27545	9473966	9350	10555241	10428
	50	3228164	27518	9464616	9430	10565669	10538
19	0	3255682	27490	9455186	9511	10576207	10648
	10	3283172	27462	9445675	9590	10586855	10760
	20	3310634	27435	9436085	9670	10597615	10872
	30	3338069	27406	9426415	9750	10608487	10984
	40	3365475	27377	9416665	9830	10619471	11097
	50	3392852	27349	9406835	9909	10630568	11210
20	0	3420201	27320	9396926	9988	10641778	11324
	10	3447521	27291	9386938	10069	10653102	11438
	20	3474812	27262	9376869	10147	10664540	11554
	30	3502074	27232	9366722	10227	10676094	11669
	40	3529306	27202	9356495	10306	10687763	11785
	50	3556508	27171	9346189	10385	10699548	11902
		Basis:	Differen:	Perpendiculū	Differen:	Hypotenusa	Differen:

CVM ANGVLO RECTO IN PLANITIE PARTIVM 10000000 PONITVR

tiū angulū rect:		Minus latus includentium angulum rectum.					
Perpend:	Differen:	Hypotenusa	Differen:	Basis.	Differen:		
2493280	30920	41335655	476525	40107809	491291	0	76
2524200	30965	40859130	465326	39616518	480098	50	
2555165	31011	40393804	454512	39136420	469289	40	
2586176	31058	39939292	444068	38667131	458850	30	
2617234	31105	39495224	433974	38208281	448762	20	
2648339	31153	39061250	424217	37759519	439011	10	
2679492	31202	38637033	414782	37320508	429581	0	75
2710694	31251	38222251	405655	36890927	420460	50	
2741945	31300	37816596	396821	36470467	411632	40	
2773245	31352	37419775	388269	36058835	403086	30	
2804597	31402	37031506	379988	35655749	394811	20	
2835999	31455	36651518	371965	35260938	386794	10	
2867454	31507	36279553	364190	34874144	379024	0	74
2898961	31560	35915363	356653	34495120	371494	50	
2930521	31614	35558710	349345	34123626	364192	40	
2962135	31668	35209365	342255	33759434	357108	30	
2993803	31724	34867110	335375	33402326	350235	20	
3025527	31780	34531735	328699	33052091	343565	10	
3057307	31836	34203036	322216	32708526	337088	0	73
3089143	31893	33880820	315920	32371438	330800	50	
3121036	31952	33564900	309805	32040638	324690	40	
3152988	32010	33255095	303861	31715948	318754	30	
3184998	32069	32951234	298085	31397194	312984	20	
3217067	32130	32653149	292469	31084210	307375	10	
3249197	32190	32360680	287007	30776835	301920	0	72
3281387	32252	32073673	281695	30474915	296614	50	
3313639	32314	31791978	276525	30178301	291451	40	
3345953	32377	31515453	271494	29886850	286428	30	
3378330	32441	31243959	266596	29600422	281537	20	
3410771	32505	30977363	261828	29318885	276776	10	
3443276	32570	30715535	257183	29042109	272139	0	71
3475846	32637	30458352	252659	28769970	267621	50	
3508483	32703	30205693	248250	28502349	263220	40	
3541186	32770	29957443	243953	28239129	258931	30	
3573956	32839	29713490	239765	27980198	254750	20	
3606795	32907	29473725	235681	27725448	250674	10	
3639702	32978	29238044	231698	27474774	246698	0	70
3672680	33048	29006346	227814	27228076	242822	50	
3705728	33119	28778532	224022	26985254	239039	40	
3738847	33191	28554510	220325	26746215	235348	30	
3772038	33264	28334185	216714	26510867	231746	20	
3805302	33338	28117471	213190	26279121	228230	10	
Basis.	Differen:	Hypotenusa	Differen:	Perpendicu:	Differen:		

CANON DOCTRINÆ TRIANGVLORVM IN QVO TRIQVETRI

		Subtendens angulum rectum				Maius latus includen-	
		Perpendicu:	Differen:	Basis	Differen:	Hypotenusa	Differen:
21	0	3583679	27142	9335804	10464	10711450	12019
	10	3610821	27111	9325340	10543	10723469	12138
	20	3637932	27080	9314797	10621	10735607	12255
	30	3665012	27049	9304176	10701	10747862	12375
	40	3692061	27018	9293475	10779	10760237	12495
	50	3719079	26987	9282696	10857	10772732	12615
22	0	3746066	26955	9271839	10937	10785347	12737
	10	3773021	26923	9260902	11014	10798084	12858
	20	3799944	26890	9249888	11093	10810942	12980
	30	3826834	26859	9238795	11171	10823922	13103
	40	3853693	26825	9227624	11249	10837025	13227
	50	3880518	26793	9216375	11326	10850252	13352
23	0	3907311	26760	9205049	11405	10863604	13476
	10	3934071	26727	9193644	11483	10877080	13602
	20	3960798	26693	9182161	11560	10890682	13729
	30	3987491	26659	9170601	11638	10904411	13856
	40	4014150	26625	9158963	11716	10918267	13984
	50	4040775	26591	9147247	11792	10932251	14112
24	0	4067366	26557	9135455	11871	10946363	14241
	10	4093923	26522	9123584	11947	10960604	14372
	20	4120445	26487	9111637	12024	10974976	14503
	30	4146932	26453	9099613	12102	10989479	14634
	40	4173385	26416	9087511	12178	11004113	14766
	50	4199801	26382	9075333	12255	11018879	14900
25	0	4226183	26345	9063078	12332	11033779	15034
	10	4252528	26310	9050746	12408	11048813	15168
	20	4278838	26273	9038338	12485	11063981	15304
	30	4305111	26237	9025853	12561	11079285	15441
	40	4331348	26200	9013292	12638	11094726	15578
	50	4357548	26163	9000654	12714	11110304	15715
26	0	4383711	26127	8987940	12789	11126019	15855
	10	4409838	26089	8975151	12866	11141874	15995
	20	4435927	26051	8962285	12941	11157869	16135
	30	4461978	26014	8949344	13018	11174004	16277
	40	4487992	25975	8936326	13092	11190281	16419
	50	4513967	25938	8923234	13169	11206700	16562
27	0	4539905	25899	8910065	13243	11223262	16707
	10	4565804	25861	8896822	13319	11239969	16852
	20	4591665	25821	8883503	13395	11256821	16998
	30	4617486	25783	8870108	13469	11273819	17146
	40	4643269	25743	8856639	13544	11290965	17293
	50	4669012	25704	8843095	13619	11308258	17443
		Basis:	Differen:	Perpendiculū	Differen:	Hypotenusa	Differen:

CVM ANGVLO RECTO IN PLANITIE PARTIVM 10000000 PONITVR

tiū angulū rect:		Minus latus includentium angulum rectum.					
Perpend:	Differen:	Hypotenusa	Differen:	Basis.	Differen:		
3838640	33413	27904281	209749	26050891	224797	0	69
3872053	33488	27694532	206388	25826094	221445	50	
3905541	33564	27488144	203106	25604649	218170	40	
3939105	33641	27285038	199899	25386479	214972	30	
3972746	33719	27085139	196765	25171507	211846	20	
4006465	33797	26888374	193702	24959661	208792	10	
4040262	33877	26694672	190710	24750869	205808	0	68
4074139	33958	26503962	187782	24545061	202889	50	
4108097	34039	26316180	184921	24342172	200036	40	
4142136	34121	26131259	182122	24142136	197247	30	
4176257	34203	25949137	179384	23944889	194517	20	
4210460	34288	25769753	176706	23750372	191848	10	
4244748	34373	25593047	174086	23558524	189237	0	67
4279121	34458	25418961	171521	23369287	186681	50	
4313579	34545	25247440	169012	23182606	184181	40	
4348124	34632	25078428	166554	22998425	181732	30	
4382756	34721	24911874	164148	22816693	179336	20	
4417477	34810	24747726	161793	22637357	176989	10	
4452287	34900	24585933	159485	22460368	174692	0	66
4487187	34992	24426448	157226	22285676	172442	50	
4522179	35084	24269222	155012	22113234	170237	40	
4557263	35176	24114210	152843	21942997	168077	30	
4592439	35271	23961367	150717	21774920	165962	20	
4627710	35367	23810650	148634	21608958	163889	10	
4663077	35462	23662016	146592	21445069	161856	0	65
4698539	35559	23515424	144591	21283213	159865	50	
4734098	35657	23370833	142628	21123348	157912	40	
4769755	35757	23228205	140704	20965436	155998	30	
4805512	35856	23087501	138816	20809438	154120	20	
4841368	35958	22948685	136965	20655318	152280	10	
4877326	36060	22811720	135149	20503038	150473	0	64
4913386	36163	22676571	133367	20352565	148703	50	
4949549	36267	22543204	131619	20203862	146965	40	
4985816	36373	22411585	129904	20056897	145260	30	
5022189	36479	22281681	128221	19911637	143587	20	
5058668	36586	22153460	126567	19768050	141945	10	
5095254	36696	22026893	124946	19626105	140333	0	63
5131950	36805	21901947	123352	19485772	138752	50	
5168755	36916	21778595	121789	19347020	137199	40	
5205671	37027	21656806	120253	19209821	135674	30	
5242698	37141	21536553	118745	19074147	134176	20	
5279839	37255	21417808	117263	18939971	132706	10	
Basis.	Differen:	Hypotenusa	Differen:	Perpendicu:	Differen:		

CANON DOCTRINÆ TRIANGVLORVM IN QVO TRIQVETRI

		Subtendens angulum rectum				Maius latus includen-	
		Perpendicu:	Differen:	Basis	Differen:	Hypotenusa	Differen:
28	0	4694716	25664	8829476	13694	11325701	17592
	10	4720380	25624	8815782	13768	11343293	17743
	20	4746004	25584	8802014	13843	11361036	17896
	30	4771588	25543	8788171	13917	11378932	18048
	40	4797131	25503	8774254	13991	11396980	18203
	50	4822634	25462	8760263	14066	11415183	18358
29	0	4848096	25421	8746197	14139	11433541	18514
	10	4873517	25380	8732058	14214	11452055	18671
	20	4898897	25339	8717844	14287	11470726	18829
	30	4924236	25296	8703557	14361	11489555	18989
	40	4949532	25255	8689196	14434	11508544	19150
	50	4974787	25213	8674762	14508	11527694	19311
30	0	5000000	25170	8660254	14581	11547005	19475
	10	5025170	25128	8645673	14654	11566480	19638
	20	5050298	25086	8631019	14727	11586118	19803
	30	5075384	25042	8616292	14801	11605921	19970
	40	5100426	24999	8601491	14872	11625891	20137
	50	5125425	24956	8586619	14946	11646028	20306
31	0	5150381	24912	8571673	15018	11666334	20476
	10	5175293	24868	8556655	15091	11686810	20647
	20	5200161	24825	8541564	15162	11707457	20820
	30	5224986	24780	8526402	15235	11728277	20993
	40	5249766	24736	8511167	15307	11749270	21169
	50	5274502	24691	8495860	15379	11770439	21345
32	0	5299193	24646	8480481	15451	11791784	21523
	10	5323839	24601	8465030	15522	11813307	21701
	20	5348440	24556	8449508	15594	11835008	21882
	30	5372996	24511	8433914	15665	11856890	22064
	40	5397507	24464	8418249	15736	11878954	22247
	50	5421971	24419	8402513	15807	11901201	22432
33	0	5446390	24373	8386706	15879	11923633	22618
	10	5470763	24327	8370827	15949	11946251	22805
	20	5495090	24280	8354878	16020	11969056	22993
	30	5519370	24233	8338858	16090	11992049	23185
	40	5543603	24187	8322768	16161	12015234	23376
	50	5567790	24139	8306607	16231	12038610	23569
34	0	5591929	24092	8290376	16302	12062179	23765
	10	5616021	24045	8274074	16371	12085944	23961
	20	5640066	23996	8257703	16441	12109905	24159
	30	5664062	23949	8241262	16511	12134064	24359
	40	5688011	23901	8224751	16581	12158423	24560
	50	5711912	23852	8208170	16650	12182983	24763
		Basis:	Differen:	Perpendiculū	Differen:	Hypotenusa	Differen:

CVM ANGVLO RECTO IN PLANITIE PARTIVM 10000000 PONITVR

tiū angulū rect:		Minus latus includentium angulum rectum.					
Perpend:	Differen:	Hypotenusa	Differen:	Basis.	Differen:		
5317094	37371	21300545	115808	18807265	131262	0	62
5354465	37487	21184737	114378	18676003	129844	50	
5391952	37605	21070359	112974	18546159	128450	40	
5429557	37724	20957385	111593	18417709	127081	30	
5467281	37844	20845792	110236	18290628	125736	20	
5505125	37966	20735556	108903	18164892	124414	10	
5543091	38088	20626653	107592	18040478	123116	0	61
5581179	38212	20519061	106304	17917362	121838	50	
5619391	38337	20412757	105037	17795524	120584	40	
5657728	38463	20307720	103791	17674940	119350	30	
5696191	38592	20203929	102567	17555590	118137	20	
5734783	38720	20101362	101362	17437453	116945	10	
5773503	38850	20000000	100178	17320508	115772	0	60
5812353	38982	19899822	99012	17204736	114620	50	
5851335	39115	19800810	97866	17090116	113485	40	
5890450	39249	19702944	96738	16976631	112370	30	
5929699	39385	19606206	95629	16864261	111273	20	
5969084	39522	19510577	94537	16752988	110193	10	
6008606	39660	19416040	93462	16642795	109132	0	59
6048266	39801	19322578	92405	16533663	108087	50	
6088067	39941	19230173	91364	16425576	107059	40	
6128008	40084	19138809	90340	16318517	106048	30	
6168092	40228	19048469	89331	16212469	105052	20	
6208320	40374	18959138	88339	16107417	104072	10	
6248694	40520	18870799	87361	16003345	103107	0	58
6289214	40669	18783438	86398	15900238	102159	50	
6329883	40820	18697040	85450	15798079	101223	40	
6370703	40970	18611590	84517	15696856	100304	30	
6411673	41124	18527073	83597	15596552	99397	20	
6452797	41279	18443476	82691	15497155	98505	10	
6494076	41435	18360785	81800	15398650	97627	0	57
6535511	41592	18278985	80920	15301023	96762	50	
6577103	41753	18198065	80055	15204261	95909	40	
6618856	41913	18118010	79201	15108352	95070	30	
6660769	42076	18038809	78360	15013282	94243	20	
6702845	42240	17960449	77533	14919039	93429	10	
6745085	42407	17882916	76715	14825610	92627	0	56
6787492	42574	17806201	75911	14732983	91836	50	
6830066	42744	17730290	75117	14641147	91057	40	
6872810	42915	17655173	74336	14550090	90289	30	
6915725	43088	17580837	73564	14459801	89533	20	
6958813	43262	17507273	72805	14370268	88788	10	
Basis.	Differen:	Hypotenusa	Differen:	Perpendicu:	Differen:		

CANON DOCTRINÆ TRIANGVLORVM IN QVO TRIQVETRI

		Subtendens angulum rectum				Maius latus includen-	
		Perpendicu:	Differen:	Basis	Differen:	Hypotenusa	Differen:
35	0	5735764	23804	8191520	16719	12207746	24967
	10	5759568	23755	8174801	16788	12232713	25174
	20	5783323	23707	8158013	16858	12257887	25382
	30	5807030	23657	8141155	16926	12283269	25592
	40	5830687	23607	8124229	16995	12308861	25803
	50	5854294	23559	8107234	17064	12334664	26016
36	0	5877853	23508	8090170	17132	12360680	26231
	10	5901361	23458	8073038	17201	12386911	26448
	20	5924819	23409	8055837	17268	12413359	26667
	30	5948228	23358	8038569	17337	12440026	26887
	40	5971586	23307	8021232	17405	12466913	27110
	50	5994893	23257	8003827	17472	12494023	27334
37	0	6018150	23206	7986355	17540	12521357	27560
	10	6041356	23155	7968815	17607	12548917	27788
	20	6064511	23103	7951208	17675	12576705	28019
	30	6087614	23052	7933533	17741	12604724	28251
	40	6110666	23000	7915792	17809	12632975	28485
	50	6133666	22949	7897983	17875	12661460	28722
38	0	6156615	22896	7880108	17943	12690182	28960
	10	6179511	22844	7862165	18008	12719142	29201
	20	6202355	22791	7844157	18075	12748343	29444
	30	6225146	22739	7826082	18142	12777787	29688
	40	6247885	22686	7807940	18207	12807475	29936
	50	6270571	22633	7789733	18273	12837411	30185
39	0	6293204	22580	7771460	18339	12867596	30436
	10	6315784	22526	7753121	18405	12898032	30691
	20	6338310	22472	7734716	18470	12928723	30947
	30	6360782	22419	7716246	18536	12959670	31206
	40	6383201	22365	7697710	18600	12990876	31467
	50	6405566	22310	7679110	18666	13022343	31730
40	0	6427876	22256	7660444	18730	13054073	31996
	10	6450132	22202	7641714	18795	13086069	32265
	20	6472334	22146	7622919	18859	13118334	32536
	30	6494480	22092	7604060	18924	13150870	32810
	40	6516572	22037	7585136	18988	13183680	33085
	50	6538609	21981	7566148	19052	13216765	33365
41	0	6560590	21926	7547096	19116	13250130	33646
	10	6582516	21870	7527980	19180	13283776	33931
	20	6604386	21814	7508800	19243	13317707	34217
	30	6626200	21759	7489557	19306	13351924	34508
	40	6647959	21702	7470251	19370	13386432	34800
	50	6669661	21645	7450881	19433	13421232	35095
		Basis:	Differen:	Perpendiculū	Differen:	Hypotenusa	Differen:

CVM ANGVLO RECTO IN PLANITIE PARTIVM 10000000 PONITVR

tiū angulū rect:		Minus latus includentium angulum rectum.					
Perpend:	Differen:	Hypotenusa	Differen:	Basis.	Differen:		
7002075	43440	17434468	72055	14281480	88053	0	55
7045515	43618	17362413	71317	14193427	87329	50	
7089133	43798	17291096	70588	14106098	86615	40	
7132931	43980	17220508	69869	14019483	85912	30	
7176911	44164	17150639	69161	13933571	85218	20	
7221075	44350	17081478	68462	13848353	84534	10	
7265425	44538	17013016	67772	13763819	83860	0	54
7309963	44728	16945244	67093	13679959	83195	50	
7354691	44920	16878151	66421	13596764	82540	40	
7399611	45113	16811730	65760	13514224	81893	30	
7444724	45309	16745970	65106	13432331	81256	20	
7490033	45508	16680864	64463	13351075	80627	10	
7535541	45707	16616401	63826	13270448	80007	0	53
7581248	45909	16552575	63199	13190441	79395	50	
7627157	46113	16489376	62580	13111046	78792	40	
7673270	46319	16426796	61968	13032254	78197	30	
7719589	46529	16364828	61366	12954057	77610	20	
7766118	46738	16303462	60770	12876447	77031	10	
7812856	46952	16242692	60182	12799416	76459	0	52
7859808	47167	16182510	59602	12722957	75895	50	
7906975	47384	16122908	59029	12647062	75339	40	
7954359	47604	16063879	58463	12571723	74790	30	
8001963	47827	16005416	57905	12496933	74248	20	
8049790	48050	15947511	57354	12422685	73713	10	
8097840	48278	15890157	56809	12348972	73186	0	51
8146118	48507	15833348	56271	12275786	72665	50	
8194625	48739	15777077	55740	12203121	72151	40	
8243364	48973	15721337	55216	12130970	71643	30	
8292337	49210	15666121	54697	12059327	71143	20	
8341547	49449	15611424	54186	11988184	70648	10	
8390996	49692	15557238	53680	11917536	70160	0	50
8440688	49936	15503558	53180	11847376	69678	50	
8490624	50183	15450378	52688	11777698	69202	40	
8540807	50433	15397690	52199	11708496	68733	30	
8591240	50686	15345491	51718	11639763	68268	20	
8641926	50941	15293773	51242	11571495	67811	10	
8692867	51200	15242531	50772	11503684	67358	0	49
8744067	51461	15191759	50307	11436326	66912	50	
8795528	51725	15141452	49847	11369414	66470	40	
8847253	51991	15091605	49394	11302944	66035	30	
8899244	52262	15042211	48944	11236909	65604	20	
8951506	52534	14993267	48502	11171305	65180	10	
Basis.	Differen:	Hypotenusa	Differen:	Perpendicu:	Differen:		

CANON DOCTRINÆ TRIANGVLORVM IN QVO TRIQVETRI

		Subtendens angulum rectum				Maius latus includen-	
		Perpendicu:	Differen:	Basis	Differen:	Hypotenusa	Differen:
42	0	6691306	21589	7431448	19495	13456327	35394
	10	6712895	21532	7411953	19559	13491721	35696
	20	6734427	21475	7392394	19621	13527417	36000
	30	6755902	21418	7372773	19683	13563417	36308
	40	6777320	21361	7353090	19745	13599725	36618
	50	6798681	21303	7333345	19808	13636343	36932
43	0	6819984	21245	7313537	19869	13673275	37248
	10	6841229	21187	7293668	19932	13710523	37569
	20	6862416	21130	7273736	19992	13748092	37893
	30	6883546	21071	7253744	20054	13785985	38219
	40	6904617	21013	7233690	20116	13824204	38549
	50	6925630	20954	7213574	20176	13862753	38883
44	0	6946584	20895	7193398	20237	13901636	39220
	10	6967479	20836	7173161	20298	13940856	39560
	20	6988315	20778	7152863	20359	13980416	39905
	30	7009093	20718	7132504	20418	14020321	40252
	40	7029811	20658	7112086	20479	14060573	40604
	50	7050469	20599	7091607	20539	14101177	40959
45	0	7071068		7071068		14142136	
		Basis:	Differen:	Perpendiculū	Differen:	Hypotenusa	Differen:

CVM ANGVLO RECTO IN PLANITIE PARTIVM 10000000 PONITVR

tiū angulū rect:		Minus latus includentium angulum rectum.					
Perpend:	Differen:	Hypotenusa	Differen:	Basis.	Differen:		
9004040	52811	14944765	48062	11106125	64760	0	48
9056851	53089	14896703	47630	11041365	64345	50	
9109940	53372	14849073	47201	10977020	63935	40	
9163312	53657	14801872	46777	10913085	63531	30	
9216969	53945	14755095	46359	10849554	63131	20	
9270914	54237	14708736	45944	10786423	62736	10	
9325151	54532	14662792	45535	10723687	62346	0	47
9379683	54830	14617257	45130	10661341	61960	50	
9434513	55133	14572127	44730	10599381	61580	40	
9489646	55437	14527397	44334	10537801	61203	30	
9545083	55746	14483063	43943	10476598	60831	20	
9600829	56059	14439120	43555	10415767	60464	10	
9656888	56374	14395565	43172	10355303	60100	0	46
9713262	56694	14352393	42793	10295203	59742	50	
9769956	57017	14309600	42418	10235461	59387	40	
9826973	57343	14267182	42048	10176074	59036	30	
9884316	57675	14225134	41680	10117038	58690	20	
9941991	58009	14183454	41318	10058348	58348	10	
10000000		14142136		10000000		0	45
Basis.	Differen:	Hypotenusa	Differen:	Perpendicu:	Differen:		