A reconstruction of
the Mathematical Tables Project’s
table of natural logarithms
(1941)

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“[f]or a few brief years, [the Mathematical Tables Project] was the largest computing organization in the world, and it prepared the way for the modern computing era.”
D. Grier, 1997 [29]

“Blanch, more than any other individual, represents that transition from hand calculation to computing machines.”
D. Grier, 1997 [29]

“[Gertrude Blanch] was virtually the backbone of the project, the hardest and most conscientious worker, and the one most responsible for the amount and high quality of the project’s output.”
H. E. Salzer, 1989 [66]

1 The Mathematical Tables Project

The present table was published by the Mathematical Tables Project, a project of the Works Progress Administration (WPA, renamed Works Projects Administration), a New Deal agency established by President Roosevelt to alleviate unemployment through public works. The purpose of the Mathematical Tables Project was to compute tables of higher mathematical functions. Because the Mathematical Tables Project was part of the WPA, much of the computation was done by hand. This project was in operation since January 1938 and its administrative director was Arnold Lowan.1 The mathematical leader of the Project was Gertrude Blanch.2

Prior to the Mathematical Tables Project, the British association for the advancement of science had started publishing volumes of tables in 1931. Between 1931 and 1946, 11 volumes were published, and a final one in 1952.19 The British group appears with hindsight to have been driven less by the production of general fundamental tables than the Mathematical Tables Project. Instead, it was more aimed at organizing earlier tables. These twelve volumes are the following ones:

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1Arnold Noah Lowan (1898–1962) [11, 5] was born Leibovici in Iasi (Romania). He graduated from the Bucharest Polytechnical Institute of Chemical Engineering in 1924, and the same year moved to the United States. He obtained a Master of Science from New York University in 1929 and a PhD from Columbia University in 1934. He was a fellow at the Institute for Advanced Study, Princeton (1928–1931), lecturer of mathematics at Brooklyn College, New York (1935–1940).

From 1938 to 1949, he was the director of the computation laboratory at the National Bureau of Standards, where he was directing the publication of a number of mathematical tables. From 1950 to 1952, he was a consultant at the US Naval Ordnance Laboratory and from 1955 to 1962, he was professor of mathematics at Yeshiva University, New York.

2Gertrude Blanch (1897–1996) was born in Poland and moved to the United States around 1907. After having graduated from high school in 1914, she first worked as a clerk for 14 years, honing her skills and knowledge of accounting, inventory, planning, risk calculations, and so on. In 1928, she fulfilled her dream to become a mathematician and matriculated to New York University. She received a BSc in Mathematics from NYU in 1932 and a PhD in mathematics from Cornell University in 1935. Around the end of 1937, while attending a continuing education class on relativity taught by Arnold Lowan, Lowan offered her the job of technical director of the Mathematical Tables Project, which she joined in February 1938. Within that project, she designed algorithms that were executed by teams of human computers. Blanch also worked regularly with the Manhattan Project, both during and after the war. In the mid-1950s, she was hired by the Air Force and continued to work on numerical analysis, in particular on Mathieu functions.
I. Circular and hyperbolic functions, exponential and sine and cosine integrals, factoral and allied functions, hermitean probability functions (1931)

II. Emden functions, being solutions of Emden’s equation together with certain associated functions (1932)

III. Minimum decompositions into fifth powers (1933)

IV. Cycles of reduced ideals in quadratic fields (1934)

V. Factor table giving the complete decomposition of all numbers less than 100,000 (1935) [56]

VI. Bessel functions. Part I. Functions of orders zero and unity. (1937)

VII. The probability integral (1939)

VIII. Number-divisor tables (1940)

IX. Table of powers, giving integral powers of integers (1940)

X. Bessel functions. Part II. Functions of positive integer order. (1952)

Part-volume A. Legendre polynomials (1946)

Part-volume B. The Airy integral, giving tables of solutions of the differential equation $y'' = xy$ (1946)

On the other hand, the Mathematical Tables Project computed a number of large tables mostly ab initio. Moreover, the purpose of the Project was not so much to complete the computations quickly, but to keep the (human) computers busy, and at the same time to conduct some useful work. At one point the Mathematical Tables Project employed 450 human computers, sometimes aided by mechanical calculating machines, a group which was reminiscent of the one set up for the famed French Tables du cadastre [61].

The main tables published between 1939 and 1949 by the Mathematical Tables Project are the following ones:3

- Table of the first ten powers of the integers from 1 to 1000, 1939
- Tables of the exponential function $e^x$, 1939 (reconstructed in [65])
- Tables of circular and hyperbolic sines and cosines for Radian arguments, 1939
- Tables of sines and cosines for Radian arguments, 1940
- Tables of sine, cosine and exponential integrals, 1940 (2 volumes)
- Table of natural logarithms, 1941 (4 volumes)
- Tables of the moment of inertia and section modulus of ordinary angles, channels, and bulb angles with certain plate combinations, 1941
- Miscellaneous physical tables, 1941

3Numbers such as MT1, MT2, etc. were given to each volume, but only at a later time. They served for a proper identification of each volume. However, the numbers given in the National Bureau of Standards’s publication list [54] and by Grier [30] do not completely coincide. It was possibly only after 1948 that a set of 28 “main tables” was presented, with numbers from MT1 to MT28. The list given here is that given by Grier.
• Table of sine and cosine integrals for arguments from 10 to 100, 1942
• Tables of probability functions, 1942 (2 volumes)
• Table of arc tan \( x \), 1942
• Table of reciprocals of the integers from 100,100 through 200,009, 1943
• Table of the Bessel functions \( J_0(z) \) and \( J_1(z) \) for complex arguments, 1943
• Table of circular and hyperbolic tangents and cotangents for radian arguments, 1943
• Tables of Lagrangian interpolation coefficients, 1944
• Table of arc sin \( x \), 1945
• Tables of associated Legendre functions, 1945
• Tables of fractional powers, 1946
• Tables of spherical Bessel functions, 1947 (2 volumes)
• Tables of Bessel functions of fractional orders, 1948 & 1949 (2 volumes)
• Tables of Bessel functions \( Y_0(x), Y_1(x), K_0(x), K_1(x), 0 \leq x \leq 1 \), 1949

Many other smaller or more specialized tables were also published by the Mathematical Tables Project. Lists of published tables are given in the appendices of each of the published volumes. The announcement published in 1941 [25] also lists the tables published so far, those for which computation had been completed or was in progress, and those which were considered for calculation. Archibald’s survey gives the status of computations by the end of 1942 [9].

The WPA was terminated in 1943, but the Mathematical Tables Project continued to operate in New York until 1948. That year, a number of members of the Mathematical Tables Project moved to Washington, DC to become the Computation Laboratory of the National Bureau of Standards, now the National Institute of Standards and Technology. But Blanch moved to Los Angeles to lead the computing office of the Institute for Numerical Analysis at UCLA, and Lowan joined the faculty at Yeshiva University in New York. Other tables continued to be computed, of which a detailed list is given by Fletcher et al. [26, pp. 718–720].

The greatest legacy of the Project is the Handbook of Mathematical Functions [1], published in 1964, and edited by Milton Abramowitz (1915–1958) and Irene A. Stegun (1919–2008), two veterans of the Project. But more broadly, the Project developed “the numerical methods of scientific computation [and demonstrated] that computation could solve practical and important problems” [29].

2 Tables of natural logarithms

In this section, we review the main tables of natural logarithms. Although sometimes called Napierian logarithms, the first table of natural logarithms was not published by Napier, but by Speidell a few years later [70, 60, 45].
An extensive table of natural logarithms was first computed by Isaac Wolfram, a Dutch lieutenant of artillery and published by Schulze in 1778 [68] (figure 1). Wolfram’s table represents six years of work [17, p. 166]. The original table omits half-a-dozen logarithms which Wolfram could not compute because of an illness. These logarithms were supplied in the Berlin Ephemeris for 1783 [69, p. 191], [24, p. 602].

As highlighted by Archibald [12, p. 193], Wolfram made use of two or three different methods for the computation of the natural logarithms. A first way is to take an accurate value of the decimal logarithm and to multiply it by $\ln 10$. Two other ways are to make use of the formulæ

\[ \ln(1 + x) = \sum_{m=1}^{\infty} (-1)^{m+1} \frac{x^m}{m} \]  
\[ \ln \frac{1}{1 - x} = \sum_{m=1}^{\infty} \frac{x^m}{m} \]

and to choose appropriate values for $x$. For instance, with $x = 1/23400$, we have

\[ 1 + x = \frac{23401}{23400} = \frac{7 \cdot 3343}{2^3 \cdot 3^2 \cdot 5 \cdot 13} \]

and the computation of $\ln(1 + x)$, combined with the values of the logarithms of 2, 3, 5, 7, and 13, provides the logarithm of 3343. With $x = \frac{1}{2651\cdot10^3}$, we have

\[ 1 - x = \frac{2651 \cdot 10^3}{2651 \cdot 10^3 - 1} = \frac{11 \cdot 241 \cdot 10^3}{13 \cdot 61 \cdot 3343} \]

and we have another way to compute $\ln 3343$. This method was in fact anticipating the work of Edward Sang [67] who proceeded much in the same way.

Apart from Wolfram’s table which was reprinted in 1794 by Vega [79] (figure 2), in 1795 by Callet (first hundred primes, to 20 places) [18], and in 1922 by Peters & Stein [55], we mention the large tables of Thiele (1908, 48 decimals) [72], Vietz (1825, 81 decimals) [78], Warmus (1954, 108 decimals) [14], Uhler (1942, 137 decimals) [76], and Uhler (1943, 155 decimals) [77]. Uhler (1940) [75] gave a few basic natural logarithms to about 330 places. The present table published in 1941 gave the logarithms to 16 places, and finally, Spenceley, Spenceley & Epperson (1952) [71] gave them to 23 places.

Mention should also be made of Kulik who had prepared a table of natural logarithms to 48 places from 1 to 11000, based on Wolfram’s table [38, 36, 37, 7, 10, 12, 13], but although this table may have been printed, no copy has been located anywhere.

A number of extensive smaller tables of natural logarithms have also been published, in particular by Barlow (1814) [15] (reprinted in Rees’s Cyclopædia (1819) [4]) and Dase (1850) [22].

For an extensive list of existing tables of natural logarithms by about 1960, see the survey published by Fletcher in 1962 [26, p. 249]

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4For more on Wolfram, see [27, 8, 12, 13]. Wolfram had also been working on factor tables [39, 40].
3 The Project’s table of natural logarithms (1941)

3.1 Description

The Project’s table of natural logarithms spans four volumes [43]. The first two volumes give the natural logarithms of the integers from 1 to 49999 and from 50000 to 99999 to sixteen decimal places. The last two volumes give the natural logarithms of the decimal numbers from 0 to 10, by steps of 0.0001, also to sixteen decimal places. The latter two volumes can be (and were) derived from the first two by subtracting \( \ln(10000) = 4 \ln 10 \), so that for instance \( \ln 0.2805 = \ln 2805 - 4 \ln 10 \).

The technical staff involved in the preparation of these volumes was made of Gertrude Blanch (1897–1996), Frederick G. King, Milton Abramowitz (1915–1958), Jack Laderman, William Kaufman, Matilda Persily, and Jacob Miller for volumes 1 and 2, in that order. In addition, volumes 3 and 4 list William Horenstein, Ida Rhodes (1900–1986) and Herbert E. Salzer (1915–2006), but this time the names are listed alphabetically.

3.2 Algorithms

The starting point of the Project’s table was Wolfram’s table of natural logarithms published by Schulze in 1778 [68]. This table gave the natural logarithms of the first 2200 integers and of primes and certain composite numbers up to 10009, all to 48 decimal places. The only omissions were the logarithms of 9769, 9781, 9787, 9771, 9783, and 9907. The Project used Vega’s reprint published in 1794 [79], where the omissions had been filled.

The logarithms of composite numbers not found in Wolfram’s table were obtained by the formula \( \ln AB = \ln A + \ln B \). Logarithms of primes greater than 10009 were computed as follows: using

\[
\argth(x) = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots \quad (3)
\]

and

\[
\argth(x) = \frac{1}{2} \ln \left( \frac{1 + x}{1 - x} \right) \quad (4)
\]

we have

\[
\argth \left( \frac{1}{2n^2 - 1} \right) = \frac{1}{2} \ln \left( \frac{1 + \frac{1}{2n^2 - 1}}{1 - \frac{1}{2n^2 - 1}} \right) = \frac{1}{2} \ln \left( \frac{n^2}{n^2 - 1} \right) = \ln n - \frac{\ln(n^2 - 1)}{2} = \ln n - \frac{\ln(n - 1) + \ln(n + 1)}{2} \quad (5)
\]

From that it follows that
\[
\ln n = \frac{\ln(n-1) + \ln(n+1)}{2} + \argth\left(\frac{1}{2n^2 - 1}\right) \tag{7}
\]

\[
= \frac{\ln(n-1) + \ln(n+1)}{2} + \frac{1}{2n^2 - 1} + \frac{1}{3 \cdot (2n^2 - 1)^3} + \frac{1}{5 \cdot (2n^2 - 1)^5} + \cdots \tag{8}
\]

Hence, \(\ln n\), where \(n\) is prime, was obtained from the logarithms of two even numbers, as well as a series. This series converges very rapidly, and when \(n > 10009\), only the first term was used, all the others being very small.

These logarithms were computed to 20 places and differencing tests were applied to ensure that the 20th place was correct.\(^5\) These 20-place values were then rounded to 16 places and further subjected to differencing tests. The 20-place worksheet values were also used to derive the last two volumes, by subtraction of \(4\ln 10\), and the values were then rounded to 16 places.

The first 20 decimal places of all of Wolfram’s logarithms were compared with those found here, and several errors were found in Wolfram’s table. Most, if not all, of these errors were actually typos, rather than genuine computation errors.\(^6\)

In order to ensure the correctness of the table, the 20-place values were added by groups of ten, and comparisons were made between the two volumes, as the difference of two corresponding sums had to be \(40\ln 10\). Moreover, the values from 10000 to 20000 and from 30000 to 100000 were compared with Thompson’s Logarithmetica Britannica [73], by adding the values in groups of ten and multiplying them by \(\ln 10\). Not a single error was found in either table. (The logarithms of integers from 20000 to 30000 had not yet been published by Thompson, whose work was only completed in 1952.)

In addition to the description of the construction of the table, Lowan also describes interpolation methods. For direct interpolations of logarithms, one might make use of Everett’s formula, which is (with the original notations)

\[
u_p = pu_1 + qu_0 + \frac{1}{6}p(p^2 - 1)d^2u_1 + \frac{1}{6}q(q^2 - 1)d^2u_0
\]

where \(p\) is the decimal part of the number, \(q = 1 - p\), \(u_0 = \ln x_0\), \(u_1 = \ln(x_0 + 1)\), \(u_p = \ln(x_0 + p)\) and \(d^2u_0\) and \(d^2u_1\) are the second central differences corresponding to \(u_0\) and \(u_1\) respectively. In order to use this formula, it is therefore necessary to compute the second central differences from the values in the table, since the original table does not give them directly. We do not go into the details of Everett’s formula here, and we direct the reader to our introduction to the reconstruction of Thompson’s table of logarithms [59].

On the other hand, it can often be dispensed with Everett’s formula and in most cases, finding the logarithm of a decimal number can be done by using the development of \(\ln(1 + x)\) and dividing the decimal number by its integral part. For instance, in order

\(^5\)No details on these tests are given, but we can expect occasional errors of one unit in the 20th place.

\(^6\)A table of erroneous logarithms is given in the original introduction, but we do not reproduce it here. The faulty values are the logarithms of 829, 1099, 1409, 1937, 1938, 2093, 3571, 4757, 6343, 7853, 8023, 8837, and 9623.
to compute $\ln 1.23456$, one can write

$$\ln 1.23456 = \ln \left( \frac{12345.6}{10000} \right)$$

$$= \ln \left( \frac{12345.6}{12345} \right) + \ln 12345 - \ln 10000$$

$$= \ln(1.0000486\ldots) + \ln 12345 - \ln 10000$$

and use the tabulated values of $\ln 10000$ and $\ln 12345$, and the series development for $\ln 1.0000486\ldots$

For inverse interpolation, the simplest method is by the linear interpolation formula

$$x = x_0 + \frac{\ln x - \ln x_0}{\ln(x_0 + 1) - \ln x_0}.$$

By linear inverse interpolation, it is possible to obtain ten significant digits in the argument. The introduction to the original volumes gives hints for more accurate methods of inverse interpolations, and sometimes it may be useful to make use of the table of exponentials [42]. But in any case, one should be aware that the number of meaningful figures in the argument is about the same as in the logarithms, so that for instance if a logarithm is given to 10 decimal places, this will often mean that there is an uncertainty of possibly $0.5 \cdot 10^{-10}$ and this uncertainly will be transferred to the argument. It is therefore usually useless to try to obtain more significant places in the arguments than there are significant places in the logarithms, except if the logarithm is considered to be exact, in which case different methods may be more appropriate anyway.

### 3.3 Notes on the layout

The layout of the tables is in general quite straightforward. There is just one idiosyncrasy. In volume 3 of the original tables, the logarithms up to 0.9999 are negative and the signs are given every five values. But when the integer part changes, the original table gives the absolute value of the logarithm, and not its real value. One has to assume that the sign should be taken from the previous full value given. This may be confusing. For instance, $\ln 0.0004$ is given as 7.82404\ldots, but it is really $-7.82404\ldots$. We have respected this peculiarity. There is only one such case where the original value is given with its sign, namely for $\ln 0.0498$, but for reasons of consistency, we have also used the absolute value in our reconstruction.

Interestingly, the original volume 4 that we had in hands uses two different types of paper, one light, the other darker, and a note printed at the beginning of the volume reads “Because of a paper shortage caused by the war, it was necessary to use two types of paper for this volume.”
Figure 1: An excerpt of Wolfram’s table as published by Schulze [68], with the six missing values added later.
Figure 2: An excerpt of Wolfram’s table as published by Vega [79].
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Figure 3: An excerpt of the Project’s table of natural logarithms (volume 1) [43].
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<th>$x$</th>
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Figure 4: An excerpt of the Project’s table of natural logarithms (volume 4) [43].
MATHEMATICAL TABLES

The tables listed below (with the exception of MT15) were prepared by the Project for the Computation of Mathematical Tables conducted by the Federal Works Agency, Work Projects Administration for the city of New York, under the sponsorship of and made available through the National Bureau of Standards. They are of special interest to physicists, engineers, chemists, biologists, mathematicians, computers, and others engaged in scientific and technical work.

The tables have been arranged in the following groups: Those obtainable from: (1) the Superintendent of Documents, Government Printing Office, (2) Columbia University Press, and (3) those available elsewhere.

(1) TABLES OBTAINABLE FROM THE SUPERINTENDENT OF DOCUMENTS

MT1. Table of the first ten powers of the integers from 1 to 1,000. $3.00.
MT2. Tables of the exponential function $e^x$. $3.00.
MT3. Tables of circular and hyperbolic sines and cosines for radian arguments. $2.50.
MT4. Tables of sines and cosines for radian arguments. $2.00.
MT5. Tables of sine, cosine, and exponential integrals, volume I. $2.75.
MT6. Tables of sine, cosine, and exponential integrals, volume II. $2.00.
MT7. Table of natural logarithms, volume I. $3.00.
MT8. Tables of probability functions, volume I. $2.00.
MT9. Table of natural logarithms, volume II. $3.00.
MT10. Table of natural logarithms, volume III. $3.00.
MT11. Tables of the moments of inertia and section moduli of ordinary angles, channels, and bulb angles with certain plate combinations. $2.00.
MT12. Table of natural logarithms, volume IV. $3.00.
MT13. Table of sine and cosine integrals for arguments from 10 to 100. $2.00.
MT15. The hypergeometric and Legendre functions with applications to integral equations of potential theory. Chester Snow, National Bureau of Standards. $1.50.
MT16. Table of arc tan $z$. $2.00.
MT17. Miscellaneous physical tables: Planck’s radiation functions, and electronic function. $1.50.
MT18. Table of the zeros of the Legendre polynomials of order 1 — 16 and the weight coefficients for Gauss’ mechanical quadrature formula. A. N. Lowan, N. Davics, and A. Levenson. 25c.
MT19. On the function $H(m,a,x) = \exp \left( -x \right) F \left( m + 1 - i\alpha, 2m + 2; i\alpha \right)$. With table of the confluent hypergeometric function and its first derivative. A. N. Lowan and W. Horenstein. 25c.
MT20. Table of integrals $\int_0^\infty J_0(t)dt$ and $\int_0^\infty Y_0(t)dt$. Arnold N. Lowan and Milton Abramowitz. 25c.
MT21. Table of $J_\nu(x) = \int_0^\infty J_\nu(t) \frac{dt}{t}$ and related functions. Arnold N. Lowan, G. Blanch, and M. Abramowitz. 25c.
MT22. Table of coefficients in numerical integration formulæ. A. N. Lowan and Herbert Salszer.
MT26. A short table of the first five zeros of the transcendental equation $J_\nu(x)Y_\nu(x) - J_\nu(2x)Y_\nu(2x) = 0$. A. N. Lowan and A. Hillman. Reprinted from Journal of Mathematics and Physics, December 1948. 2 p. 25c.

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MT27. Table of coefficients for inverse interpolation with central differences.
Reprinted from Journal of Mathematics and Physics, December 1948. 16 p. 25c.

MT28. Table of \( f_n(x) = \left( \frac{d^n}{dx^n} \right)^n J_0(x) \) ......... The Mathematical Tables Project
Reprinted from Journal of Mathematics and Physics, February 1944. 16 p. 25c.

MT29. Table of coefficients for inverse interpolation with advancing differences.
Reprinted from Journal of Mathematics and Physics, May 1944. 28 p. 25c.

MT30. A new formula for inverse interpolation ............... H. E. Salzer
Reprinted from Bulletin of the American Mathematical Society, August 1944. 4 p. 25c.

MT31. Coefficients for interpolation within a square grid in the complex plane.
A. N. Lowan and H. E. Salzer
Reprinted from Journal of Mathematics and Physics, August 1944. 11 p. 25c.

MT32. Table of coefficients for differences in terms of the derivatives. H. E. Salzer
Reprinted from Journal of Mathematics and Physics, November 1944. 4 p. 25c.

MT33. Table of coefficients for numerical integration without differences.
A. N. Lowan and H. E. Salzer
Reprinted from Journal of Mathematics and Physics, February 1945. 21 p. 25c.

MT34. Inverse interpolation for eight-, nine-, ten-, and eleven-point direct interpolation
Reprinted from Journal of Mathematics and Physics, May 1945. 4 p. 25c.

MT35. Table of coefficients for double quadrature without differences, for integrating second order differential equations
Reprinted from Journal of Mathematics and Physics, November 1945. 6 p. 25c.

MT36. Formulas for direct and inverse interpolation of a complex function tabulated along equidistant circular arcs
Reprinted from Journal of Mathematics and Physics, November 1945. 8 p. 25c.

Coordinate conversion tables.
Published as Technical Manual TM 4-238 of the War Department. March 25, 1943. 388 p. 5½ by 8½ in. 42c.

Hydraulic tables (2d ed.).
Published by the Corps of Engineers, War Department. (1944) 565 p. Blue imitation leather flexible cover, 4½ by 8½ in. $1.50.

(2) TABLES OBTAINABLE FROM THE COLUMBIA UNIVERSITY PRESS

The following four tables can be obtained from the Columbia University Press, Morningside Heights, New York 27, N. Y.

Table of reciprocals of the integers from 100,000 through 200,000.
(1948) 201 p. Buckram cover. $4.00.

Table of Bessel functions \( J_0(z) \) and \( J_1(z) \) for complex arguments.

Table of circular and hyperbolic tangents and cotangents for radian arguments.
(1948) 410 p. Buckram cover. $5.00.

Tables of Lagrangian interpolation coefficients.
(1944) 392 p. Buckram cover. $5.00.

Table of arc sin \( z \).
(1940) 121 p. Buckram cover. $3.50.

Tables of associated Legendre functions.
(1948) 262 p. Buckram cover. $5.00.

(3) TABLES AVAILABLE ELSEWHERE

The eight tables listed below can be consulted in libraries maintaining a file of mathematical and technical journals. No reprints of them are obtainable from the Bureau.

On the computation of second differences of the \( Si(z) \), \( Ei(z) \), and \( Ci(z) \) functions.
Arnold N. Lowan

On the distribution of errors in the \( n \)th tabular differences.
Arnold N. Lowan and Jack Laderman

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Figure 6: The list of mathematical tables available from the National Bureau of Standards in 1948 (2/3) [54].
| Note on the computation of the differences of the $\text{Si}(x)$, $\text{Ci}(x)$, $\text{Ei}(x)$ and $-\text{Ei}(-x)$ functions | Milton Abramowitz | Bulletin of the American Mathematical Society, vol. 49, No. 4, pp. 392-393 (April 1949). |
| Roots of $\sin z = x$ | A. P. Hillman and H. E. Salzer | Gives the first 10 nonzero roots of $\sin z = x$ in the first quadrant to six decimal places. |
| Roots of $\sin z = x$, where $z = x + iy$. Philosophical Magazine, Series 7, vol. XXXIV, p. 575 (August 1949). |

Figure 7: The list of mathematical tables available from the National Bureau of Standards in 1948 (3/3) [54].
BRITISH ASSOCIATION MATHEMATICAL TABLES


II Emden Functions, being Solutions of Emden’s Equation together with Certain Associated Functions. 1932

III Minimum Decompositions into Fifth Powers. Prepared by L. E. Dickson. 1933


V Factor Table, giving the Complete Decomposition of all Numbers less than 100,000. Prepared independently by J. Peters, A. Lodge and E. J. Ternouth, E. Gifford. 1935


VIII Number-divisor Tables. Designed and in part prepared by J. W. L. Glaisher. 1940. Reprinted 1966


PART-VOLUME A Legendre Polynomials. Prepared by L. J. Comrie. 1946

B The Airy Integral, giving Tables of Solutions of the Differential Equation $y'' = xy$ Prepared by J. C. P. Miller. 1946

(Auxiliary tables I and II are included with Part-Volume B.)

AUXILIARY TABLES

Number I Coefficients in the Modified Everett Interpolation Formula. 1946

II Table for Interpolation with Reduced Derivatives. Coefficients for Function and for First Derivative. 1946

Note. In July 1948 the Royal Society assumed responsibility for the work on mathematical tabulation formerly undertaken by the British Association.
References

The following list covers the most important references related to the Mathematical Tables Project’s table. Not all items of this list are mentioned in the text, and the sources which have not been seen are marked so. We have added notes about the contents of the articles in certain cases.


\[Note on the titles of the works:\] Original titles come with many idiosyncrasies and features (line splitting, size, fonts, etc.) which can often not be reproduced in a list of references. It has therefore seemed pointless to capitalize works according to conventions which not only have no relation with the original work, but also do not restore the title entirely. In the following list of references, most title words (except in German) will therefore be left uncapitalized. The names of the authors have also been homogenized and initials expanded, as much as possible.

The reader should keep in mind that this list is not meant as a facsimile of the original works. The original style information could no doubt have been added as a note, but we have not done it here.

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[38] Jacob Philip Kulik. *Canon logarithmorum naturalium in 48 notis decimalibus pro omnibus numeris inter 1 et 11000 denuo in computum vocatus*. 1826. [perhaps not published, no copy known]
[39] Johann Heinrich Lambert. *Zusätze zu den Logarithmischen und Trigonometrischen Tabellen zur Erleichterung und Abkürzung der bey Anwendung der Mathematik vorfallenden Berechnungen.* Berlin: Haude und Spener, 1770. [the table of factors was reconstructed in [63]; [40] is a Latin translation of this book]

[40] Johann Heinrich Lambert and Anton Felkel. *Supplementa tabularum logarithmicarum et trigonometricarum.* Lisbon, 1798. [Latin translation of [39]; the table of factors was reconstructed in [62]]


[42] Arnold Noah Lowan, editor. *Tables of the exponential function* e^x. New York, 1939. [we have only seen the 1961 edition; reconstructed in [65]]

[43] Arnold Noah Lowan, editor. *Table of natural logarithms.* New York: Federal Works Agency, Work Projects Administration, 1941. [4 volumes, some copies seem to have been reproduced in a reduced size]


[52] Jeffrey Charles Percy Miller. The mathematical tables project VI. *The Mathematical Gazette*, 33(303):70–72, February 1949. [reviews the table of natural logarithms, as well as three other volumes; only the last lines of page 72 actually describe the table of natural logarithms]


[57] Denis Roegel. A reconstruction of Edward Sang’s table of logarithms (1871). Technical report, LORIA, Nancy, 2010. [This is a reconstruction of [67].]

[58] Denis Roegel. A reconstruction of the tables of Napier’s *descriptio* (1614). Technical report, LORIA, Nancy, 2010. [This is a recalculation of the tables of [53].]

[59] Denis Roegel. A reconstruction of the tables of Thompson’s *Logarithmetica Britannica* (1952). Technical report, LORIA, Nancy, 2010. [This is a reconstruction of the tables in [73].]


[62] Denis Roegel. A reconstruction of Lambert and Felkel’s table of factors (1798). Technical report, LORIA, Nancy, 2011. [This is a reconstruction of the table of factors in [40].]

[63] Denis Roegel. A reconstruction of Lambert’s table of factors (1770). Technical report, LORIA, Nancy, 2011. [This is a reconstruction of the table of factors in [39].]

[64] Denis Roegel. A reconstruction of the table of factors of Peters, Lodge, Ternouth, and Gifford (1935). Technical report, LORIA, Nancy, 2011. [This is a recalculation of the tables of [56].]

[65] Denis Roegel. A reconstruction of the Mathematical Tables Project’s tables of the exponential function $e^x$ (1939). Technical report, LORIA, Nancy, 2017. [This is a reconstruction of the tables in [42].]

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[70] John Speidell. *New logarithmes: the first invention whereof, was, by the honourable Lo. John Nepair, Baron of Marchiston, and printed at Edinburg in Scotland, anno 1614*, in whose use was and is required the knowledge of algebraicall addition and subtraction, according to + and -. 1619. [partially reproduced in [45]]


Note

The four volumes of reconstructed tables are given in four separate documents.