Slonimsky's Multiplying Device,
an impressive Example for Applied Mathematics

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Summary

This article presents multiplying devices from the middle of the 19th century, which are based on the so called Theorem of Slonimsky. With his theorem Slonimsky succeeded in solving the problem of tens carry in a simple device with no gears. He did it in a surprising way, but at the same time the multiplying device is an interesting and impressive example of application and usage of a rather complicated statement in numerical mathematics.

The inventor and the base problem

Hayyim Selig Slonimsky[1] (*1810 in Byelostok or Belostok, Russian Empire, now Bialystok, Poland , †1904 in Warsaw) was a knowledgeable Talmudist and author of science books about mathematics and astronomy in Hebrew for Jewish people. As a publisher he produced the science magazine Hazefirah (Ha-Tsefira) from 1862 on, also in Hebrew, which continued until 1931. Furthermore Slonimsky invented at least two calculating aids, an adding device and a multiplying device. The article in the Leipziger Illustrierte Zeitung from 1845 [2] mentions a third calculating instrument, similar to an abacus. It is worth noting that the clockmaker and inventor of a calculating machine Abraham Jacob Stern was his father-in-law.

For a better understanding of what is new in Slonimsky's invention, I consider it useful to repeat briefly some historic methods for tens carry in multiplying devices.
In his *Rabdologia* John Napier (1550 – 1617) broke a simple multiplying table into vertical columns. With these stripes, mounted on square rods which were called *Napier's rods* or *bones*, multiplication tables for any multiplicand may be composed (fig. 1, to ease comparison here I continuously use the multiplicand 274). Figures of the partial products are arranged in triangles so that the user may obtain the required product only digit by digit by diagonally addition from right to left [8, 15].

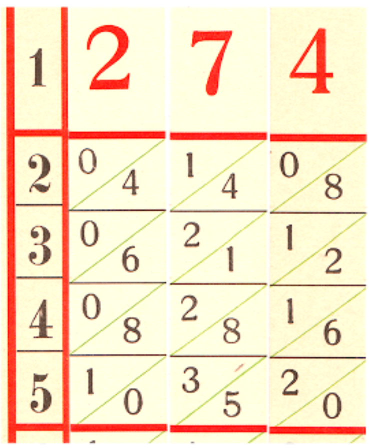


Fig. 1: *Napier's rods*, here used in *Theutometer* (Germany), about 1910, shortened

1 Chaim Zelig Slonimsky in Polish, also written Slonimski or Slonimskii.

In 1885 Henri Genaille and Edouard Lucas published their *Réglettes multiplicatrices* in France. The partial products are arranged on stripes or rods similar to Napier's. A partial product is replaced by a black triangle or arrow with a column of figures on the right side. The right side of an arrow covers the unit figures of a partial product added to a possible carry from right. The left corner of the arrow is placed in height corresponding with the tens figure of the partial product (fig. 2). Instead of making additions the user only has to follow these arrows and to read the figures which are indicated [12, 14].

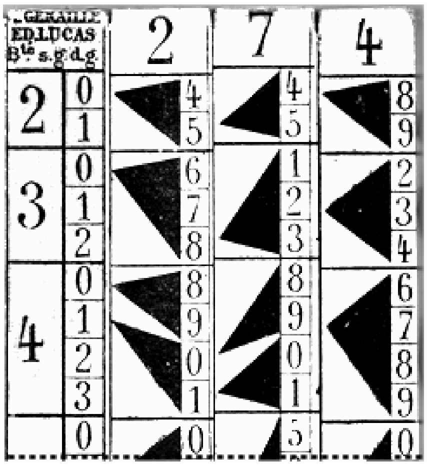


Fig. 2: *Réglettes multiplicatrices* by Genailles and Lucas, 1885, shortened

Those systems mirror our way to multiply: "3 times 4 makes 12, write 2, keep in mind 1. 3 times 7 makes 21 plus 1 makes 22..." and so on. Other inventors tried to arrange a system of masks which display only valid figures of multiples – and failed due to the carry already 3 places later at the thousands [16].

Slonimsky's statements and conclusions

To understand Slonimsky's statements easier we write any number followed downwards by its multiples with 2, 3,... to 9. Such a multiplication table for 274 is shown in fig. 3.

(0)	**2**	**7**	**4**	
0	5	4	8	*2
0	8	2	2	*3
1	0	9	6	*4
1	3	7	0	*5
1	6	4	4	*6
1	9	1	8	*7
2	1	9	2	*8
2	4	6	6	*9

Fig. 3: Simple multiplication table for 274

The column with 4 on top contains from top downwards the unit figures of the partial products 4*2=8, 4*3=12 to 4*9=36. The tens figures of the products are carried to the left. The next column to the left with 7 on top contains the unit figures of the partial products with 7 increased by the carries coming from the right. The same applies to the column with 2 on top. Finally the last column on the left holds carries only. In every column the figures form a sequence. The table in fig. 4 shows the same multiplication table as before with the figures separated to partial products plus carries. The units of altered partial products are printed in bold, the carries are underlined.

(0)	2	7	4	
00+0=0**0**	04+1=0**5**	14+0=1**4**	08+0=0**8**	*2
00+0=0**0**	06+2=0**8**	21+1=2**2**	12+0=1**2**	*3
00+1=0**1**	08+2=1**0**	28+1=2**9**	16+0=1**6**	*4
00+1=0**1**	10+3=1**3**	35+2=3**7**	20+0=2**0**	*5
00+1=0**1**	12+4=1**6**	42+2=4**4**	24+0=2**4**	*6
00+1=0**1**	14+5=1**9**	49+2=5**1**	28+0=2**8**	*7
00+2=0**2**	16+5=2**1**	56+3=5**9**	32+0=3**2**	*8
00+2=0**2**	18+6=2**4**	63+3=6**6**	36+0=3**6**	*9

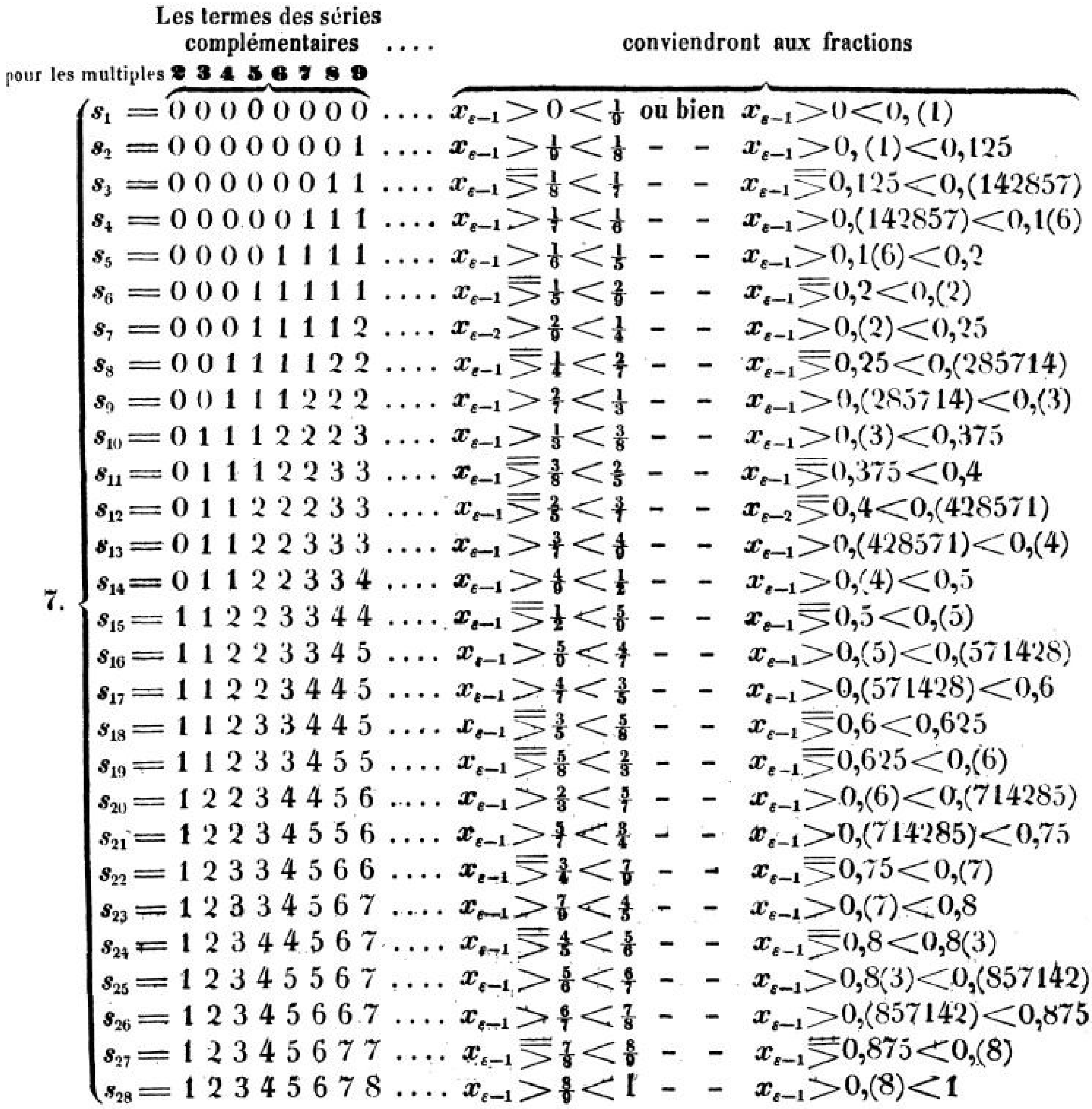
Fig. 4: Expanded multiplication table for 274

Now Slonimsky distinguishes two sequences. In every column the carries themselves form a sequence of figures. For example in the column with 7 on top we find the sequence of carries (0,1,1,2,2,2,3,3). The column on the left with 0 as thousands and with the partial products 00, 00,... holds the sequence of carries (0,0,1,1,1,1,2,2). To the column on the right side the carry sequence (0,0,0,0,0,0,0,0) is associated, because no carry really occurs. Crelle, of whom we will hear later, names the sequences of carries *séries complémentaires* in French which I translate with completion sequences or carry sequences. The figures in the columns of a multiplication table of type shown above are determined by

(1) the units of the partial products, which are known because the figure on top of that column is known and
(2) the carry sequence coming from the right, which is unknown unless we calculate it.

In the next step Slonimsky asks a leading question: how many different carry sequences may occur, regardless of the multi-digit number on top of the table, which may be as large as one likes? Or more generalized, how many unique columns can occur in such a multiplication table? I encourage the reader to stop reading here and to guess, the answer surprises.

It seems that Slonimsky never published an explanation of his answer – his theorem – and that is why August Leopold Crelle (1780 – 1855) in 1846 gave a proof [5]. Crelle was founder and editor of the specialist publication *Journal für die reine und angewandte Mathematik*, shortly named *Crelles Journal*. He deduces what Slonimsky already had found, namely 28 different carry sequences can occur, not more. They are horizontally arranged displayed in the left half of fig. 5, which is copied from his original article. Crelle marks the carry sequences with '**s**', completed by an index 1 to 28. The right half of fig. 5 he uses for his explanations, they are repeated abridged in the appendix.



7.

Les termes des séries complémentaires .... pour les multiples 2 3 4 5 6 7 8 9		conviendront aux fractions	
s_1 =	0 0 0 0 0 0 0 0	$x_{\varepsilon-1} > 0 < \frac{1}{9}$	ou bien $x_{\varepsilon-1} > 0 < 0,(1)$
s_2 =	0 0 0 0 0 0 0 1	$x_{\varepsilon-1} > \frac{1}{9} < \frac{1}{8}$	- - $x_{\varepsilon-1} > 0,(1) < 0,125$
s_3 =	0 0 0 0 0 0 1 1	$x_{\varepsilon-1} \geqq \frac{1}{8} < \frac{1}{7}$	- - $x_{\varepsilon-1} \geqq 0,125 < 0,(142857)$
s_4 =	0 0 0 0 0 1 1 1	$x_{\varepsilon-1} > \frac{1}{7} < \frac{1}{6}$	- - $x_{\varepsilon-1} > 0,(142857) < 0,1(6)$
s_5 =	0 0 0 0 1 1 1 1	$x_{\varepsilon-1} > \frac{1}{6} < \frac{1}{5}$	- - $x_{\varepsilon-1} > 0,1(6) < 0,2$
s_6 =	0 0 0 1 1 1 1 1	$x_{\varepsilon-1} \geqq \frac{1}{5} < \frac{2}{9}$	- - $x_{\varepsilon-1} \geqq 0,2 < 0,(2)$
s_7 =	0 0 0 1 1 1 1 2	$x_{\varepsilon-2} > \frac{2}{9} < \frac{1}{4}$	- - $x_{\varepsilon-1} > 0,(2) < 0,25$
s_8 =	0 0 1 1 1 1 2 2	$x_{\varepsilon-1} \geqq \frac{1}{4} < \frac{2}{7}$	- - $x_{\varepsilon-1} \geqq 0,25 < 0,(285714)$
s_9 =	0 0 1 1 1 2 2 2	$x_{\varepsilon-1} > \frac{2}{7} < \frac{1}{3}$	- - $x_{\varepsilon-1} > 0,(285714) < 0,(3)$
s_{10} =	0 1 1 1 2 2 2 3	$x_{\varepsilon-1} > \frac{1}{3} < \frac{3}{8}$	- - $x_{\varepsilon-1} > 0,(3) < 0,375$
s_{11} =	0 1 1 1 2 2 3 3	$x_{\varepsilon-1} \geqq \frac{3}{8} < \frac{2}{5}$	- - $x_{\varepsilon-1} \geqq 0,375 < 0,4$
s_{12} =	0 1 1 2 2 2 3 3	$x_{\varepsilon-1} \geqq \frac{2}{5} < \frac{3}{7}$	- - $x_{\varepsilon-2} \geqq 0,4 < 0,(428571)$
s_{13} =	0 1 1 2 2 3 3 3	$x_{\varepsilon-1} > \frac{3}{7} < \frac{4}{9}$	- - $x_{\varepsilon-1} > 0,(428571) < 0,(4)$
s_{14} =	0 1 1 2 2 3 3 4	$x_{\varepsilon-1} > \frac{4}{9} < \frac{1}{2}$	- - $x_{\varepsilon-1} > 0,(4) < 0,5$
s_{15} =	1 1 2 2 3 3 4 4	$x_{\varepsilon-1} \geqq \frac{1}{2} < \frac{5}{9}$	- - $x_{\varepsilon-1} \geqq 0,5 < 0,(5)$
s_{16} =	1 1 2 2 3 3 4 5	$x_{\varepsilon-1} > \frac{5}{9} < \frac{4}{7}$	- - $x_{\varepsilon-1} > 0,(5) < 0,(571428)$
s_{17} =	1 1 2 2 3 4 4 5	$x_{\varepsilon-1} > \frac{4}{7} < \frac{3}{5}$	- - $x_{\varepsilon-1} > 0,(571428) < 0,6$
s_{18} =	1 1 2 3 3 4 4 5	$x_{\varepsilon-1} \geqq \frac{3}{5} < \frac{5}{8}$	- - $x_{\varepsilon-1} \geqq 0,6 < 0,625$
s_{19} =	1 1 2 3 3 4 5 5	$x_{\varepsilon-1} \geqq \frac{5}{8} < \frac{2}{3}$	- - $x_{\varepsilon-1} \geqq 0,625 < 0,(6)$
s_{20} =	1 2 2 3 4 4 5 6	$x_{\varepsilon-1} > \frac{2}{3} < \frac{5}{7}$	- - $x_{\varepsilon-1} > 0,(6) < 0,(714285)$
s_{21} =	1 2 2 3 4 5 5 6	$x_{\varepsilon-1} > \frac{5}{7} < \frac{3}{4}$	- - $x_{\varepsilon-1} > 0,(714285) < 0,75$
s_{22} =	1 2 3 3 4 5 6 6	$x_{\varepsilon-1} \geqq \frac{3}{4} < \frac{7}{9}$	- - $x_{\varepsilon-1} \geqq 0,75 < 0,(7)$
s_{23} =	1 2 3 3 4 5 6 7	$x_{\varepsilon-1} > \frac{7}{9} < \frac{4}{5}$	- - $x_{\varepsilon-1} > 0,(7) < 0,8$
s_{24} =	1 2 3 4 4 5 6 7	$x_{\varepsilon-1} \geqq \frac{4}{5} < \frac{5}{6}$	- - $x_{\varepsilon-1} \geqq 0,8 < 0,8(3)$
s_{25} =	1 2 3 4 5 5 6 7	$x_{\varepsilon-1} > \frac{5}{6} < \frac{6}{7}$	- - $x_{\varepsilon-1} > 0,8(3) < 0,(857142)$
s_{26} =	1 2 3 4 5 6 6 7	$x_{\varepsilon-1} > \frac{6}{7} < \frac{7}{8}$	- - $x_{\varepsilon-1} > 0,(857142) < 0,875$
s_{27} =	1 2 3 4 5 6 7 7	$x_{\varepsilon-1} \geqq \frac{7}{8} < \frac{8}{9}$	- - $x_{\varepsilon-1} \geqq 0,875 < 0,(8)$
s_{28} =	1 2 3 4 5 6 7 8	$x_{\varepsilon-1} > \frac{8}{9} < 1$	- - $x_{\varepsilon-1} > 0,(8) < 1$

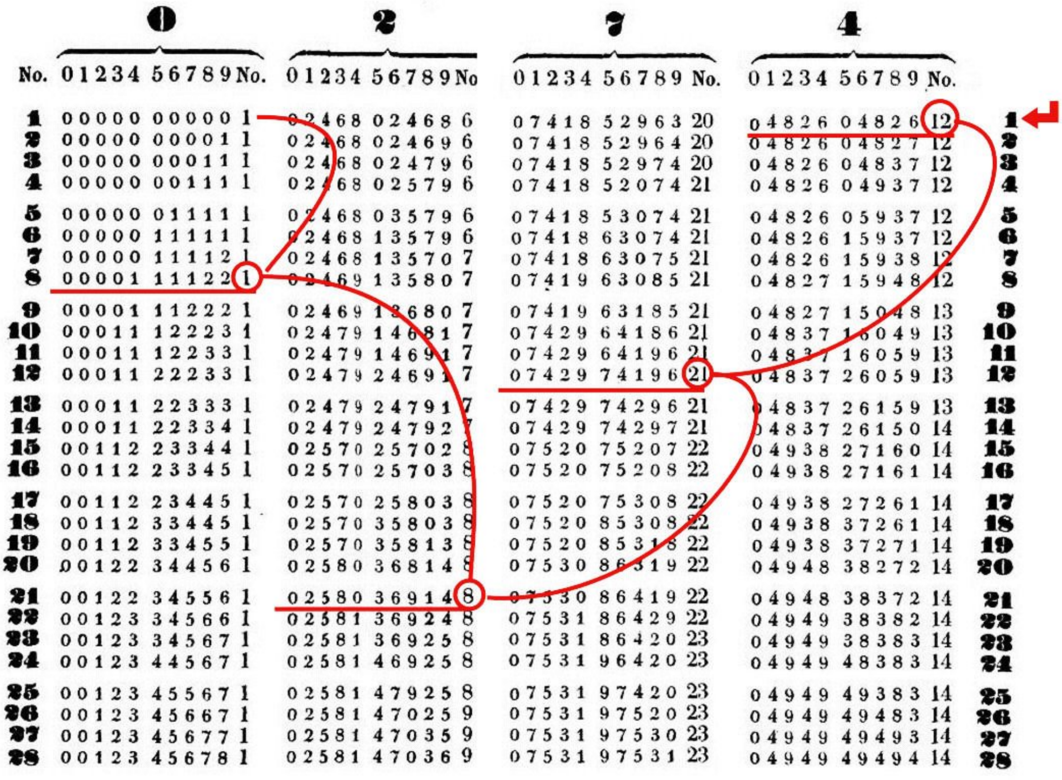
Fig. 5: Part of Crelle's proof, on the left side all 28 carry sequences, horizontally arranged

There are 10 different sequences that contain units of the partial products, one for each figure 0 to 9. To these sequences 28 carry sequences can be added, so that in a multiplication table only 10*28=280 unique columns of figures may occur, not more, regardless of the value for the number on top of the table. This relatively small amount of all columns in a simple multiplication table leads to the idea to calculate them all and make them accessible in an array. From such an array the user could copy his desired table or only the multiple of a given number. Usually such an aid is called a multiplication table too, the term array I use here to avoid confusion with the small table shown before.

The multiplying array

Each of the 280 unique columns in a simple multiplying table is clearly determined by two values: (1) by the figure on top of it and (2) by the index of the carry sequence which is transferred to it from the right. These two values must serve as entries to the multiplying array. Crelle added such a multiplying aid several pages after his article [5]. Although the array includes only 2 pages it is too big to be reproduced here, parts of it are used in fig. 6 for an calculating example later.
For my next explanations I use Crelle's original identifiers. '**Z**' denotes the multi-digit number whose multiples are to be found and '**s**' with an index 1 to 28 denotes one of the carry sequences. The two entries into the array are (1) the figures of the multi-digit number **Z.** They are given on top of the table above a curly bracket and (2) one of the indexes for the carry sequence **s**, given in bold on the left and right borders.

Below a curly bracket we find horizontally arranged from line to line the unit sequences of the partial products for the figure on top to which are added a carry sequence with its index on the sides. In contrast to fig. 3 and 5 the multiplier don't start with 2 but with 0. On the right end of a line below heading 'No.' we find a new number. This one indicates the index of the carry sequence caused by this line and that way this number explicitly links to the next result. In fig. 6 the procedure to gain the result for our example with number 274 is marked in red.



	0		2		7		4		
No.	01234 56789	No.	01234 56789	No	01234 56789	No.	01234 56789	No.	
1	00000 00000	1	02468 02468	6	07418 52963	20	04826 04826	12	1
2	00000 00001	1	02468 02469	6	07418 52964	20	04826 04827	12	2
3	00000 00011	1	02468 02479	6	07418 52974	20	04826 04837	12	3
4	00000 00111	1	02468 02579	6	07418 52074	21	04826 04937	12	4
5	00000 01111	1	02468 03579	6	07418 53074	21	04826 05937	12	5
6	00000 11111	1	02468 13579	6	07418 63074	21	04826 15937	12	6
7	00000 11112	1	02468 13570	7	07418 63075	21	04826 15938	12	7
8	00001 11122	1	02469 13580	7	07419 63085	21	04827 15948	12	8
9	00001 11222	1	02469 13680	7	07419 63185	21	04827 15048	13	9
10	00011 12223	1	02479 14681	7	07429 64186	21	04837 16049	13	10
11	00011 12233	1	02479 14691	7	07429 64196	21	04837 16059	13	11
12	00011 22233	1	02479 24691	7	07429 74196	21	04837 26059	13	12
13	00011 22333	1	02479 24791	7	07429 74296	21	04837 26159	13	13
14	00011 22334	1	02479 24792	7	07429 74297	21	04837 26150	14	14
15	00112 23344	1	02570 25702	8	07520 75207	22	04938 27160	14	15
16	00112 23345	1	02570 25703	8	07520 75208	22	04938 27161	14	16
17	00112 23445	1	02570 25803	8	07520 75308	22	04938 27261	14	17
18	00112 33445	1	02570 35803	8	07520 85308	22	04938 37261	14	18
19	00112 33455	1	02570 35813	8	07520 85318	22	04938 37271	14	19
20	00122 34456	1	02580 36814	8	07530 86319	22	04948 38272	14	20
21	00122 34556	1	02580 36914	8	07530 86419	22	04948 38372	14	21
22	00123 34566	1	02581 36924	8	07531 86429	22	04949 38382	14	22
23	00123 34567	1	02581 36925	8	07531 86420	23	04949 38383	14	23
24	00123 44567	1	02581 46925	8	07531 96420	23	04949 48383	14	24
25	00123 45567	1	02581 47925	8	07531 97420	23	04949 49383	14	25
26	00123 45667	1	02581 47025	9	07531 97520	23	04949 49483	14	26
27	00123 45677	1	02581 47035	9	07531 97530	23	04949 49493	14	27
28	00123 45678	1	02581 47036	9	07531 97531	23	04949 49494	14	28

Fig. 6: Example for reading the multiples of 274

The reading starts with the column for unit figure 4. We take the first line, indicated with an arrow. We have to take the carry sequence s_1, because here no carry comes from the right. The sequence found is (0,4,8,2,6,0,4,8,2,6). On the right end of this sequence stands No. 12, in fig. 6 marked with a circle. It represents the index of that carry sequence s_{12} which is transferred to the next sequence of partial products on the left. We go into the column for the tens figure 7 and there to the line marked with bold 12 on the borders. We read (0,7,4,2,9,7,4,1,9,6) and 21 as the next index. From here we are led to the sequence (0,2,5,8,0,3,6,9,1,4) belonging to the hundreds figure 2. From here and with index 8 we are led to the last sequence (0,0,0,0,1,1,1,1,2,2) which is a sole carry sequence belonging to zero thousands in (0)274. Its index 1 indicates, that no carry follows, the sequence is (0,0,0,0,0,0,0,0,0,0). If we still continue reading we only get leading zeroes in the result. Next we take the read sequences, write them down vertically and get the same small multiplying table for 274 shown before – without any addition. The only additional effort lies in the selection of a second entry to the array. Having understood the procedures the readings from the array run faster than any description. The array has advantages: it covers only 2 pages and can be used for numbers of any value. In contrast the largest table of that kind, Crelle's *Erleichterungs-Tafel* [6] from 1836, which gives products 1(1)10,000,000 * 1(1)9, contains 1,000 pages with size 28 x 23 cm. Although having published his large *Erleichterungs-Tafel* 10 years before, Crelle didn't hesitate to make the small array public. As additional advantages the array gives optionally 1 to eight 8 multiples simultaneously whereas with Napier's and Genaille's rods or derivates we only get one multiple at the end of a reading process.

In 1847 so called *Multiplication Tablets Derived from a Theorem of S. Slonimski* [9] were published in Great Britain – except for Crelle's array shown here the only table of this kind known to me. I inspected the copy of this rare book from British Library. The columns are arranged quite similar to those by Crelle.

The multiplying rods of type Slonimsky

Arrangement and content of a multiplying table are immobile and useful only for a single special calculation. Breaking the table into moveable parts, a simple way of mechanization, can ease handling and reading. The first and obvious solution is the use of rods.

About 1990 Peter Roubos from Emmen (Netherlands) constructed a set of such rods named *Staafjes van Slonimsky* (*Multiplying Rods Slonimsky,* fig. 7). They are made of square wooden rods with paper pasted on all sides. For 280 different sequences of figures 70 rods are needed.

Fig. 7: *Multiplying Rods Slonimsky* by Peter Roubos

More important for the selection of the rods during use are not the figures 0 to 9 in the number **Z** but the indexes of the carry sequences **s**. That is why they are set on top of each rod. Downwards the sequences from Crelle's array follow. On bottom the index of the next completion sequence is added. Fig. 8 shows the arrangement of the rods for a multiplying table of 274.

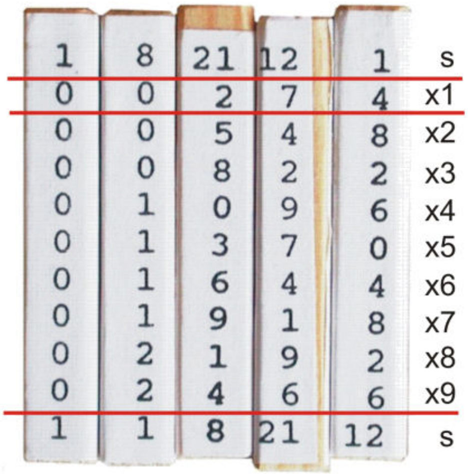


Fig. 8: Multiplying Rods Slonimsky, arranged for multiples of 274

While using these rods I found out that they are much more complicated to handle than a multiplication table because the search for the proper rods needs much time. From where Peter Roubos took his template I couldn't discover. In the literature I found no note that Slonimsky built or even suggested such a system under his name, whereas Apokin [3] reports about quite similar *Joffe's Counting Bars*. The inventor Joffe (also written Jofe or Yofe) presented his invention in 1881. In 1882 his calculating bars received an honorable mention at the All-Russian exhibition. The basic principle is based on Slonimsky's theorem. The whole set consisted of 70 rectangular bars with totally 280 columns on all sides. Furthermore Apokin reports:
"Each bar and each of its sides were marked. Both Roman and Arabic numbers, and Latin letters were used. The Latin letters and Roman numbers were used to arrange the bars correctly, while each multiplication of the multiplicand by one rank of the factor naturally require a new combination... Thus the number of products was equal to the number of ranks in the factor. Similarly to Slonimsky's method, the obtained products were added together on paper."

It is obvious that the bars were marked with the indices of the carry sequences, not with numbers like Roubos did, but with other markers. Within the second half of this quote Apokin speaks of multiplication of two multi-digit factors which can be performed of course with one and the same arrangement of the rods. Apokin cites Bohl [4, p. 197], who in 1896 wrote the first systematized description of calculation appliances in Russia and came to the conclusion "The Joffe bars simplify multiplication even more than Napier sticks or their later modifications." It is reported that the *Joffe Bars* were quite common in Russia [10]. Despite their advantages they remained unknown in the Western World.

The multiplying device

Connatural with the principle of rods is the use of turnable cylinders. Slonimsky designs such a multiplying device and presents it on August 8, 1844, together with his adding device to the Prussian Academy of Sciences in Berlin [1]. In the same year he publishes his description of the device [13]. Therein he elaborates on the history of calculating machines and on the advantages of his device, the description itself is less meaningful. In the middle of the 19th century very few calculating machines were available and moreover they wereregarded to be unreliable. Their mass production started years later. This situation explains why simpler devices stood in focus of attention. Slonimsky points out that his device is reliable because of its design and that it gives all multiples of a number the same time without adding. He argues Napier's rods and derivates cannot do that and multiplication tables for the same task are too big.

One year later, in 1845 Slonimsky presents his device to the Royal Academy in St Petersburg where the instrument and its mathematical principle were examined by the mathematician Bunjakowski and the scientific secretary Fuss (Voss). Both give their report[2] on April 10, 1845. Slonimsky is granted the Demidov Prize with 2000 (other sources 2500) rubles. The report contains a detailed description of the instrument, which I give here due to its authenticity unabridged in my free translation.
"The whole instrument is made of a flat wooden box, similar to a chessboard, 16 *Zoll* long, 13 *Zoll* wide and 2 *Zoll* high.[3] On the upper plate are 11 rows of openings. During use in each opening not more than one figure and one letter are displayed. The letters only appear in the second and third row, counted from bottom upwards. They are the key to the intrument and they show the manipulations during the preparation of a calculation. All other rows are solely dedicated for the numbers. Inside the box and parallel to its length 8 cylinders or small rollers are mounted, on their surface rows of figures and letters are written due to a certain rule. All these cylinders, except of the small one on the right side, are manufactured in a special way. Except of the turning movement the surface of each cylinder also has a linear movement forwards and backwards in direction of the axis according to the 10 rows ahead. The second movement is transferred to the cylinders with help of seven screws, which are mounted on the surface of the plate between the second and the third row of openings, counted from bottom upwards. The letters with their different exponents, displayed in these two rows, determine to which distance the cylinders must be moved..."

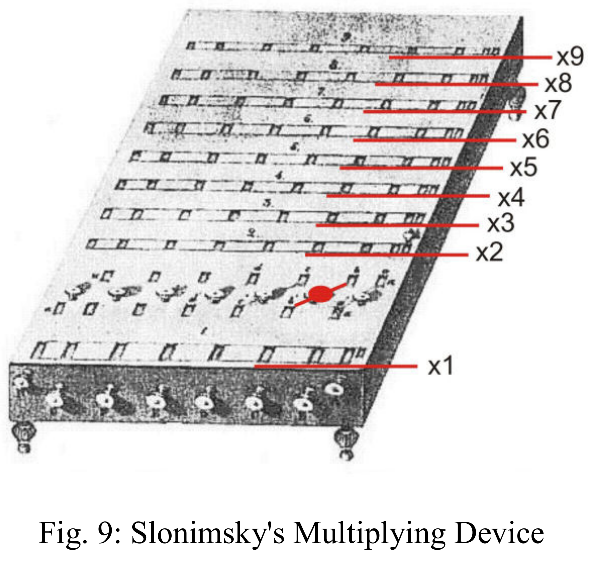


Fig. 9: Slonimsky's Multiplying Device

Until today only one picture of the multiplying device is known which already appeared in Leipziger Illustrierte Zeitung and which is copied again in several scientific publications. In fig.

2 The whole minutes of the St. Petersburg Academy of Sciences from April 10, 1845, regarding Slonimsky's instrument, are published in German language in Leipziger Illustrierte Zeitung [2].

3 Russian *Zoll* (*Djuim*), about 40 x 32 x 5 centimeter.

9 this only picture is completed with markers that indicate the rows of multiples and one of the small wheels that moves its cylinder up and down. The advantage of cylinders instead of rods lies in the availability of number series in a moment. To set the figures of a number, which is the first entry, the wheels on front of the instrument are turned. For the second entry the wheels on the plate must be used. They move the appropriate cylinder up and down. The inscription of the cylinders can be regarded to be an array transformation of Crelle's multiplication array.

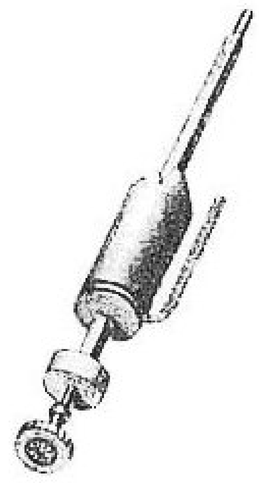


Fig. 10: A single cylinder from the multiplying device

Fig. 10 shows one of the moveable cylinders how it may have looked. In front the turning wheel is fixed. Parallel to the axis a gear rack is mounted to move the cylinder up and down. The figure only shows a principal construction because the proportions do not match with the positions of the openings.
To each wheel on the upper plate two diagonally arranged openings are assigned. As the St. Petersburg report [2] reads both openings display a combination of 7 figures 1...7 and 4 letters a, b, c, d which gives 28 keys 1a, 1b, 1c, 1d, 2a...,7d and 28 is the amount of carry sequences. While turning the cylinder to set a figure such a key has to be copied from the (already set) right cylinder to the next moving cylinder left to set the correct carry sequence simultaneously. These movements correspond to the selection of a line in the flat array. This assumption is confirmed by a second short description in the Berlin report [1] that reads "that two marked with letters) horizontal rows exist, whose displays have to be copied identically from roller to roller". At the same moment when all figures are set, the multiples of that number can be read from the openings. Mental additions are replaced by copying a key and, to repeat it again, there are no gears for the tens carry at all. For more than 7 places in the number to be multiplied, two or more instruments can be put together, as long as some rules are obeyed.
Slonimsky tells us that with his instrument one can perform multiplications, divisions and square roots [13]. He is right in a sense that the instrument gives us multiples of a number when they are needed during the calculations. The Berlin report mentions a facility to ease divisions, unfortunately it is not explained.
Russian sources report that he was granted a patent for the period of ten years for this machine on November 24, 1845 [11]. He also applied for patents in the United States and Great Britain, but was apparently unsuccessful.

Slonimsky had only little profitable success with his multiplying device. Despite its good properties the device was never manufactured industrially. The inventor only built a single specimen for presenting purposes [10]. On the other hand, for us today it is a part in the long chain of aids for multiplying and, as the St. Petersburg report states[4] "The device in question is so simple that it could hardly be named 'a machine'. The theoretical foundations implemented in it are the most important features of the invention..."

Appendix: Slonimsky's theorem explained

In the minutes of the Public Meeting of Royal Academy of Sciences, St. Petersburg, on April 10, 1845, Slonimsky's theorem is explained thus (free translation by me):
"The theorem found by Mr. Slonimsky deals with a highly strange property of simple numbers: assumed an integer number with as many figures as you like is multiplied by 2, 3 up to 9 and the results are written down one below the other without moving the results from place to place, then we get whole vertical columns with nine figures. Shortly we name the sequence of numbers in such a column a form. Due to Slonimsky's theorem the amount *p* of different forms is expressed with the formula $p = 10(q + 1)$ with *q* as the amount of all proper and different fractions with denominators 2, 3 to 9. In that case a simple calculation shows that there will never be more than 280 columns which are different in their form. This relatively small number led Slonimsky to the idea to build a calculating device and the success complied his hypothesis thoroughly."

$$
1. \quad \left\{
\begin{array}{llllll}
\frac{1}{9} & \frac{2}{9} & \frac{4}{9} & \frac{5}{9} & \frac{7}{9} & \frac{8}{9} \\
\frac{1}{8} & \frac{3}{8} & \frac{5}{8} & \frac{7}{8} & & \\
\frac{1}{7} & \frac{2}{7} & \frac{3}{7} & \frac{4}{7} & \frac{5}{7} & \frac{6}{7} \\
\frac{1}{6} & \frac{5}{6} & & & & \\
\frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} & & \\
\frac{1}{4} & \frac{3}{4} & & & & \\
\frac{1}{3} & \frac{2}{3} & & & & \\
\frac{1}{2} & & & & &
\end{array}
\right.
$$

Fig. 11: The twenty seven fractions used by Crelle

The mathematician August Leopold Crelle profed Slonimsky's theorem one year later [5]. For his explanations he uses the 27 fractions[5] shown in fig. 11 or, ordered by value 1/9, 1/8, 1/7, 1/6, 1/5, 2/9, 1/4, 2/7, 1/3, 3/8, 2/5, 3/7, 4/9, 1/2, 5/9, 4/7, 3/5, 5/8, 2/3, 5/7, 3/4, 7/9, 4/5, 5/6, 6/7, 7/8, 8/9.

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4 Cited from Apokin [3].

5 Also called Farey sequence *F(n=9)* without the members 0/1 and 1/1. "The Farey sequence of order *n* is the sequence of completely reduced fractions between 0 and 1 which, when in lowest terms, have denominators less than or equal to *n*, arranged in order of increasing size" (from Wikipedia.org).

Shortened Crelle's proof runs as follows: each carry sequence is determined by all figures that are placed right of it within the number **Z**. So if one writes down any decimal fraction *f* with $0 < f < 1$, followed from top to bottom by its products $2*f$ to $9*f$, then the integer parts of all multiples on the left side form a carry sequence that belongs to the figure sequence in the first line. In fig. 12 the complete table is listed for the decimal fraction $f = 0.274$, which gives (0,0,0,1,1,1,1,2,2) as carry sequence. This one equals $\mathbf{s_8}$ in fig. 5.

0.	**2**	**7**	**4**	
0.	5	4	8	*2
0.	8	2	2	*3
1.	0	9	6	*4
1.	3	7	0	*5
1.	6	4	4	*6
1.	9	1	8	*7
2.	1	9	2	*8
2.	4	6	6	*9

Fig. 12: Table to isolate a carry sequence

Next Crelle shows, that all decimal fractions $0 < f < 1$ can be replaced by 28 intervals between the 27 proper fractions given before (see fig. 5 right half) and that every interval stands for a carry sequence that doesn't change. The integer parts of the multiples of these fractions give the carry sequences looked for. The replacement of decimal fractions with proper fractions eases to estimate the integer part of the result when multiplying with 2, 3,...

The example for $\mathbf{s_2}$:
for $1/9 \leq x < 1/8$ we get int(2*1/9) = 0 up to int(8*1/9) = 0; 9*1/9 = 1;

The example for $\mathbf{s_{28}}$:
for $8/9 \leq x < 1$ we get int(2*8/9) = 1; int(3*8/9) = 2; int(4*8/9) = 3 and so on.

The multiplier 10 in the formula above equals the amount of figures 0 to 9, because to all partial products of each figure 0 to 9 all 28 completion sequences can be added, which gives not more than 280 different columns in the multiplication table.

Sources

Fig. 1, 2, 7, 8: from the author's collection,
fig. 5, 11: Crelle [5],
fig. 9, 10: Majstrov [10],
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