Stephan Weiss

Reconstruction and Background of Gaspar Schott's *Tabula Sexagenaria* (1661)

Sexagesimals belong to a place-value system with sixty as the base. It had been invented by the Sumerians in the 2000s BC, was then transmitted to the Babylonians, to the Islamic World [10, 11, 14], from there to the Western World and is still in use for measuring time and angles. Before Schott's table is introduced, a short overwiew of sexagesimals in post-medieval times in the Western World may help to understand his work and intentions.
The work with sexagesimal numbers was brought in connection to and named with astronomy. Varel (1533 – 1599) [27] writes in his easy to understand educational book about that subject:

> "What is astronomic calculation? It's a unique and special certain arithmetic, or principle of calculation, used by astronomers and cosmographers for calculating locus of points, times, movements in the sky and similar things." (*Quid est Logistice Astronomica? Est Singvaris & peculiaris quædam Arithmetica , seu ratio computandi, qua Astronomi & Cosmographi in computatione locorum, temporum, motuum coelestium, & similium rerum utuntur.)*

In addition sexagesimals were in use to represent fractions in general.
The base 60 involves to bear in mind sixty different numerals. In the Western World however decimal numbers are used to replace sexagesimal digits. For example 6° 18' 43'' equals 6 + 18/60 + 43/3600 degrees. This notation is not a pure place-value system, but a derived one, in which the figures within the whole number are distinguished by signs used as place-identifiers. With such a notation there is no need to fill an empty place with zero.
For place-identifiers (lat. *denominatio, -onis*) the authors used similar names but different symbols or indices, placed over or near the right side of the figures [22 p. 234].[1] Some of used signs are listed in fig. 1.
Although this system has advantages – 60 can be divided by 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 without remainder – there is a severe disadvantage. The smallest multiplication table holds 3600 products (0..59) × (0..59), in consideration of commutative law[2] at least about 1800 products. Without such a table one has to

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1 This notation for sexagesimal or astronomic numbers should not be mixed up with the same notation used for decimal fractions, called geometric numbers.

2 $a \times b = b \times a$

perform a multiplication followed by at least one division by 60. That is why multiplication tables were essential and in many historical manuscripts and books a sexagesimal multiplication table is added.

Pwr of 60	Identifier	sign 1)	sign 2)	sign 3)
…				
60^3	*sexagenae tertiae* ("third sixties")	3æ	3æ	///æ
60^2	*sexagenae secundae* ("second sixties")	2æ	2æ	//æ
60^1	*sexagenae primae* ("first sixties")	1æ	1æ	/æ
60^0	*gradus, dies, integrum* ("degree, day, whole number")	°	0	°
60^{-1}	*scrupula prima* ("first parts")	' or I	1a	/a
60^{-2}	*scrupula secunda* ("second parts")	" or II	2a	//
60^{-3}	*scrupula tertia* ("third parts")	''' or III	3a	///
…				

Fig. 1: Place-identifiers in sexagesimal numbers
1) as used by **Schott** [21] or Strauch [24] and others
2) as used by Peucer [15] and Varel [27]
3) as used by Theodoricus [26]

Over centuries we have to distinguish two types of multiplication tables in the Western World: the complete table and the abridged triangular sexagesimal table.

The complete Table

A complete table holds all products within the range of base for both factors. Such a table is very large and therefore must be split into several parts and printed on successive pages. The table is named *Tabula Sexagenaria, Tabula Sexagesimorum, Canon Sexagenarum* or *Canon Sexagenarum et Scrupulorum Sexagesimorum,* seldom *Tabula Proportionalis*. Here a selection of authors, who give a complete table:

Fine	1532 [7]	(1×1)..(60×60)	
Schoner, J.	1536 [20]	(1×1)..(60×60)	
Fine	1555 [6]	(1×1)..(60×60)	In earlier editions of Fine's *Arithmetica* he adds a description of the abridged table (see next section) instead of the complete table.
Theodoricus	1564 [26]	(1×1)..(60×60)	
Calvisius	1629 [4]	(1×2)..(59×60)	
Strauch	1662 [24]	(1×1)..(59×60)	

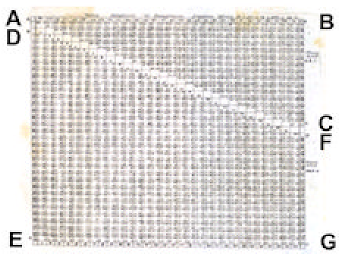
Argoli	1677 [2]	(1×1)..(60×180)	
Jeake	1696 [9]	(1×1)..(60×60)	Printed on a single folded sheet of paper, a rare exception.
Grueneberg	1700 [8]	(1×2)..(59×60)	
Lorenz	1800 [13]	(1×2)..(59×59)	

I skipped sexagesimal tables adapted for special purposes like Bernoulli 1779 [3] or Taylor 1780 [25] for proportions.
Factors and products are given without any place-identifier. The products are always written with two adjacent places.
In relation to sexagesimal multiplication tables Samuel Reyher (1635 – 1714) should be mentioned. He adapted Napier's calculating rods for use with sexagesimal numbers [19 German, 18 Latin, 29].

The abridged Table

Some authors don't give a complete but an abridged sexagesimal tabe that is smaller in size and therefore needs only a folded page to be printed. Furthermore its arrangement allows an easier overview. The used allocation of products is understood better in comparison with a complete table. A full table with its entries 1..60 on a horizontal side and 1..60 on a vertical side contains the four parts a), b), c), d), (see sketch below):

1 … 30	**a**) (1..30) × (1..30)	**b**) (1..30) × (31..60)
31 … 60	**c**) (31..60) × (1..30)	**d**) (31..60) × (31..60)
	1 … 30	31 … 60

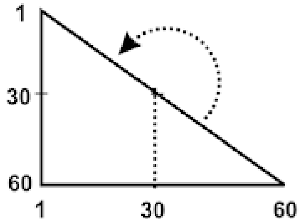


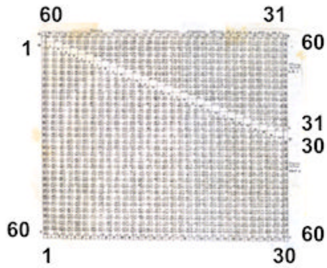
Parts b) and c) are equal due to commutative law and can be replaced by one of them. In the abridged table part d) is reduced to the triangle A B C, parts a) and b) are contracted to trapezoid D E F G.

Lazarus Schoner uses the poetical title

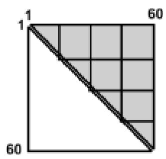
> *Tabula, sive Canon Sexagesimorum, qui multiplicatione, divisione, lateris quadrati investigatione, caelum, terras, maria mensurat.* ("Table or Canon of sixties that measures the sky, the earth, the sea by multiplication, division and extracting square root").

He derives the shape of the table from cutting through a triangular table at entry 30 on the base (see sketch below) and moving the rotated triangular part on top. Varel [27] uses the same explanation. Triangular tables avoid commutative products and have been in use as a multiplication aid for decimal numbers.





Among other authors an abridged table is given by

Schoner, J.	1536 [20]	(1×1)..(60×60)	
Reinhold	1571 [17]	(1×1)..(60×60)	Before section *Logistice Scrupulorum Astronomicorum.*
Schoner, L.	1586 [16]	(1×1)..(59×59)	In section Lazari Schoneri *De Logistica Sexagenaria liber.*
Alsted	1630 [1]	(1×1)..(60×60)	In *Arithmeticae Pars II, cap. Xi & XII*
Schott	1661 [21]	(1×1)..(60×60)	
Lansberg	1663 [12]	(1×1)..(60×60)	Appended to section *tabulae motuum coelestium.*
Wallis	1693 [28]	(1×1)..(60×60) 	In cap. VII *de partibus sexagesimalibus.* A very rare arrangement: the triangular table is cut and reproduced in 10 individual parts.

At the end of his *Tabulae Arithmeticae* Johannes Schoner adds two different abridged tables of equal shape, one configured for sexagesimal numbers (*tabula proportionum ad LX. minuta*) and the other one, very rare, with sexagesimal results of three places for astronomical calculations (*tabula proportionum ad XXIIII horas. pro motu horario &c.planetarum*).
Gaspar (Caspar) Schott (1608 – 1666), a German Jesuit and scientist, specialized in the fields of mathematics and physics, published in his *Cursus Mathematicus (*1661), section *De Arithmetica Astronomica* an abridged

Tabula Sexagenaria vel Sexagesimorum Scrupulorum ("Table of sixties or of sixtieth parts").

The reconstruction of Schott's table and his own associated rules as well as instructions for use are reproduced in the supplement to this article.

Some authors, like Reinhold [17] or Fine in earlier versions of his *Arithmetica* [6], only mention or describe the abridged table without reproducing. Up to now I couldn't find the inventor of this special arrangement.

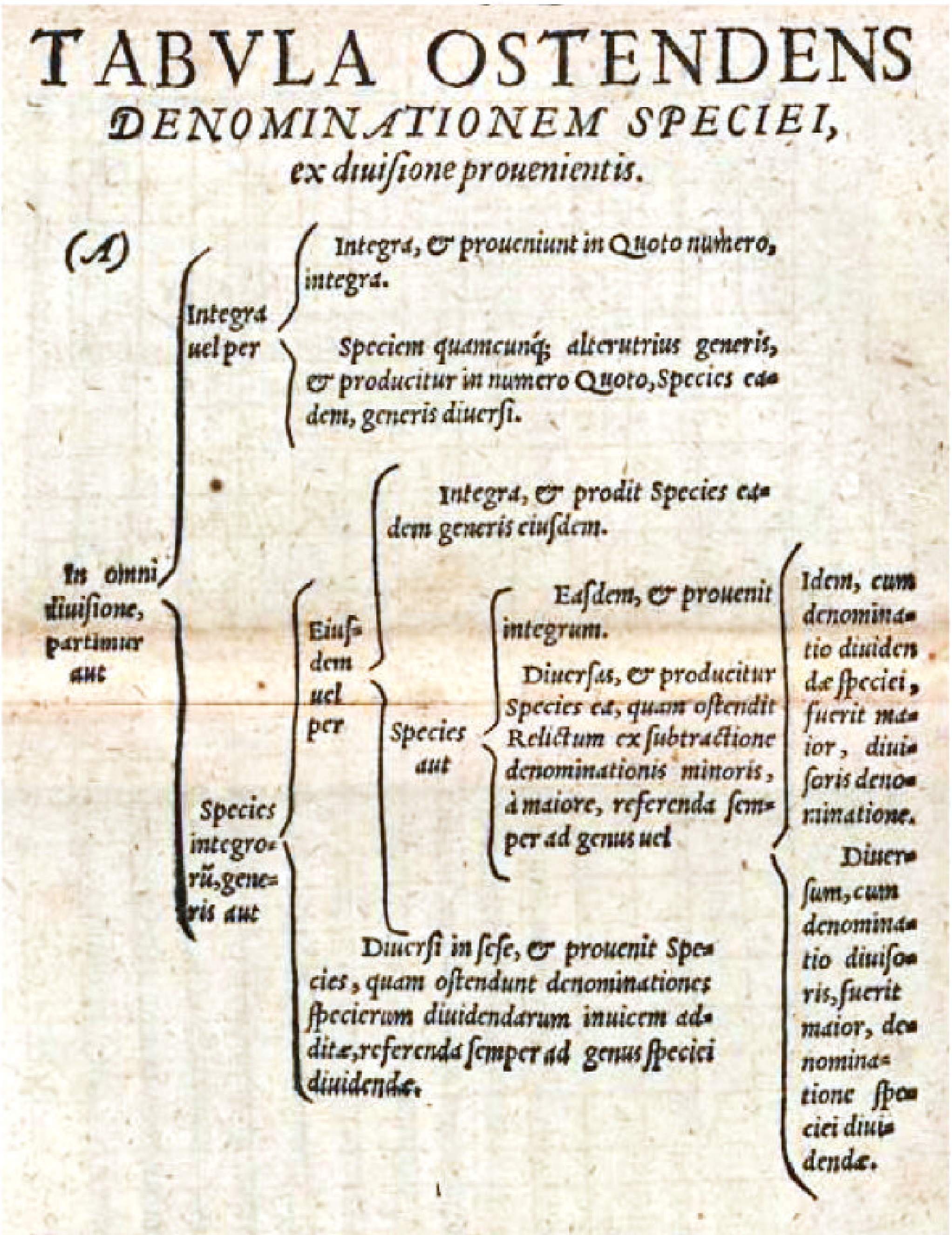
Calculating with Place-Identifiers

Sexagesimal numbers are an arrangement of digits 1..59 connected with their ordering signs that act as place-identifiers. In addition or subtraction the numbers are treated by corresponding places. For multiplication and division instead both parts, digit and sign, must undergo the same arithmetical operation. With other words and in an example 12 *sexagenae secundae* divided by 4 *scrupula tertia* equals 12 divided by 4 plus *sexagenae secundae* divided by *scrupula tertia*.[3]
To solve this problem the authors teach different procedures. For help in defining the new sign or order to be processed, they offer either rather clumsy rules or graphical aids. The rules in Theodoricus' *Canon Sexagenarum* [30] and those Schott teaches are translated into English to give an impression to the reader. In his *Compendium* Theodoricus [26] summarizes his rules in a modern looking decision tree (see fig. 2a and my translation fig. 2b).
Within their rules the authors add or subtract place-identifiers as if they were exponents, but with restrictions: in both ranges, *sexagenae* and *scrupula,* they only have positive values. Therefore the range for a new place-identifier must be specified by comparing the magnitudes of the two identifiers in calculation. Moreover the order *gradus* resp. *integrum* (60^0) doesn't represent zero, instead it is regarded to be of extraordinary state and treated individually. Those procedures make the rules sometimes hard to decode. In contrast Wallis is an exception. He works with negative indices:

> "IV' multiplied with III´´ gives XII´. That means 4 *Sexagena* multiplied with 3 *secunda minuta* give 12 *minuta prima* because of +1–2=–1."
> (*IV' in III´´ facit XII´. Hoc est, 4 Sexagena in 3 secunda minuta, faciunt 12 minuta prima: propter +1–2=–1.* [28, cap. VII, p. 25]).

Wallis' approach is new but doesn't surprise. Not before the end of 17th c. he was one of the first who generalized the usage of exponents, negative and fractional ones included.

3 In modern notation $(12 / 4) \times (60^2 / 60^{-3})$.



TABVLA OSTENDENS
DENOMINATIONEM SPECIEI,
ex diuisione prouenientis.

(A)

In omni diuisione, partimur aut

Integra uel per
- Integra, & proueniunt in Quoto numero, integra.
- Speciem quamcunque alterutrius generis, & producitur in numero Quoto, Species eadem, generis diuersi.

Species integrorũ, generis aut
- Eiusdem uel per
  - Integra, & prodit Species eadem generis eiusdem.
  - Species aut
    - Easdem, & prouenit integrum.
    - Diuersas, & producitur Species ea, quam ostendit Relictum ex subtractione denominationis minoris, à maiore, referenda semper ad genus uel
      - Idem, cum denominatio diuidendæ speciei, fuerit maior, diuisoris denominatione.
      - Diuersum, cum denominatio diuisoris, fuerit maior, denominatione speciei diuidendæ.
- Diuersi in sese, & prouenit Species, quam ostendunt denominationes specierum diuidendarum inuicem additæ, referenda semper ad genus speciei diuidendæ.

Fig. 2a: Theodoricus' summary to define orders in division

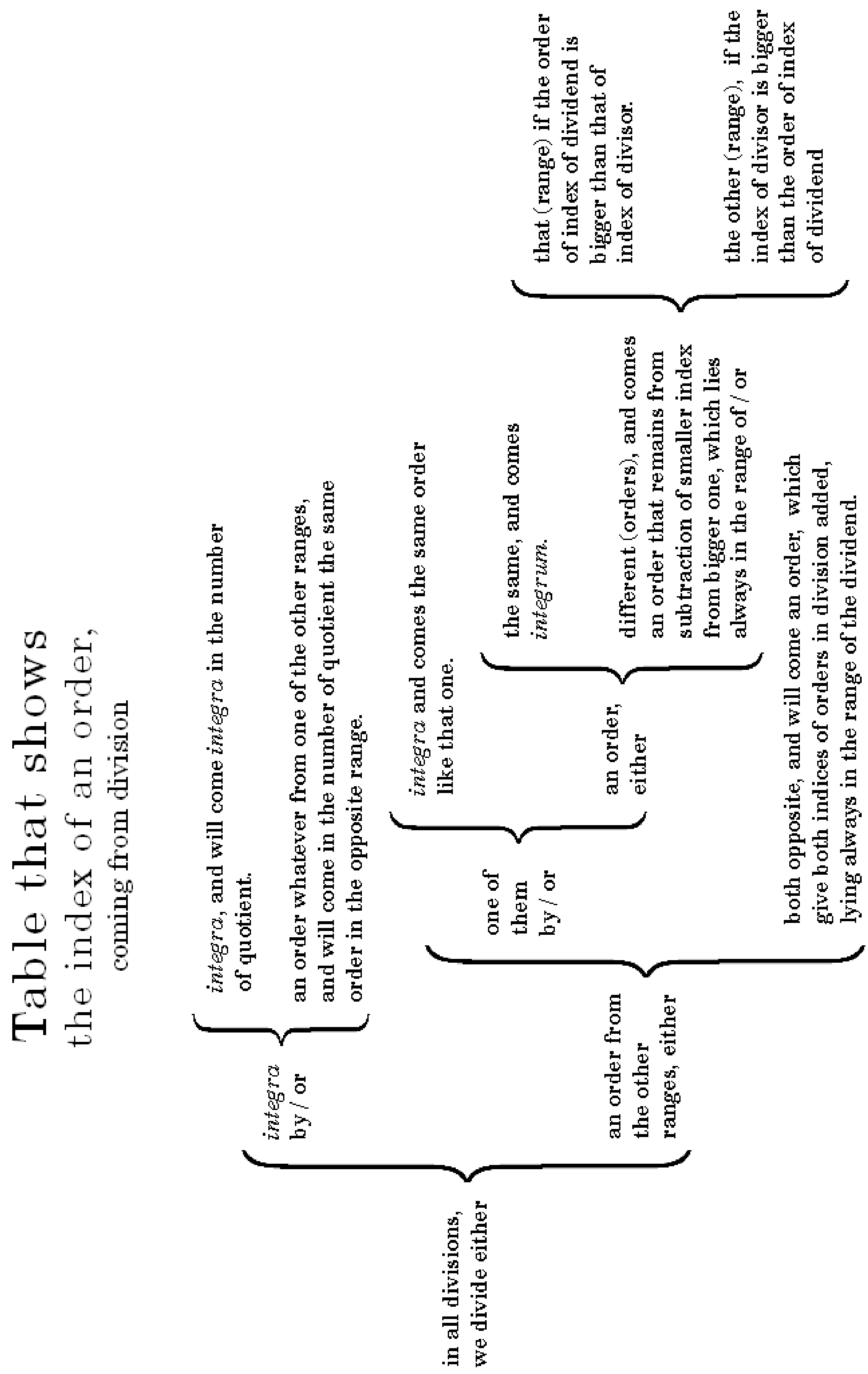


Fig. 2b: Theodoricus' decision tree translated

In a graphical aid called *Tabula denominationum* ("table of denominations", fig. 3) Johannes Schoner combines lists of names for indices to generate sentences that read
"<name> (multiplied) by <name> will give <name> with <name>".

Tabula denominationum.

gra.		gra.		ſigna		gra.
gra.		minu.		gra.		minu.
gra.		ſecun.		minu.		ſecun.
gra.		tert.		ſecun.		tert
gra.		quart.		tert.		quar.
minu.		minu.		minu.		ſecun.
minu.	per	ſecun.	prodibunt	ſecun.	cum	tert.
minu.		tert.		tert.		quart.
minu.		quart.		quart.		quint.
ſecun.		ſecun.		tert.		quart.
ſecun.		tert.		quart.		quint.
ſecun.		quart.		quint.		ſext.
tert.		tert.		quint.		ſext.
quart.		quart.		ſept.		octa.

Fig. 3: A table to define place-identifiers in multiplication, given by Schoner [20]

The result covers two places in case its numerical value exceeds 59.
Other authors add small tables with two entries only for signs (fig. 4).

CANON SEXAGENARUM.

Tabula pro ſpecie emergente ex Multipl.
Multiplicandus.

4æ	3æ	2æ	1æ	o	/	//	///	////	
8æ	7æ	6æ	5æ	4æ	3æ	2æ	1æ	o	4æ
7æ	6æ	5æ	4æ	3æ	2æ	1æ	o	/	3æ
6æ	5æ	4æ	3æ	2æ	1æ	o	/	//	2æ
5æ	4æ	3æ	2æ	1æ	o	/	//	///	1æ
4æ	3æ	2æ	1æ	o	/	//	///	////	o
3æ	2æ	1æ	o	/	//	///	////	V	/
2æ	1æ	o	/	//	///	////	V	V/	//
1æ	o	/	//	///	////	V	V/	V//	///
o	/	//	///	////	V	V/	V//	V///	////

Multiplicans.

Tabula pro ſpecie emergente ex Divif.
Dividendus.

4æ	3æ	2æ	1æ	o	/	//	///	////	
o	/	//	///	////	V	V/	V//	V///	4æ
1æ	o	/	//	///	////	V	V/	V//	3æ
2æ	1æ	o	/	//	///	////	V	V/	2æ
3æ	2æ	1æ	o	/	//	///	////	V	1æ
4æ	3æ	2æ	1æ	o	/	//	///	////	o
5æ	4æ	3æ	2æ	1æ	o	/	//	///	/
6æ	5æ	4æ	3æ	2æ	1æ	o	/	//	//
7æ	6æ	5æ	4æ	3æ	2æ	1æ	o	/	///
8æ	7æ	6æ	5æ	4æ	3æ	2æ	1æ	o	////

Diviſor.

Fig. 4: Tables to define place-identifiers given by Grueneberg [8]

In these tables the two given signs are selected at their borders. In the common field of column and row the new valid sign can be read.

References

1 J.H. Alsted. *Encyclopaedia septem tomis distincta*... 2 vols 1630, 4 vols 1649 (digitalized copy 1630 Google Books).

2 Andrea Argoli. *Ephemerides exactissimae caelestium motuum*. 1677 (digitalized copy e-rara.ch)

3 John Bernoulli. *A sexcentenary table exhibiting, at sight, the result of any proportion*... 1779

4 Sethus Calvisius. Opus Chronologicum. 1629 (digitalized copy Staatsbibliothek Berlin)

5 Florian Cajori. *A History of Mathematical Notations*. 1928, 1993

6 Oronce Fine. *Arithmetica practica.* 1536, 1544, 1555 (digitalized copy of ed. 1544 Google Books), (digitalized copies of ed. 1544, 1555 Bayerische Staatsbibliothek Muenchen).

7 Oronce Fine. *Protomathesis.* 1532 (digitalized copy Google Books).

8 Christian Grueneberg. *Pandora Mathematica Tabularum Universae Mathesis nempe Sinuum, Tangentium et Logarithmorum.* 1700 (digitalized copy GDZ)

9 Samuel Jeake. *Logistikelogia, or Arithmetick surveighed and reviewed.* 1696 (digitalized copy EEBO).

10 David A. King. *On medieval Islamic multiplication tables*. Historia Mathematica 1,3 (1974).

11 D.A. King, J. Samsò & B.R. Goldstein. *Astronomical Handbooks and tables from the Islamic World (750-1900): an Interim Report.* Suhayl 2

12 Philipp Lansberg. *Opera omnia*. 1663 (digitalized copy e-rara.ch).

13 Johann Friedrich Lorenz. *Grundriß der reinen und angewandten Mathematik*. 1800 (digitalized copy GDZ)

14 Paul Luckey: *Die Rechenkunst bei Gamsid b. Mas'ud a-Kasi*. Abhandlungen für die Kunde des Morgenlandes XXXI,1. 1951.

15 Kaspar Peucer. *Logistice astronomica Hexacontadon*. 1556 (digitalzed copy Bayerische Staatsbibliothek Muenchen).

16 Petrus Ramus and Lazarus Schoner (Schöner). *Arithmeticae Libri duo*... 1586, 1592 and 1599. (digitalized copies Staats- und Universitaetsbibliothek Dresden).

17 Erasmus Reinhold. *Prutenicae tabulae coelestium motuum.* 1571 (digitalized copy Staats- und Universitaetsbibliothek Dresden or e-rara).

18 Samuel Reyher. *Bacilli Sexagenales i.e. Descriptio Logisticae Sexagenariae per Bacillos exercendae,...*1688

18 Samuel Reyher. *Kurtze Beschreibung der Sechzig-theiligen Rechnung.* 1688 (digitalized copy GDZ)

20 Johannes Schoner (Schöner). *Tabulae Astronomicae.* 1536 (digitalized copy Bayerische Staatsbibliothek Muenchen).

21 Gaspar Schott. *Cursus Mathematicus*. 1661 and 1674 (digitalized copy 1674 Google Books or http://www.uni-mannheim.de/mateo/camenaref/schott.html).

22 D. E. Smith. *History of Mathematics*, vol. II. New York 1958.

23 Michael Stifel. *Arithmetica Integra.* 1544 (digitalized copy Staats- und Universitaetsbibliothek Dresden).

24 Aegidius Strauch (II). *Tabulae per universam mathesin summopere necessariae*. 1662 (digitalized copy Google Books).

25 Michael Taylor. *A sexagesimal Table, exhibiting at sight, the result of any proportion...* 1780 (digitalized copy Goole Books)

26 Sebastian Theodoricus *Breve, perspicuum, et facile compendium logisticae astronomiae*. 1563 (digitalized copy Bayerische Staatsbibliothek Muenchen).

27 Edo Hilderich von Varel. *Logistice Astronomica.* 1568 (digitalized copy Universitaets- und Landesbibliothek Sachsen-Anhalt).

28 Johannis Wallis. *De algebra tractatus... operum mathematicorum volumen alterum*. 1693 (digitalized copy Goole Books)

29 Stephan Weiss. *Die Rechenstäbe von Neper, ihre Varianten und Nachfolger.* 2007 http://www.mechrech.info/publikat/Neper85Au.pdf

30 Stephan Weiss. *Reconstruction of Sebastian Theodoricus: Canon Sexagenarum* (1564) (Dec. 2011) http://www.mechrech.info/publikat/TheodoricusCanonRecon.pdf

Jan. 2012
http://www.mechrech.info

Supplement: Translation and Reconstruction

The following text is an excerpt of Gaspar Schott's *Cursus Mathematicus*[4] in so far as it concerns multiplication or division with sexagesimal numbers in conjunction with use of his *Tabula Sexagenaria*. My translation tries to follow the original text closely to give a feeling of its distinctive flow without loss of readability. Comments may help to clarify hardly unintelligible phrases. Comparisons of Schott's sometimes sketchy rules with modern rules for calculating with exponents turned out to be helpful in detecting their meaning.
A copy of the original Latin text is added for comparison purposes and because some of the historical letters and abbreviations are difficult to be expressed in electronic text.
The table itself has been calculated and drawn with the graphical programming environment *Processing*[5] and is appended on the last page. Printed on paper of size ISO A3 (297 × 420 Millimeters) the table will get approximately its original size. The purpose of an additional number 60 in the upper right corner of the table is unclear, it has been omitted in the reconstruction.

4 I inspected a digital copy of the 1st ed. 1661 (European Cultural Heritage Online ECHO) and an original book from edition 1674. Both contain the named *Tabula Sexagenaria*.

5 URL http://processing.org

Gaspar Schott. Cursus mathematicus.
Liber II. Arithmetica Practica. Pars II. Caput II.

(p. 45)

Articulus V.

De Multiplicatione Aſtronomica.

INtricatiſſima eſt praxis multiplicandi numeros aſtronomicos ſexagenariam proportionem ſervantes, præſertim quando diverſæ ſpecies per diverſas ſpecies multiplicandæ ſunt, nempe gradus per minuta, ſecunda, &c: aut ſexagenæ primæ per ſecundas, tertias, &c. Conabor tamen quàm ordinatiſſimè procedere. Igitur

Primò. Commodioris operationis gratiâ ſcribe majorem numerum (qui nimirum ex plurib. ſpecieb. compoſitus eſt) ſuprà pro Multiplicando. minorem verò, ſeu pauciorum ſpecierum, infra pro Multiplicante, ita tamen ut ultima ad dexterã multiplicantis ſpecies ſubjiciatur ultimæ Multiplicandi, ſive ambæ ultimæ ſint ejuſdem ſpeciei, ſive diverſæ, ut in exemplis infra apparet. Quòd ſi uterque numerus æq; multas ſpecies continet, perinde eſt qui ſupernè, & qui infernè ponatur.

Secundò. Ductâ lineâ infra numeros collocatos, à dextera incipe, & duc ſingulas Multiplicantis ſpecies, in ſingulas Multiplicandi, more conſueto in multiplicatione ordinaria, productum, ſi ſexagenarium numerum excedit, divide per 60, reſiduum colloca ſub Multiplicante, Quotum verò productum ex diviſione adjice ſpeciei antecedenti, uti in iiſdem exemplis factum vides.

Tertiò. Peractâ totâ multiplicatione, nota ac diſtingue ritè in ſpecies numeros ex multiplicatione emergentes, tali pacto: ſi notæ utriuſq; numeri, multiplicandi videlicet & multiplicantis, ſunt ejuſdẽ ſpeciei, hoc eſt, ſi uterq; habeat notas tantùm tales, 0, I, II, III, &c. eas adde inter ſe, & producto ſupraſcribe: ſi diverſas, ut 0 & I, 0 & II, &c. itẽ minuta & ſexagenas; ſubtrahe minorẽ ex majore & reſiduum ſcribe pro nota ſupra productũ.

Section V.

About Astronomical Multiplication.

Most complicated is the process of multiplying astronomic numbers while keeping the sexagesimal proportions[6], especially when different orders[7] are to be multiplied by different orders, be it *gradus* by *minuta*, *secunda* and so on or *sexagenae primae* by *secundae*, *tertiae* and so on. Nevertheless I'll try to go on most systematically. Therefore

First. For a more convenient operation write the bigger number (which may be composed by several orders) on top as the multiplicand, the smaller or that (number) with less orders, beneath as multiplier, so that the very right order of the multiplier is set under the very (right order) of the multiplicand, may it be that both are of the same order or of a different (order), as it occurs in the examples below. And if each number holds many orders, it's the same way how to set down above and below.

Second. Draw a line below the collected numbers, start at the right side and multiply each order of the multiplier with each of the multiplicand, as usual in normal multiplication. If a product exceeds 60, divide it by 60, set the remainder below the multiplier, set the quotient from this division to the preceding order,[8] like you see it done in these examples.

Third. When the whole multiplication is done, note and distinguish in the right way the orders of the numbers, that come from multiplication, which is done this way: if the signs of both numbers, of course of multiplicand and multiplier, have the same order, which means, if both bear the signs 0, I, II, III, and so on, add them and write them over the product: when they are different, like 0 & I, 0 & II, and so on, also *minutae* and *sexagenae*; subtract the smaller from the bigger one[9] and write the result as sign over the product.

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6 To bundle place values to the next higher place if they exceed 59.

7 *Species* is an often used word with different meanings. Mostly the order of a place itself within a number where a sexagesimal figure stands is meant. In those cases I use the word "order" for translation, because from a modern point of view we can regard *species* with this meaning to be an exponent of base 60. In the example for division however *species* denotes the figure on the place with specified order. In some rules to denote sexagesimal figures with signs, *species* is set for ranges of signs.

8 For example if partial product = 125, then 125 / 60 = quotient 2 + remainder 5.

9 This text is a good example for Schott's sketchy and sometimes superficial way of teaching, that tends to become wrong. The rule is misinterpretative in formulation and in my opinion incomplete. He really wants to express the term $60^m \times 60^n = 60^{m+n}$ with positive and negative exponents (see fig. 1). His way to calculate with ordering signs is discussed more detailed at sexagesimal division.

Exemplum. Sint multiplicanda 4°.$13'$.$42''$.$50^{\bullet}$. per $38''$. Colloca numeros ut vides, & duc 38 in 50, producentur 1960, hæc divide per 60, producentur 31, & remanebunt 40, ſcribe ergo 40 infra multiplicantem, & 31 pone infra antecedentem ſpeciem, adjicienda ſummæ ex multiplicatione ſequenti producendę, ut vides factum in exemplo. Iterum duc 38 in 42, producentur 1596, hæc diviſa per 60, dant 26, & remanent 36, ſcribe 36 infra 42, & infra 31, ſed 26 pone infra 13. Iterum duc 28 in 13, productumque 494 divide per 60; habebis 8, & remanebunt 14, hæc ſcribe infra 13, illa infra 4. Tandem duc 38 in 4, producentur 152, quæ diviſa per 60, dant 2, & 32: ſcribe 32 infra 4, & 2 ſcribe in loco anteriori. His peractis, collige ſummas infra primam lineam poſitas in ſummam totalem, modo dicto Artic. 3. & habebis ſummam infra ſecundam lineam, notatam ut vides juxta tertium præcedens præceptũ, quoniam ſecunda ducta in tertia, dant quinta: in ſecunda, quarta: in prima, tertia, in integra, ſecunda, & hæc diviſa per 60, dant prima.

	4°.	$13'$.	$42''$.	$50^{\bullet}$.
				$38''$.
	8.	26.	31.	4°
2.	32.	14.	36.	
2.	$40''$.	$41'''$.	$7''''$.	40^{v}.

Simili prorſus modo procedendum eſt in omnibus aliis exemplis, ſive motus per motum multiplicentur, ſive tempora per tempora: & etiamſi Multiplicans contineat plures ſpecies. Exempla tu ipſe tibi ſtatue.

Example[10]. Has to be multiplied 4°.13'.42".50"'. by 38". Compose the numbers as you see, multiply 38 by 50, comes 1960[11], this divide by 60, comes 31, remaining 40, so write 40[12] below the multiplier and set 31 below the preceding order, whereby the sums produced by following multiplication will be increased, as you see in the example. Again multiply 38 and 42, comes 1596, divide it by 60, makes 26 and 36 remaining, write 36 below 42 and below 31, but set 26 below 13. And again multiply 28[13] and 13, comes 494, divide by 60, you will get 8, remaining 14, this (14) write below 13, that (8) below 4. At last multiply 38 and 4, comes 152, which divided by 60, makes 2, and 32: write 32 below 4, and 2 in a preceding place. When this is done, collect the sums below the first line and put them together in a total sum in the way given in section 3[14]. And you will get the sum below the second line, marked as you see in accordance to the third preceding rule, which means *secunda* multiplied by *tertia* gives *quinta*: by *secunda* (gives) *quarta*: by *prima* (gives) *tertia*, by *integra* (gives) *secunda*, and these divided by 60 give *prima*.[15]

The same procedure has to be done in all other examples, in case there has to be multiplied motion by motion or times by times[16] and even if the multiplicand may have several orders.[17] Give examples to yourself.

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10 In this example a multi-digit number is multiplied by a single-digit number in written form.

11 Wrong result, $38 \times 50 = 1900$. He continues his calculation the right way.

12 In the written display 40 is erroneously printed as 4°.

13 Wrong value, should read 38.

14 He refers to section III, *De Additione astronomica*, about astronomical addition.

15 First the numbers are multiplied and the partial products are written down and when finished the orders are defined and noted. Here Schott enumerates the orders of all partial products from right to left.

16 Mathematicians distinguished between astronomical numbers for motion with multiples and parts of degrees, based on proportion 60 and numbers for time that hold years, months, days, hours, minutes, seconds and their parts (see *Articulus I*, p. 43).

17 Schott passes over a multiplication with two multi-digit factors, which is in consideration of the orders of all partial products more difficult to perform. A century before authors give such an example. Systematically they arrange figures in columns for orders and thus ease calculation. See Stifel *Arithmetica Integra,* fol. 67r or Theodoricus *Compendium*.

Annotatio.

De Tabula Sexagenaria, pro multiplicatione, divisione, in numeris astronomicis.

Quoniam res laboris ac tadii plena est productum ex multiplicatione, quoties sexagenarium numerum superat, dividere per 60, & quotum inventum ad anteriorem speciem rejicere, retento solùm residuo; ordinarunt Artifices, magno ingenio, Tabulam quam Canonem Sexagenarium appellant, seu sexagesimorũ scrupulorum, ex qua statim & uno quasi intuitu colligitur quid ex qualibet multiplicatione producatur ad diversas species spectans; quam propterea huic loco inserere volui. Constat ea trianguli *ABC*, & trapezij *DEFG* formâ.

Usus Tabulæ hic est. Si tam multiplicandus, quàm multiplicator, tricenario major sit, quæratur major in trianguli latere dextro *BC*, minor verò in superiori transversali *AB* : Si verò alteruter tricenario minor sit, quæratur major in trapezii latere sinistro *DE*, minor in transversali obliquo *DF*. Cum his duobus numeris in utrolibet casu, perge ad areolam eorum communem, & invenies in ea productum sub duabus speciebus, antecedente, & consequente: & sinistimus quidẽ numerus dicta areola significat quotum ad speciem antecedentem reijciendã, dextimus verò residuum ex multiplicatione. Quã porrò notam hoc residuum habere debeat, disces ex tertio præcepto paulò antè posito. (p. 46)

Exemplum. Sint ut antea, multiplicanda $50^{'''}$ per $38^{''}$: quære 50 in trianguli latere dextro *BC*, & 38 in latere superiore *AB*; invenies in areola seu quadratulo communi, 31, 40; quæ significant in casu posito $31^{''''}$, & 40^{v}. Item sint multiplicanda $13^{'}$ per $38^{''}$: quære 38 in trapezij latere sinistro *DE*, 13 verò in diagonali *DF*; invenies in areola seu quadrangulo concursus 8, 14; quæ significant in posito casu $8^{''}$, & $14^{'''}$.

Hæc autem Tabula servit etiam pro divisione, ut ex dicendis Articulo sequenti patebit. Examen multiplicationis fit per divisionem, de qua nunc agemus.

Annotation.

About the sexagesimal table for multiplication, division with astronomical numbers.

Since it means trouble and dislike to divide by 60 a product from a multiplication whenever the number exceeds 60 and to diminish the found quotient to the preceding order to get only the remainder,[18] Artists constructed with much ingenuity a table, which they call Canon *Sexagenarium vel Scrupulorum Sexagesimorum* (Table of sixties or of sixtieth parts), from which at once and almost at a glance can be brought together, what from a multiplication with different orders is expected to be created; and that is why I intended to insert it (the table) here. The shape consists of a triangle ABC and a trapez DEFG.

Follows usage of the table. If both multiplicand and multiplier are bigger than 30, the bigger one has to be searched on the right side BC of the triangle, the smaller in the horizontal side AB above. If one of them is smaller than 30, the bigger one has to be searched in the left side DE of the trapez, the smaller one in the diagonal line DF. With these two numbers whatever go to their small shared place and here you will find the product with two orders (figures), the preceding and the consequential one: and the left number in this named small place marks the quotient, reduced to the preceding order, the right one the remainder of multiplication.[19] What a sign the remainder has to have you will get from the third rule placed short before.[20]

Example. As before, 50''' are to be multiplied by 38'': search for 50 on the left side BC of the triangle, and 38 in the upper side AB, you will find in this small place or shared small quare 31,40; which (numbers) in this case mean 31'''' and 40^{V}. In the same way are to be multiplied 13' and 38'': search for 38 on the left side DE of the trapez, 13 in the diagonal DF; you will find in the small space or shared small square 8,14; which (numbers) in this case mean 8'' and 14'''.

But this table is useful for division too, as will be explained in the following article. The study of multiplication leads to division, which we now cover.

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18 An example for what he says: $31° \times 46 = 1426°$; $1426° / 60 = 23^{ae}$ R 46° with calculation or, much more easier, $31 \times 46 = 23.46$ from the table.

19 Schott refers to an intermediate division by 60 if the partial product exceeds 59, not necessary by use of the table, see preceding footnote.

20 In the same column on top of the page.

Articulus VI.

De Divisione Astronomica.

Divisio numerorum astronomicorum per astronomicos, rarum habet usum, & per Tabulam Sexagenariam difficulter peragitur, sine Tabula difficillimè. Qui ea destituitur, resolvat tam Dividendum, quàm Divisorem, per continuã multiplicationem Sexagenariam, in ultimas species quas continent, & tum divisionem more vulgato instituat, quotumque inventum rursus per continuam divisionem sexagenariam more vulgato in suas species colligat.

Exemplum. Sint dividenda Sexagesimæ secundæ 15, Sexagesimæ primæ 5, gradus 9, Minuta 12, Secunda 17, Tertia 16. per Sexagesimas primas 42, Gradus 23, Minuta 35, Secunda 46. Reduc totũ dividẽdum ad Tertia, per continuã multiplicationem per 60, & habebis Tertia 1250838 8236. Similiter reduc totum Divisorẽ ad Secunda, per eandem continuam multiplicationẽ per 60, & habebis Secunda 9156946. Divide jam Tertia per Secunda, divisione vulgari, & habebis pro Quoto Prima 1366, uti patebit ex Regulis paulò pòst adsignandis. Hæc rursus divide divisione vulgari per 60, & produces Gradus 22, Minuta 46, uti ex iisdem Regulis patebit.

2.æ 1.æ 0
16. 5 9. $12'$. $17''$. $16'''$,
1.æ 0
42. 23. $35'$. $46''$.

Tertia. 12508388236
Secunda. 9156946
Prima. 1366
Gradus 22. Minuta 46,

Section VI.

About Astronomical Division.

The division of astronomic numbers by astronomic (numbers) is seldomly used and with the Sexagesimal Table difficult to perform, without the table most difficult. Who is left alone without it (the table), enlarges dividend and divisor with repeated sexagesimal multiplication to the farest orders they hold and then performs a division the normal way, and going back that way puts together their orders with normal sexagesimal divisions.
Example. Should be divided *Sexagesimae secundae* 15[21], *Sexagesimae primae* 5, *gradus* 9, *Minuta* 12, *Secunda* 17, *Tertia* 16 by *Sexagesimae primae* 42, *Gradus* 23, *Minuta* 35, *Secunda* 46. Reduce the whole dividend to *Tertia* with continuous multiplication by 60, and you will get 12508388236 *Tertia*. In the same way reduce the whole divisor to *Secunda* with the same continuous multiplication by 60, and you will get *Secunda* 9156946 . Divide *Tertia* by *Secunda* with normal division and you will get a quotient of 1366 *Prima* which will be specified with the near below following rules. Going back divide these (*Prima)* with usual division by 60 and you produce *Gradus* 22, *Minuta* 46 that will come from the same rules.

21 Wrong value, should read 16 like in written example.

Per Tabulam Sexagenariam ſic inſtitues diviſionem. *Primò*. Totum Dividendum cum ſuis ſpecieb. ſcribe loco ſuperiori, eique ſubjice, incipiendo à ſiniſtra, totum Diviſorem cum ſuis etiam ſpecieb. ita ut vel ſiniſtima figura diviſoris ſubjiciatur ſiniſtimæ Dividendi, ſi minor ſit Diviſor quàm membrum Dividendi cui reſpondere debet; vel proximè ſequenti, ſi ſit major. Deinde poſt utrumque numerum ad dexteram forma ſemilunulam, cui Quotus inſcribatur , prout in Diviſione ordinaria fit, & prout in exemplo factum vides in A & B.

Secundò. Si utraq; prima ſpecies, tam Diviſoris, quàm Dividendi, excedat triginta , adi tabulam triangularem, & quære Diviſorē in latere ſupremo tranſverſali AB, Dividendū verò in ſubjecta columna perpendiculari; vel ſi Dividendus in dicta columna præcisè nō reperitur, quære numerum proximè minorem; & ab hoc numero perge ad latus dextrum BG trianguli, invenieſque Quotum poſt lunulam ſcribendum. Si verò alterutra, ſive Dividendi, ſive Diviſoris, prima ſpecies minor eſt tricenario, adi tabulam quadrangularem, & quære Diviſorem in latere ſiniſtro DE dictæ tabulæ, Diviſorem verò in columna tranſverſali, vel ipſo proximè minorem numerum, à quo numero ſi aſcendas rectà ad caput tabulæ, invenies in ejus latere obliquo DF Quotum poſt ſemilunulam ſcribendum, ut vides in L factum.

(To even out page count the corresponding original sketch is added at the end of text)

With help of the Sexagesimal Table you will set up a division that way. First. Write the whole dividend on top with its orders, downwards set, beginning from the left, the whole divisor with his orders too, so that either the leftmost figure of the divisor is set below the leftmost (figure) of dividendus, if divisor is smaller than the corresponding part of dividend, or, if it is bigger, under the next following (figure).Next behind one of the two numbers to the right side form a small half moon[22], in which the quotient will be written in the sequence of the division and like you see in A and B.

Second.If the first order[23] of divisor or dividend exceeds thirty, go to the triangular table and look for the divisor on the horizontal side AB on top, and look for the dividend in the rectangular column below, if the dividend cannot be found exactly in the named column, look for the next smaller number; and from this number go to the right side BG in the triangle and you find a quotient that is written behind the left parenthesis. But if in dividendus or divisor the first order is smaller than thirty, go to the square table and look for divisor in the left side DE of the named table, (look for) divisor[24] or the next smaller number in the horizontal row, from this number go up to top of table and you will find in this oblique side DF the quotient that is written behind the parenthesis as you see done in L.

22 A nice replacement for left parenthesis.

23 Precisely the figure in the first order.

24 Rectified dividend

Tertiò.Quotum inventum multiplica in totum Diviſorem,modo dicto in Articulo V.de Multiplicatione aſtronomica,ut vides factum in C.

Quartò.Productum ex multiplicatione collige in unam ſummam, diſtinctã ritè in ſuas ſpecies,ut vides in D:eamque ſummam ſubtrahe à Dividen- (p. 47) do,cui ſubſcriptus eſt diviſor,& reſiduum ſcribe infrà,ut in ſequenti exemplo vides in E.

Quintò. Reſiduo infrà notato adjunge aliam ſpeciem Dividendi, ut factum vides in F.Diviſoreq; promoto, ut fieri ſolet in diviſione ordinaria, & ut vides factum in G,repete operationes omnes ut antè,& ut factum vides in H,in I,in K, & in M.

Sextò.Si plures ſuperſunt ſpecies Dividendi,repete eundem operandi modũ, donec omnes abſumpſeris.Si promoto Diviſore,is major eſt quã Dividendus cui ſubſcriptus eſt,pone cifram poſt lunulam,& iterum promove Diviſorẽ,operareque ut antea,Si quid ex diviſione reſiduũ ſit, placeatque ulterius per eundem Diviſorem dividere;tunc reſiduo à dextris loco ulterioris ſpeciei adjunge cifram ſemel,aut iterum,prout libuerit, numerumque iſtum ulterius partire modo antea dicto. Sic ad minima pervenire poteris.

Third. Multiply the quotient by the whole divisor, as told in section V. About Astronomical Multiplication, as you see done in C.

Fourth. Collect the product from multiplication in one sum, parted as usual in its orders, as you see in D: this sum subtract from dividend, which holds the divisor below, the remainder write below as you see in the following example in E.

Fifth. This remainder written below combine with the other order[25] in the dividend, as you see it done in F. Repeat all operations as before with the moved divisor how it is done in normal division and as you see it done in G, and as you see it done in H, in I, in K and in M.

Sixth. If more orders in the dividend are left, repeat this operation until you have removed all of them. If a moved divisor is bigger than that dividend it is written below, place a zero behind parenthesis and operate with the moved divisor like before. If something remains from division and it pleases going on to divide with the same divisor, then add to the remainder zero at the right in the next order once, or repeatedly, as you like, and part this number again and again in the way told before. Thus you will come to the least.

25 Orders that are not treated yet, 16''' in his example.

Sit dividẽdus & diviſor ut ſupra. Colloca illos ut vides in A & B. Et quoniã prima dividendi ſpecies, 16, minor eſt tricenario, quære diviſorẽ, 42, in latere DE trapezij, & perge dextrorſum in eadem columna; in qua quia non occurrunt 16, 5; accipe proximè minorem numerum 15, 24, & aſcendendo invenies 22. Scribe ergo 22 pro Quoto, ut vides in L. eumque multiplica in diviſorem B; invenies numeros C; quos collige in ſummam D, & ſubtrahe ab A, eritque reſiduus numerus E. Hinc adde ultimam dividendi ſpeciem, nimirum 16^{m}, & habebis novum dividendum F, cui ſubſcribes diviſorem, ut vides factum in G. Et quia prima ſpecies tam dividendi, quàm diviſoris major eſt tricenario, quære primam diviſoris ſpeciem, 42. in fronte AB trianguli, & deſcendendo invenies 32, 12, à quo numero perge detrorſum, & in latere BC trianguli invenies 46. Scribe hæc poſt lunulam pro Quoto, ut vides in M: eundem multiplica in G, numeros H productos collige in ſummam I, eamque ſubtrahe ab F, & nihil remanebit. Erit itaque Quotus, 22, 46.

Septimò. Peractâ diviſione totâ, ſigna notis convenientibus ſpecies Quoti ex diviſione emergentis, obſervando ſequentes Regulas.

Dividendus and divisor are as above.[26] Set them as you see in A and B. And because the first order of dividendus, 16, is smaller than thirty, search the divisor, 42, on the side DE of trapez, go to the right in the same row, in which 16,5 don't occur, take the next smaller number 15,24 and going down you find 22. So write 22 for quotient as you see in L. This (number) multiply by divisor B, you will find numbers C, which you collect to sum D and subtract from A, will come remainder E. Here add the last order of dividend, it's 16'' and you will get the new dividend F, to which you write below the divisor, as you see done in G. And because first order of dividend and of divisor is bigger than thirty, look for the first order of divisor, 42, in front AB of triangle and going down you will find 32,12 from where you go to the right and you will find 46 in the side BC of the triangle. Write it behind the parenthesis as quotient, as you see in M: this multiply with G, the produced numbers H collect in sum I, which you subtract from F and nothing will remain. So the quotient will be 22,46.

Seventh. When the whole division is done, mark the orders of quotient with signs that follow from division, observing the following rules.

26 Without any connection an example for division with given numbers follows.

Regulæ ad ſignandas ſpecies Quoti.

PRimò. Cùm diviſor & dividendus habent notas ejuſdem ſpeciei, & quantitatis, proveniunt integra. *Sic ſi dividas gradus per gradus, Sexagenas primas per primas, Secundas per ſecundas, &c. Scrupula prima per prima, ſecunda per ſecunda, &c. emergunt gradus, qui integrum conſtituunt.*

Secunda. Cùm diviſor & dividendus habent notas ejuſdẽ ſpeciei, ſed nota dividendi ſuperat notã diviſoris: ſubtrahe minorem notam ex majori, eritque reſiduum nota Quoti ejuſdem ſpeciei ut antea. *Sic ſi dividas ſcrupula* $36'''$ *per* $6''$, *fiet Quotus* $6'$: *Si ſexagenas* 12^{3as} *per* 5^{2as}, *fiet Quotus* 2^{1a}.

Tertia. Cùm diviſor & dividendus habent notas ejuſdem ſpeciei, ſed nota diviſoris ſuperat notam dividendi; ſubtrahe ſimiliter minorẽ ex majore, eritq; reſiduum nota Quoti diverſæ ſpeciei quàm antea: nam ex ſcrupulis fiunt Sexagenæ, & ex Sexagenis ſcrupula. *Sic ſi dividas ſcrupula* $12''$ *per* $6'''$,

Rules to indicate orders of the Quotient.

First. If divisor and dividend have signs of the same range and quantity[27] come *integra*. So if you divide *gradus* with *gradus*, *sexagenae primae* by *primae*, *secundae* by *secundae* and so on, *scrupula prima* by *prima*, *secunda* by *secunda* and so on, will come *gradus*, which become *integra*.

Second. If divisor and dividend have signs of same range, but the sign of dividend is bigger than the sign of divisor: subtract the smaller sign from the bigger one, the remaining sign of quotient will belong to the same range as before. So if you divide *scrupula* 36''' by 6'', the quotient will be 6': if (you divide) *sexagenae* 12^{3as} by 5^{2as}, the quotient will be $2^{1æ}$[28].

Third. If divisor and dividend have signs of same range, but the sign of divisor is bigger than the sign of dividend; subtract in the same way the smaller from the bigger (one), the remaining sign of quotient will be in the opposite range than before: from *scrupula* come *Sexagenae* and from *Sexagenae* *scrupula*. So if you divide *scrupula* 12'' by 6''',

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27 Here *species* is used in the sense of "range" and *quantitas* is the value of a sign. Schott distinguishes the three ranges *sexagenae, gradus, scrupula*, from modern view negative or positive or zero exponents.

28 The superscript signs "as" and "æ" are equal, but they follow Latin grammar: *sexagen**ae*** in nominative case and *sexagen**as*** in accusative case.
Furthermore *Scupula tertia* are bigger than *scrupula secunda* and *sexagenae tertiae* are bigger than *sexagenae secundae*. From view of powers, exponents have no sign in sense of positive or negative, their value is absolute. Schott uses eight rules to avoid zero or negative differences between signs.

fiunt 2^{ae} *Sexagenæ : ſi vero ſexagenas* 16^{2as} *per ſexagenas* 4^{3as} *, fiunt ſcrupula* $4'$.

Quarta. Cùm diviſor & dividendus habent notas diverſæ ſpeciei, adde notas diviſoris notis dividendi, & ſumma erit nota Quoti ſub ea ſpecie, ſub qua dividendus erat *Sic ſi dividas* 20^{2as} *ſexagenas* $4'''$ *ſcrupula, emergunt 5 ſexagenæ quintæ: & ſi dividas ſcrupula* $16''$ *per 4 Sexagenas tertias, emergunt 4 ſcrupula quinta*

Quinta. Cùm gradus dividuntur per ſcrupula, proveniunt Sexagenæ ejuſdē ſpeciei, cujus ſunt ſcrupula diviſoris. *Sic ſi dividas* 16° *per* $4'$ *ſcrupula, habebis in Quoto* 4^{1as} *ſexagenas.*

Sexta Cùm ſcrupula per gradus dividuntur, proveniunt ſcrupula ejus ſpeciei, cujus eſt dividendus. *Sic ſi dividas* $16''$ *ſcrupula per* 4° *, habebis in Quoto* $4''$ *ſcrupula.*

Septima. Cùm gradus ſexagenas dividuntur, proveniunt in Quoto ſcrupula ejus ſpeciei, quam habet diviſor. *Sic diviſis* 16° *per* 4^{2as} *ſexagenas, erit Quotus* $4''$ *ſcrupula.*

Octava. Cùm Sexagenæ per gradus dividuntur, proveniunt ſexagenæ ejus ſpeciei, cujus eſt dividendus. *Sic diviſis* 16^{2is} *ſexagenis per* 4°*, erit Quotus* 4^{2ae} *ſexagenæ.*

Annotatio.

HÆ regulæ valent, cùm dividendus primus eſt major diviſore : ſi enim minor eſt, producitur ſpecies in Quoto uno loco inferior illa, quam Regulæ docent; ſcilicet non gradus, ſed ſcrupula; non ſcrupula, ſed ſecunda. Si tamen minorem dividendum reducas ad ſequentem ſpeciem minorem, v.g. gradus ad ſcrupula, ſcrupula ad ſecunda, &c. valent Regulæ.

come $2^{1æ}$ *Sexagenae*: in contrast *sexagenae* 16^{2as} by *sexagenae* 4^{3as} will come *scrupula* 4'.

Fourth. If divisor and dividend have signs of opposite range, add the signs of divisor and the signs of dividend, and the sum will be the sign of quotient in that range the dividend had. So if you divide 20^{2as} *sexagenae* (by) 4''' *scrupula*, will come 5 *sexagenæ quintæ*: and if you divide *scrupula* 16'' by 4 *Sexagenae tertiae* will come 4 *scrupula quinta*.

Fifth. If *gradus* are divided by *scrupula*, come *Sexagenae* with the same order *scrupula* of the divisor have. So if you divide 16° by 4' *scrupula*, you will get in the quotient 4^{1as} *sexagenae*.

Sixth. If *scrupula* are divided by *gradus* come *scrupula* with the same order of dividend. So if you divide 16'' *scrupula* by 4° you will get in quotient 4'' *scrupula*.

Seventh. If *gradus* are divided by *sexagenae* come in quotient *scrupula* with the same order the divisor has. So if you divide 16° by 4^{2as} *sexagenae* will come a quotient of 4'' *scrupula*.

Eighth. If *sexagenae* are divided by *gradus*, will come *sexagenae* with the same order the dividend has. So if you divide 16^{2is} *sexagenae* by 4°, will come a quotient of $4^{2æ}$ *sexagenae*.

Remark.

These rules are valid as long as the dividend is bigger than the divisor: if it is smaller, an order in the quotient comes that is one place lower than that the rules give; which means, not *gradus* but *scrupula*; not *scrupula* but *secunda*. But if you reduce the smaller dividend to the next smaller order, for example *gradus* to *scrupula*, *scrupula* to *secunda*, and so on, the rules apply.[29]

29 The named case is already covered in the rules to perform a written division. What he really wants to say is avoid a numerical quotient smaller than 1.
Background of this rule is the term
$(a\times 60^m) / (b\times 60^n) = a/b\times 60^{m-n}$ for a>b and $= (60a/b)\times 60^{m-n-1}$ for a<b.

Exemplum.

I.	Dividendus.	A.	2.ª 16.	1.ª 5.	0 9.	12′.	17″.	16‴.	0 (L.22.
	Divisor.	B.		1.ª 42.	0 23.	35′.	46″.		
	Numeri ex multiplicatione Quoti in Divisorem orti.	C.	15.	8. 24.	12. 26.	16. 50.	52.		
	Summa subtrahenda ex Dividendo.	D	2.ª 15.	1.ª 32.	0 39.	′6.	5″2.		
	Residuum ex subtractione.	E.	00	1.ª 32.	0 30.	5′.	25″.		
II.	Novus Dividendus	F.		1.ª 32.	0 30.	5′.	25″.	16‴.	0 (M.22.46′.
	Divisor promotus.	G.			1.ª 42.	0 23.	35′.	46″.	
	Numeri ex multiplicatione novi Quoti in Divisorem orti.	H.		32.	17. 12.	26. 38.	35. 50.	16.	
	Summa subtrahenda ex novo Dividendo.	I.		1.ª 32.	0 30.	5′.	25″.	16‴.	
	Residuum ex subtractione.	K.		00.	00.	0.	00.	00.	

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Follows *Tabula*

Tabula Sexagenaria Vel Sexagesimorum scrupulorum. Inseratur lib. 2 par. 2 cap. 2 Artic. 5 pag. 45.

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Triang. Forma. A.B.C

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