A reconstruction of Kulik's "Magnus Canon Divisorum" (ca. 1825–1863) Introduction

Denis Roegel

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The chief objection to Kulik's scheme is, of course, the extreme difficulty of comparing it with other tables, where a different notation has been used.
D. N. Lehmer [73, p. IX]

1 Jakob Philipp Kulik (1793–1863)

Jakob Philipp Kulik was born in 1793 in Lemberg (now Lviv in Ukraine) which was then part of the Austrian Empire. He first studied philosophy, then law, and finally mathematics. In 1814, he applied for a position of professor of elementary mathematics in Olomouc and in 1816, he became professor of physics at the Lyceum in Graz. In 1822, he was given the title of doctor for a thesis on the rainbow. Four year later, in 1826, he became professor of mathematics at the University of Prague, where he remained till his death in 1863. He is buried in the Vyšehrad Cemetery.

Kulik devoted a large part of his work to the construction of mathematical tables, so that when Kulik died, the mathematician Studnička wrote of him: *Er hat aufgehört zu rechnen und zu leben.*[1] [93, p. 310]

Kulik published his *Handbuch mathematischer Tafeln*, a collection of mathematical tables in 1824 [52, 6]. He worked on other tables, sometimes only publishing them many years later. This was the case for his table of squares and cubes which was computed in 1828, but only published in 1848.

His first collection of tables already contained tables of primes and factors, but he devoted his next book published in 1825 entirely to this topic [51]. A second part of the *Handbuch* with the title *Vollständige Sammlung mathematisch-physikalischer Tafeln* was announced, but it was apparently never published.

Other tables followed, in particular conversion tables in 1833 [54], tables of squares and cubes in 1848 [59], tables of hyperbolic sectors and elliptic arcs [61] and a table of multiplication [60], both in 1851. Kulik also published a calculus textbook in 1831 [53], with a second edition in two volumes in 1843 and 1844, a textbook on mechanics in 1846 [58] as well as perpetual calendars.

But Kulik's most important achievement in tablemaking was his table of factors for all integers up to 100 millions. Before analyzing his table in depth, we will review the most important tables of factors which had been computed prior to Kulik.

2 Factor tables before Kulik's tables

Good summaries of the history of factor tables have been published by Lehmer [73], Henderson (in Peters' table [88]), Palamà [85], Depman [32] and others. A survey from 1657 to 1817 was published recently by Bullynck [15]. Here, we sketch only the main developments.

Factor tables started to be developped in the 17th century, especially when Brancker computed a table of smallest factors up to 100000 in 1668 [96, 97, 76]. In 1770, Lambert published a table of factors to 102000 [66]. He extended the table to 102000, because

[1]"He stopped calculating and living."

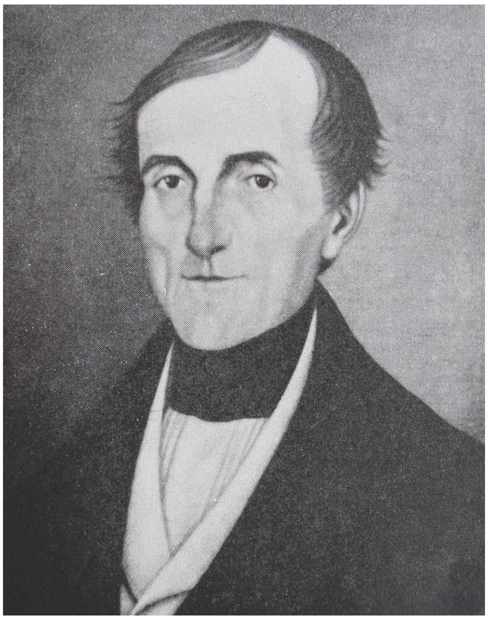


Figure 1: Jakob Philipp Kulik (1793–1863). (picture from [95] who took it from [32])

two pages of his table covered a range of 3000 and $102000 = 3000 \times 34$. In 1772, Marci published a list of primes to 400000, and in 1776 Felkel published a table giving the complete decompositions of all integers not divisible by 2, 3, or 5 up to 408000 [34, 36, 37].

Some tables gave the smallest factor on greater ranges, but they were not always published. This is for instance the case of Schenmark's table (ca. 1780) [123].

In 1811, Chernac was the first to publish complete (and clear) decompositions of all integers not divisible by 2, 3, or 5 up to a million [23], albeit at the cost of a bulky volume. Then, between 1814 and 1817, Burckhardt published his tables giving the smallest factor for the first three millions [18]. He had also computed the table further, but the fourth to ninth millions were only computed later by Dase [28, 29, 30] and Glaisher [39, 40, 41].

Crelle had also computed tables for the fourth, fifth and sixth millions, probably in the 1830s, and these tables were deposited in the Archives of the Academy of sciences in Berlin [24]. But they were never published.

3 Kulik's first tables of factors (1824–1825)

Kulik published his first table of primes and factors in 1824 [52, 6]. An extension of this table was published in 1825 [51]. This extension gave the smallest factors, and sometimes more, for all integers not divisible by 2, 3, 5, or 11, from 1 to one million. In a number of cases, Kulik used symbols either in order to save space, or in order to give a greater number of factors. This table was probably computed anew and not based on an earlier table table such as Chernac's [23].

In the preface of his table, Kulik wrote moreover that he had constructed a manuscript table going to $30030000 = 300 \times 77 \times 1300$, in which the pages contained 77 columns and 80 lines (and therefore a range of 300), and that this table covered 1300 pages.

According to Kulik, the 1300 printed pages bore the factors 7 and 11, but also the factors 13, 17, 19, and 23. It is however not clear how this was achieved, as there are in fact $13 \times 17 \times 19 \times 23 = 96577$ different combinations and each of the 1300 pages would in fact be different. It is also hard to imagine that the pages were printed in layers, namely first the multiples of 13, then those of 17, and so on, because some of the factors should not be printed if there is a smaller factor at that position.[2]

Kulik also wrote that the factors 29 to 503 were inserted with matrices, and that the larger factors were obtained by the "multiple method" [51, p. V] [9]. Both of these methods were used in the *Magnus Canon Divisorum* described in the next section.

In any case, it is likely that Kulik already had a table extending to 30 millions by 1825, but we cannot be sure that this table was complete.

It is particularly interesting that the matrices used in the *Magnus Canon Divisorum* contain at least one older sheet (figure 2) which possibly goes back to this first table of factors.[3]

It is likely that Kulik discarded his first table when he started work on the *Magnus Canon Divisorum*.

[2]The only reasonable explanation is that that table did not show the smallest factors, but several factors, and that they were printed. The four factors 13 (b), 17 (c), 19 (d), and 23 (e) were certainly only printed for numbers not multiples of 7 and 11, and at places such that they would not cover themselves. This assumes that a cell was divided in four areas, one receiving the symbol for 13, another the symbol for 17, still another the symbol for 19, and a fourth one the symbol for 23. We can for instance imagine that each cell had a virtual 'bcde' content, and that these symbols could have been printed independently. Although still complex, this is possible by printing first the symbols for 13, requiring 13 different sets of positions, then printing the symbols for 17, requiring 17 different sets of positions, and so on. If this is the method used by Kulik, it does of course require a careful positioning of the printing page. On the other hand, the preliminary page shown in figure 2 does not seem adapted to such a process, as the cells can hardly accomodate more than two symbols.

[3]Other such sheets may exist, but we have not gone through all matrices.

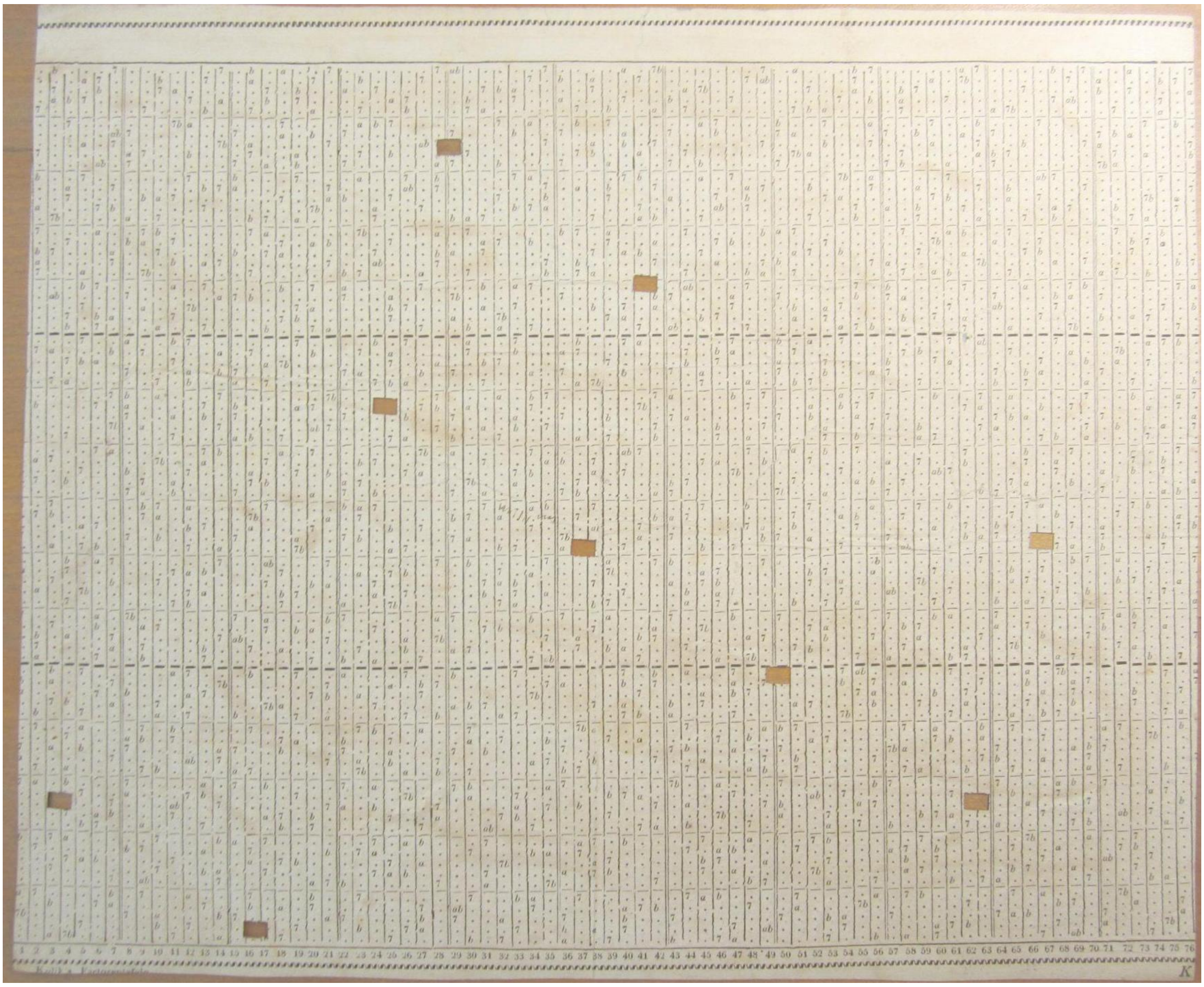


Figure 2: The back of one of the pages for the 643 (dd) matrix of the *Magnus Canon Divisorum* contains a preliminary version of Kulik's preprinted pages which is possibly from 1825. Here, not only "a" and 7 were preprinted, but also "b," "ab" and "7b." The meaning of the factors "a," "b" is the same as in the "Magnus Canon Divisorum" and the letters are located as on pages $13k$ (for instance page 4212, see figure 6). In certain cases, Kulik gave more than one factor. This is not the solution that he eventually adopted. Note also that at the bottom left there is the inscription "Kuliks Factorentafeln" and a "K" at the lower right. (AÖAW, Nachlass Kulik, reproduced by the author)

4 Kulik's *Magnus Canon Divisorum* (ca. 1825–1863)

It is not known exactly when Kulik started to work on his table of factors to 100 millions, but it may well be that this project matured in the mid-1820s.[4]

Given that Kulik had at least a partial table to 30 millions, the first step was probably to copy the old table on the new printed forms. This may have taken place in the 1830s or around 1840. At that time, Kulik was apparently particularly interested in factoring methods and in 1841 he described a method based on tables of squares [55] and another method for determining the number of primes smaller than a given number [56].

Interestingly, Kulik seems mentioned in a letter written on 23 November 1847 by Carl Gustav Jacobi to Peter Andreas Hansen about some calculation work which might be given to Dase. Jacobi added that "tables of factors for the fourth, fifth, sixth millions lie here unprinted" [50, p. 440], referring to Crelle's unpublished table of factors [24]. Then Jacobi added that "a Mr. K. even proposed to me to compute them up to 25 millions, without compensation, provided they are printed."

In 1856, Kulik mentioned his tables to 100 millions as being mostly complete [62]. In 1860, he wrote that "[he has] a manuscript which contains the continuation of Burckhardt's table from 3 millions until 100 millions on 4212 dense folio pages"[5] [63, p. 25]. Kulik offered to copy part of the table for anyone interested. This statement, as observed by Nový, created the impression that Kulik's table was complete [84, pp. 328–329].

From the above, it is most likely that Kulik worked on his table from around 1825 to 1863, and it would not make much sense to separate the work on the final form of the table from the work on the table to 30 millions.

The table constructed by Kulik aims at giving the smallest factor of all numbers not divisible by 2, 3, and 5, up to 100 millions. Contrary to what he did in his first table to a million, Kulik did not exclude multiples of 11. Instead, as we will see, he included them at fixed positions, so that he wouldn't have to worry about them. The full title of the table, as given in the first volume, is *Magnus Canon Divisorum pro omnibus numeris per 2, 3 et 5 non divisibilibus, et numerorum primorum interjacentium ad millies centena millia accuratius ad 100330201 usque* (figure 3).

Kulik did not start his table at 1, but he started where Burckhardt finished. Burckhardt's table gave the smallest factor of the first three millions, stopping at 3035999. Kulik started his table at 3033001. We do not know the reason for this overlap.

4.1 The encoding of prime numbers

In order to save space, Kulik used a notation for the primes, so that all the primes appearing as factors would be represented by at most two characters. Kulik goes up to 100 330 200 and he therefore needed to name all prime factors up to 10009. Symbols for primes had been used before Kulik, in particular by Felkel. After Kulik, we can mention

[4]Kulik's tables are part of his *Nachlass* deposited at the Archives of the Academy of sciences in Vienna (Archiv der Österreichischen Akademie der Wissenschaften, AÖAW). There are only very few detailed descriptions of Kulik's tables. The most useful descriptions are those of Lehmer [73, 74], Depman [32], Nový [83, 84], and Porubský [93].

[5]"Ich besitze ein Manuskript, welches die Fortsetzung der Burckhardtschen Tafel von 3 Millionen an bis 100 Millionen auf 4212 eng geschriebenen Folioseiten enthält."

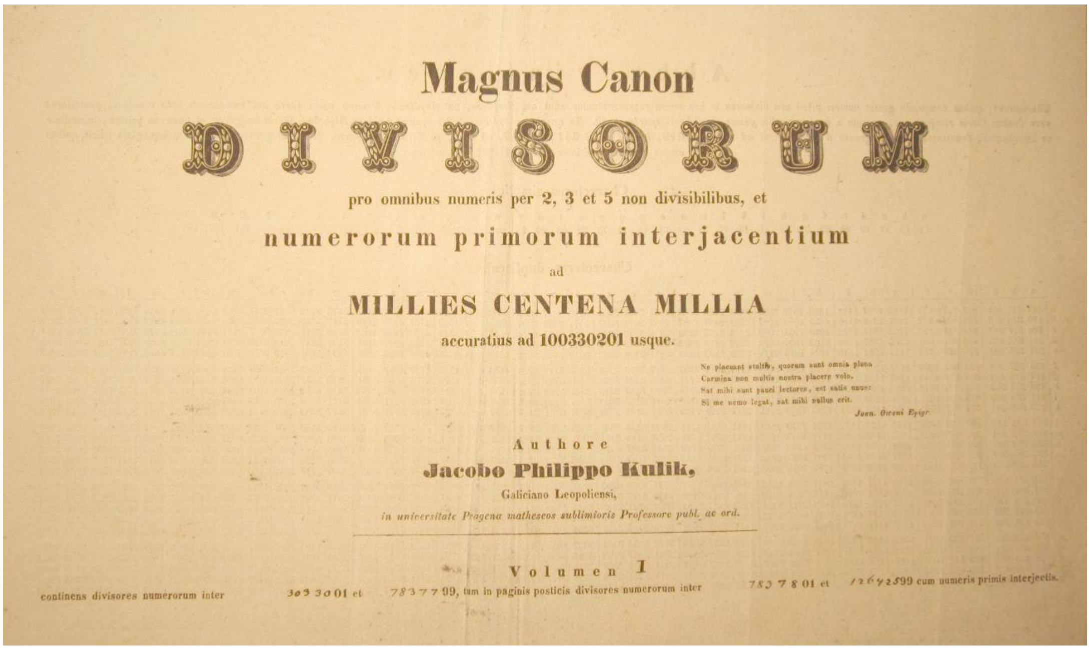


Figure 3: The first page of the first volume. This is the only cover page, although it is obvious that one was planned for each volume. The volume number and the range (except the last two digits 01 and 99) were not printed, but added by hand. Note that the extent of the entire title is not 100330201, as indicated, but 100330199. (AÖAW, Nachlass Kulik, reproduced by the author)

Lebesgue [68] who used in fact a system similar to one of the systems used by Kulik in 1825.

In Kulik's encoding, all primes from 7 to 163 are represented by a unique character: 7 → 7, a → 11, b → 13, c → 17, d → 19, e → 23, f → 29, g → 31, h → 37, i → 41, k → 43, l → 47, m → 53, n → 59, o → 61, p → 67, q → 71, r → 73, s → 79, t → 83, u → 89, v → 97, w → 101, x → 103, y → 107, z → 109, 1 → 113, 2 → 127, 3 → 131, 4 → 137, 5 → 139, 6 → 149, 8 → 151, 9 → 157, 0 → 163. As observed by Lehmer [73, p. x], the letter "o" and the digit 0 are distinguished in the manuscript by a stroke through the digit (see for instance on figure 27). We can also note that at least 7, 11, and 13 were represented as in the manuscript table to 30 millions, assuming that the preliminary page found corresponds to this table.

All primes after 163 are represented with two characters. The primes are given in an auxiliary table at the back of the title page of the first volume. This table however stops at 8059.

In our reconstruction (see the end of this document), we continued the table up to 10091, and we split it in three parts, with 16 columns of primes in each table, ending with 10091 which is encoded by 'zr'.

Since Kulik's plans were to reach 100 millions right at the beginning, the limit of 8059 appears insufficient, but Kulik had probably planned to add a title page and a list of symbols to each volume. In that case, an extension of the list would only have been needed after $8069^2 = 65108761$, that is, only in volumes 5 to 8. The list of symbols would then certainly have been adapted. But Kulik did not complete his table, and did not insert factors larger than 8059.

4.2 The structure of the table

Kulik's table spanned eight volumes and 4212 pages, but the second volume is lost.[6] The overall structure would have been the one given in table 1.[7] The sizes of the pages are slightly larger than an A3 page.

The two sequences in each volume are interleaved, the front and back pages forming two different sequences. The pages of the first volume are actually numbered 1, 209, 2, 210, ..., 208, 416. Those of the second volume were certainly numbered 417, 638, 418, 639, ..., 637, 858. And so on. Some of the sequences have anomalies, and the continuation of the first sequence in volume 8 is for instance found in volume 7. Whether this served a real purpose or not is not clear.

[6] This volume was already missing when Lehmer worked on the first volume of the tables. In his table of factors, Lehmer writes that there are six volumes, but this is likely to be a typo or a mistake copied from Petzval's account, as the correspondence between Lehmer and the Academy of sciences mentions seven volumes [73]. About the loss of the second volume, see the (wrong) anecdote cited by Ribenboim [98, pp. 233-234].

[7] Lehmer gave a description of the extent of the volumes in 1914, but his list contained several errors [74, p. xi]. These errors were later corrected by Joffe [48]. At least one of Lehmer's errors comes from the spine of volume 4 where Kulik writes the incorrect end value 35626799. On the spine of volume 8, we also find the incorrect end value 100330201, which has been taken over by several authors, including Nový [84, p. 332]. We should however observe that Kulik himself put this incorrect limit on the main title of his tables.

volume	side	first page	last page	first number	last number
1	front	1	208	3 033 001	7 837 799
	back	209	416	7 837 801	12 642 599
2	front	417	637	12 642 601	17 747 699
	back	638	858	17 747 701	22 852 799
3	front	859	1135	22 852 801	29 251 499
	back	1964	2240	48 378 301	54 776 999
4	front	1136	1411	29 251 501	35 627 099
	back	2241	2516	54 777 001	61 152 599
5	front	1412	1687	35 627 101	42 002 699
	back	2517	2792	61 152 601	67 528 199
6	front	1688	1963	42 002 701	48 378 299
	back	2793	3068	67 528 201	73 903 799
7	front	3069	3353	73 903 801	80 487 299
	back	3641	3925	87 117 001	93 700 499
8	front	3354	3640	80 487 301	87 116 999
	back	3926	4212	93 700 501	100 330 199

Table 1: Structure of the eight volumes of Kulik's tables. The structure of volume 2 is extrapolated, the only uncertainty being that the front and back sequences might have to be swapped, although this is unlikely.

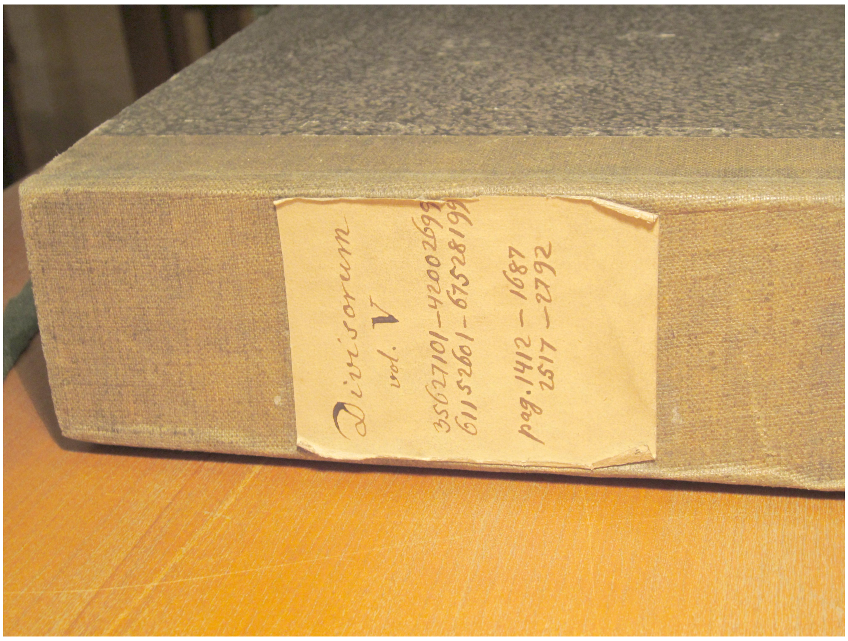


Figure 4: The spine of volume 5. (AÖAW, Nachlass Kulik, reproduced by the author)

It should also be noted that the number of pages is not the same in all volumes. There are 208 sheets in the first volume, 221 in the second one, then 277, 276, 276, 276, 285, and finally 287 sheets in the eighth volume.

The sheets in volume 1 are bound. Those in the six remaining volumes are not.

The reason why Kulik chose to interleave the pages becomes clear when we consider the methods used to fill the tables. With his first method, it is useful to put one page next to each other, and this would not have been possible if the page numbers ran continuously.

Our reconstruction follows the above layout exactly, with the correct interleaving of front and back sequences.

4.3 The layout of the table

Burckhardt's tables, as well as Dase's and Glaisher's, use a grid with 30 columns and 80 lines. Such a grid covers an interval of 9000 integers. The layout used by Crelle in his unpublished tables of the 4th, 5th, and 6th millions is not known, but was possibly the same.

Kulik, instead, used almost the same layout, but with 77 columns and 80 lines, hence covering 23100 integers. The advantage of 77 columns over 30 is that the factors 7 and 11 can be preprinted, as they do always occur in the same positions. Kulik appears to have used this scheme in his manuscript table to 30 millions which he mentioned in 1825.

Each column covers a range of 300 integers, but only those which are not divisible by 2, 3 and 5 are marked. A number n is split in three parts, as $n = a \times 10^4 + b \times 100 + c$, with $b < 100$ and $c < 100$. The value of a is written on the first line of each page, first at the upper left, then whenever it changes.[8] On the first page of Kulik's table, which covers the range 3033001 to 3056100, the value $a = 303$ is written at the upper left. On the same line we find the subsequent values 304 and 305. The first cell of the first page is at column 30 (the hundreds) and line 01 (the units), which means that it corresponds to number 303.30.01.

If a number is sought with a number of hundreds not appearing at the top, this number must be sought in the middle or lower part of the table. Each page is divided in three parts by thick lines, but these lines do not exactly correspond to the three hundreds in each column, but appear at the end of a group of five lines. A little care must therefore be taken to locate a given value.

In addition, the factor 13 (b) is preprinted on a number of pages, but this requires 13 differents pages. Indeed, an examination of volumes 1, 3, 4, 5, and 6, reveals that most front pages have a number at the lower right, and this number is between 1 and 13. This does not appear to be the case for volumes 7 and 8, where some or possibly all pages have handwritten 'b's, and no numbers in the corner.[9] These numbers are printed cyclically, in decreasing order starting with 3: (3), (2), (1), 13, 12, 11, 10, ..., (1), 13, 12, etc. We have put some numbers between parentheses, as they do not, or not always, seem to be printed.[10] Page p bears the number $13 - (p + 9) \bmod 13$ and these numbers

[8] In the actual tables, this value sometimes contains errors. A note dated 16 April 1973 gives a list of such errors. This note is inserted in the envelope left by Lehmer.

[9] We have checked for instance pages 3069, 3353, 3354, 3640, 3641, 3925, 3926, and 4212.

[10] Page 1963, for instance, has printed 'b's, but the corresponding number 4 does not appear in the corner. The same applies to page 3068. Still, there may be a pattern of pages having printed 'b's and

therefore certainly identify the layout of the factors 13. These numbers appear on front and back pages.

no number in the corner.

Figure 5: The first page of Kulik's third volume. It is not clear why some column numbers are crossed out on the last line. (AÖAW, Nachlass Kulik, reproduced by the author)

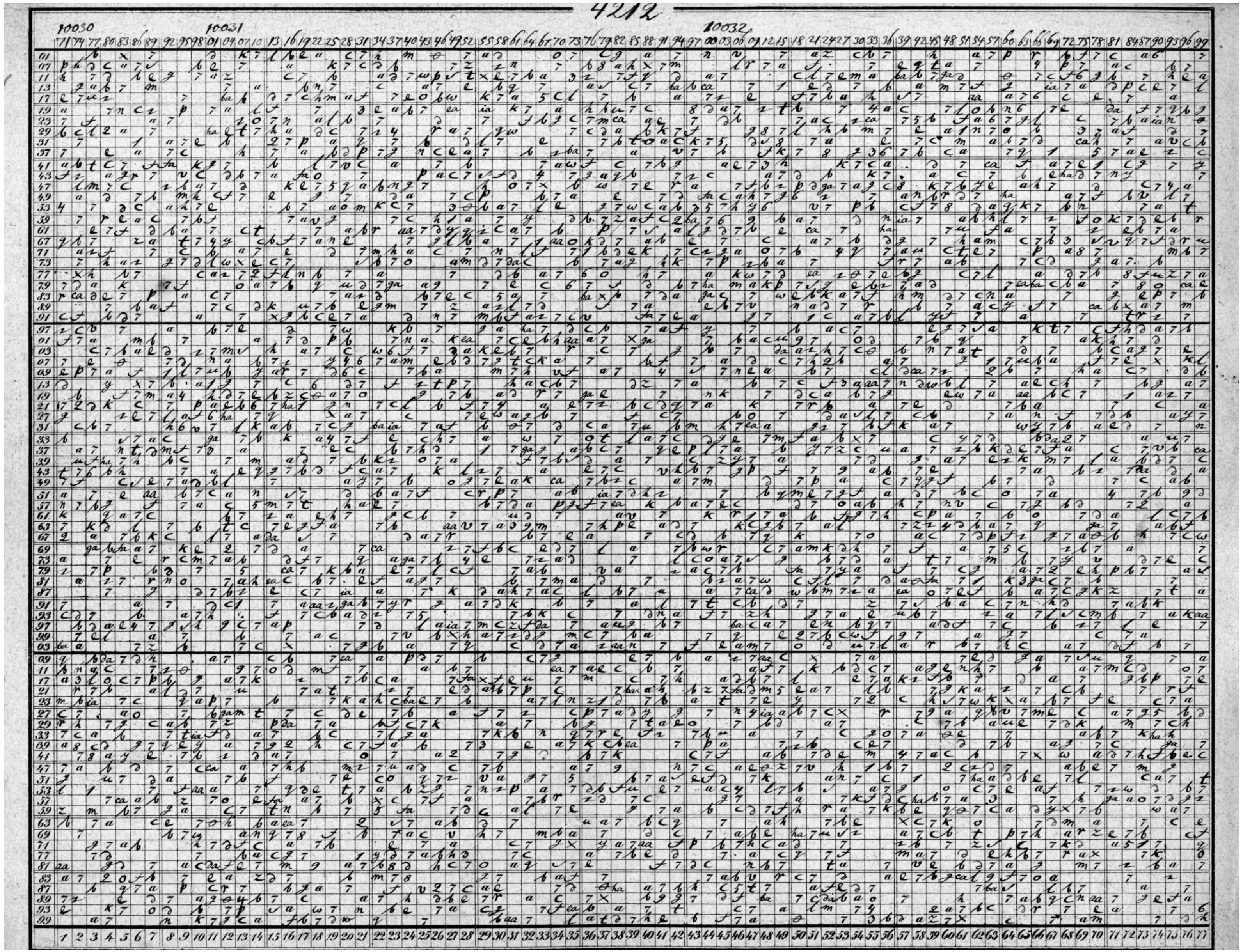


Figure 6: The last page of Kulik's table. (AÖAW, Nachlass Kulik, reproduced by the author)

5 Kulik's methods

Kulik's *Nachlass* at the Archives of the Academy of sciences in Vienna contains a number of papers related to the construction of his big table. Two different methods seem to have been employed, one for the primes up to 1000, and another for primes greater than 1000. Kulik's methods were first described by Lehmer [73, 74] and more recently by Nový [83, 84]. In addition, Kulik has obviously had his first table to 30 millions copied on the new sheets, and this did not require any new computations. This also makes it likely that the encoding of the primes was the same in 1825 and in the *Magnus Canon Divisorum*.

Kulik's two methods are really totally disjoint and they can be applied in any order. The only requirement is that the matrices be applied sequentially.

5.1 Matrices

The *Nachlass* contains a number of matrices[11] for the multiples of primes. The matrices for 37 (h), 41 (i), and for the primes 619 (ad) to 929 (ue) are extant.[12] It is likely that there were initially such matrices for all primes from 13 to 997. In fact we know that there were initially two volumes of matrices, and that only one is now found in the *Nachlass*. The missing volume probably contained the matrices 13 to 31, 43 to 617, and 937 to 997. The matrices 37 and 41 are probably no longer needed, and were used until the end of the table, but the other extant matrices were still needed when Kulik stopped working on the table.

As observed above, Kulik used the matrices 29 to 503 in his table to 30 millions. This means that these matrices were only needed in the new table after 30 millions, as the first 30 millions could be copied from the first table.

The matrix for n covers n columns, and one or more pages of the original grids. Some of the grids are printed like the final grids (with "a" and "7", and sometimes "b"), and others have no characters preprinted. There are also two different paper colors.

The matrix for 37 covers for instance only 37 columns and fits on one sheet. But the matrix for 619 covers 619 columns, and therefore eight pages of 77 columns and three additional columns. These three columns were glued to the right of the eighth page. The pages used in the 619 matrix were of alternate colors, but Kulik did not always alternate the colors. The last two pages of matrix for 683 (ld), for instance, have the same color.

The holes all have the same rectangular shape and were obviously punched with a special tool.

This method was probably only used up to 997, as it was obviously more and more cumbersome and errorprone.[13]

[11] These matrices were called "stencils" by Lehmer.

[12] However, some pages seem to be missing, in particular for 643 (dd) and 647 (ed). It is also possible that they are currently in the wrong order.

[13] It is interesting in this context to read how such matrices were used by James Glaisher for the the computation of the 4th, 5th and 6th millions. The organization of the various sheets appears very similar to the one used quite independently by Kulik. There was in particular no need to glue the sheets together [43, p. 134].

In order to facilitate the location of a hole, the holes of the matrices 37 and 41 were numbered 1, 2, 3, 4, 5, 1, 2, 3, 4, 5, etc., from top to bottom. This sequence was certainly chosen, as the lines of the main grids are divided in groups of five.

Each matrix contains exactly 80 holes, one per line. There may however be several holes in one column. For primes greater than 80, some columns do not contain holes at all.

The matrices were used as follows. First, let us define the first hole as the first hole met when going column by column from left to right and top to bottom. This first hole corresponds to a certain type of number. The 37 matrix, for instance, has its first hole for numbers $n = (37 \times 300) \times p + 37$. The second hole in the same column is for numbers $n = (37 \times 300) \times p + 7 \times 37$. The last hole is for numbers $n = (37 \times 300) \times p + 299 \times 37$. The 41 matrix starts with $n = (41 \times 300) \times p + 41$. The 619 matrix starts with $n = (619 \times 300) \times p + 619$. This scheme was probably systematically used, but we have not checked every case. More generally, every matrix M will have its top left corner at position $M \times 300 \times p + 1$.

In order to use the 37 matrix, one can superimpose the first hole with a number of the form $n = (37 \times 300) \times p + 37$ (for instance 3041437), and then add the symbol h (37) in any empty cell seen through the matrix, assuming matrices 13 to 31 had been applied before, of course. Once the matrix 37 was completed, it was shifted towards the right by 37 columns, continuing on the next page, if necessary. This process would have to be done until page 4212, but it is obviously something that could have been parallelized, if starting positions were computed in advance.[14] Since only the smallest factor is given in the table, it is however necessary to first locate all the multiples of 13 before locating those of 17, before locating those of 19, and so on. Nevertheless, the process can greatly be sped up if several persons are involved.

It was of course important to position properly the matrices and to avoid any drift. Kulik computed the positions of the first page of every matrix. He also computed which page of each matrix starts on page 1 of the tables. Such an auxiliary table exists for primes 761 to 1051, and for a few values between 1061 and 1201 (figures 7 and 8).

For instance, for 761 (wd), this table gives pairs of values: $(1, c14)$, $(8, 5)$, $(17, 73)$, $(27, 64)$, etc. The first pair means that page c of matrix 761 (which has pages "a" to "k", "j" being not used[15]) should have its first column *immediately to the right of* column 14 of page 1. In other words, the first column of the matrix should be over column 15. All other pairs concerns page a of the matrix 761, and therefore a new sequence starts on column 6 of page 8, column 74 of page 17, and so on.[16] These columns are separated by exactly 761 columns ($5 + 761 - (17 - 8) \times 77 = 73$) and one can check that this is the case for all the pairs (except the first one) in the column corresponding to 761. One can also note that the list of pairs has been extended until the columns repeat.

[14] Nový [84, p. 342] mentions a handwritten note by Kulik at the beginning of the third volume, and stating that the matrices (*Fabrikeln*) were in someone's home for treatment. We can therefore assume that some of the work was delegated. We have unfortunately not located this handwritten note, but it may be hiding between pages of the third volume.

[15] In fact, the first page does not bear any letter, but all the others do.

[16] Page 1 starts with 3 033 001 and page 8 with 3 194 701. The 6th column of page 8 starts with $3\,196\,201 = 761 \times 300 \times 14 + 1$. The first multiple of 761 afterwards is $761 \times 300 \times 14 + 761 = 3\,196\,961$ in column 8.

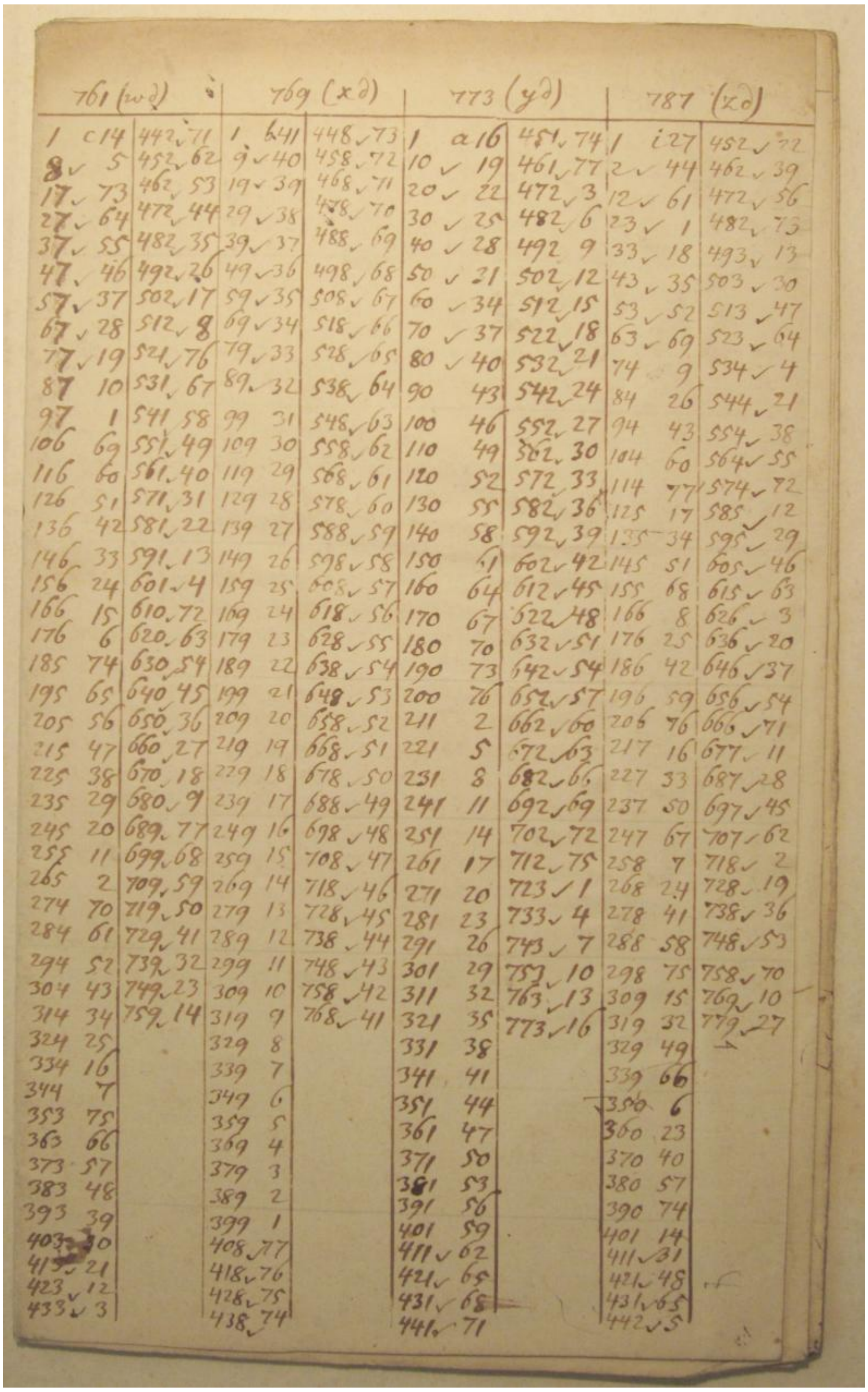


Figure 7: A page giving the positions of the matrices for primes 761 to 787. (AÖAW, Nachlass Kulik, reproduced by the author)

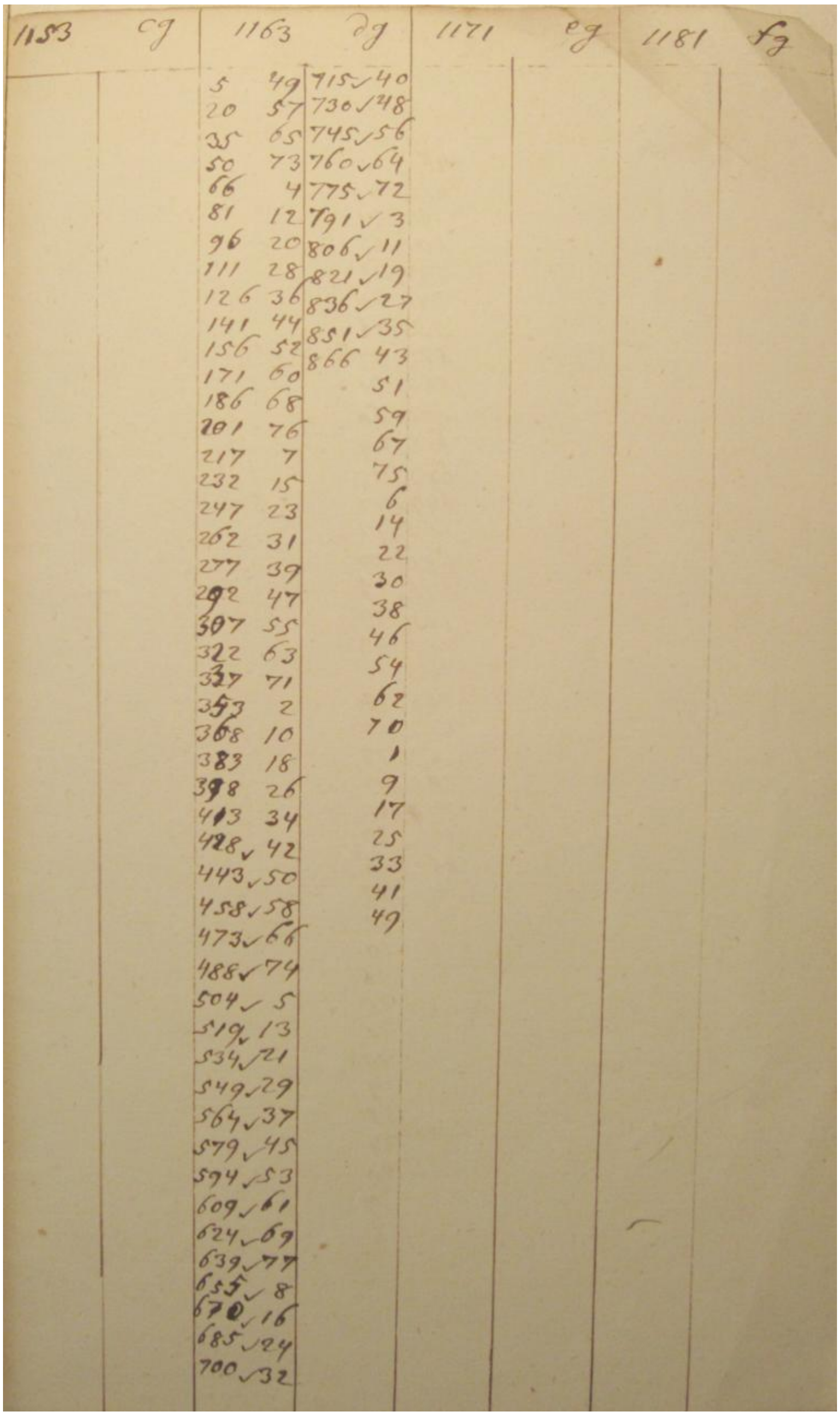


Figure 8: A page giving the positions of the matrices for prime 1163, with the positions for 1153, 1171, and 1181 not yet computed. (AÖAW, Nachlass Kulik, reproduced by the author)

3 033 001 – 3 056 100

1

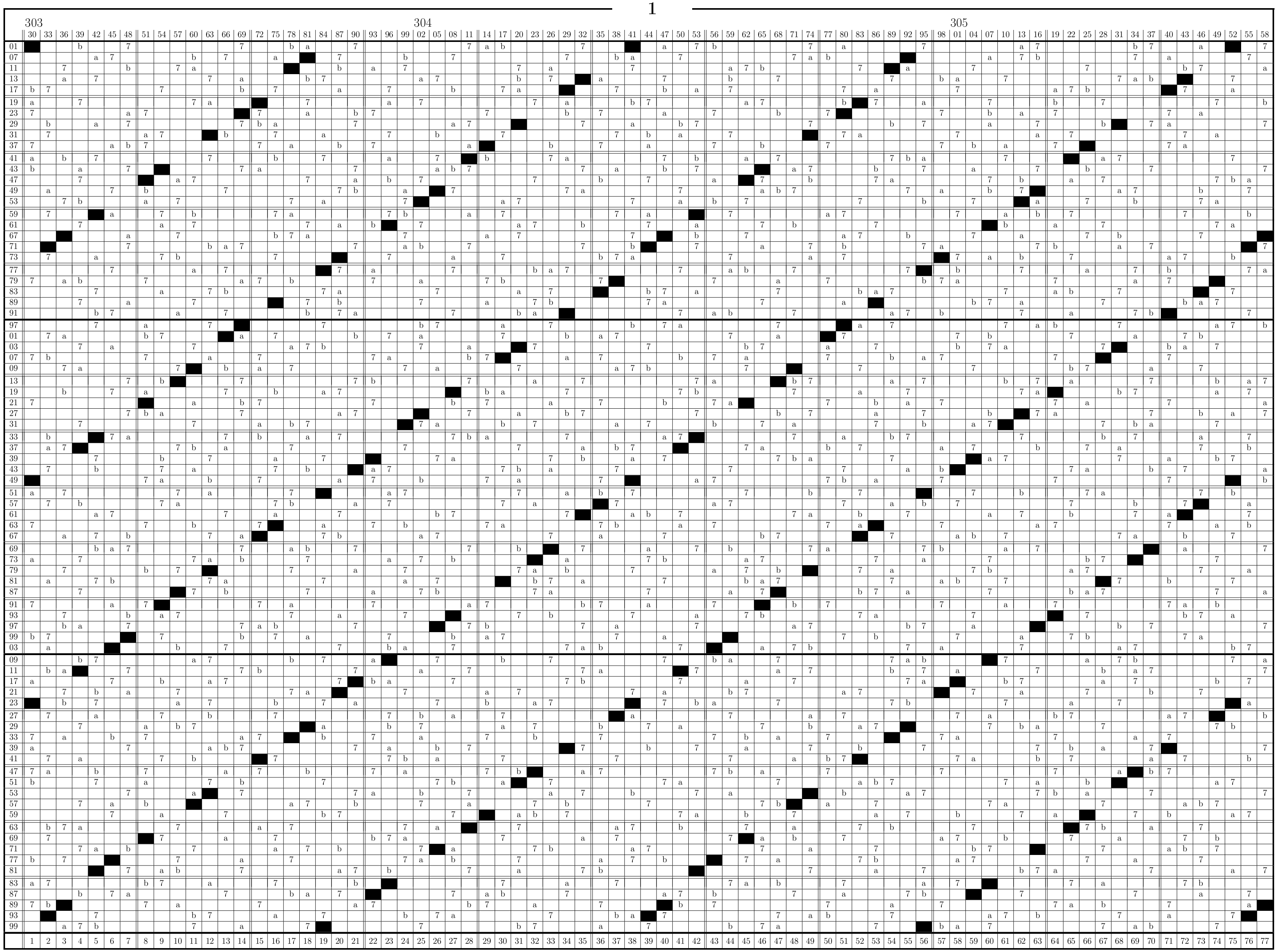


Figure 9: The multiples of 37 on the first page. The symbols “7,” “a,” and “b” (13) were preprinted, but the positions these preprinted values is not significant here.

3 033 001 – 3 056 100

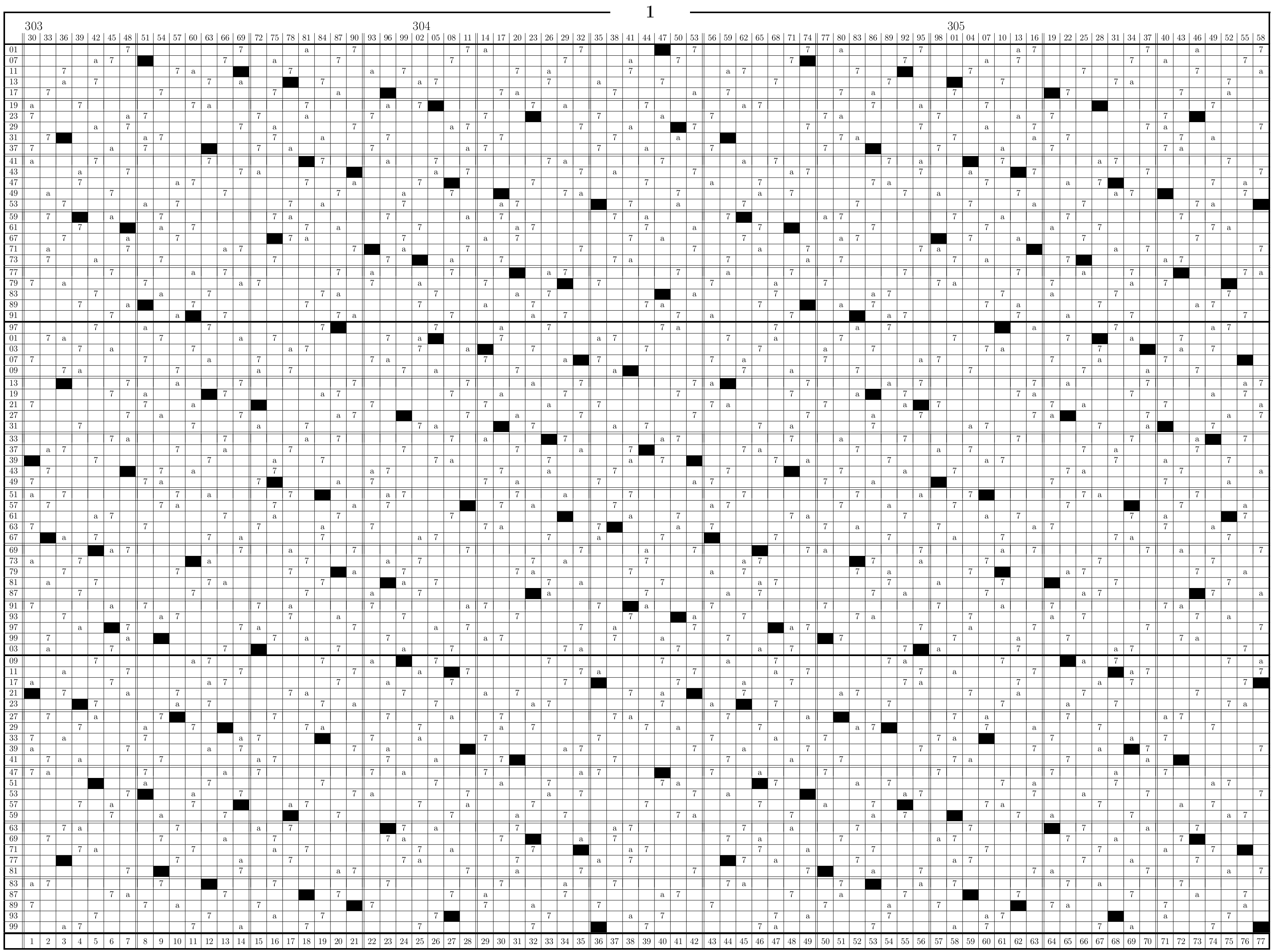


Figure 10: The multiples of 41 on the first page. The symbols "7" and "a" were preprinted.

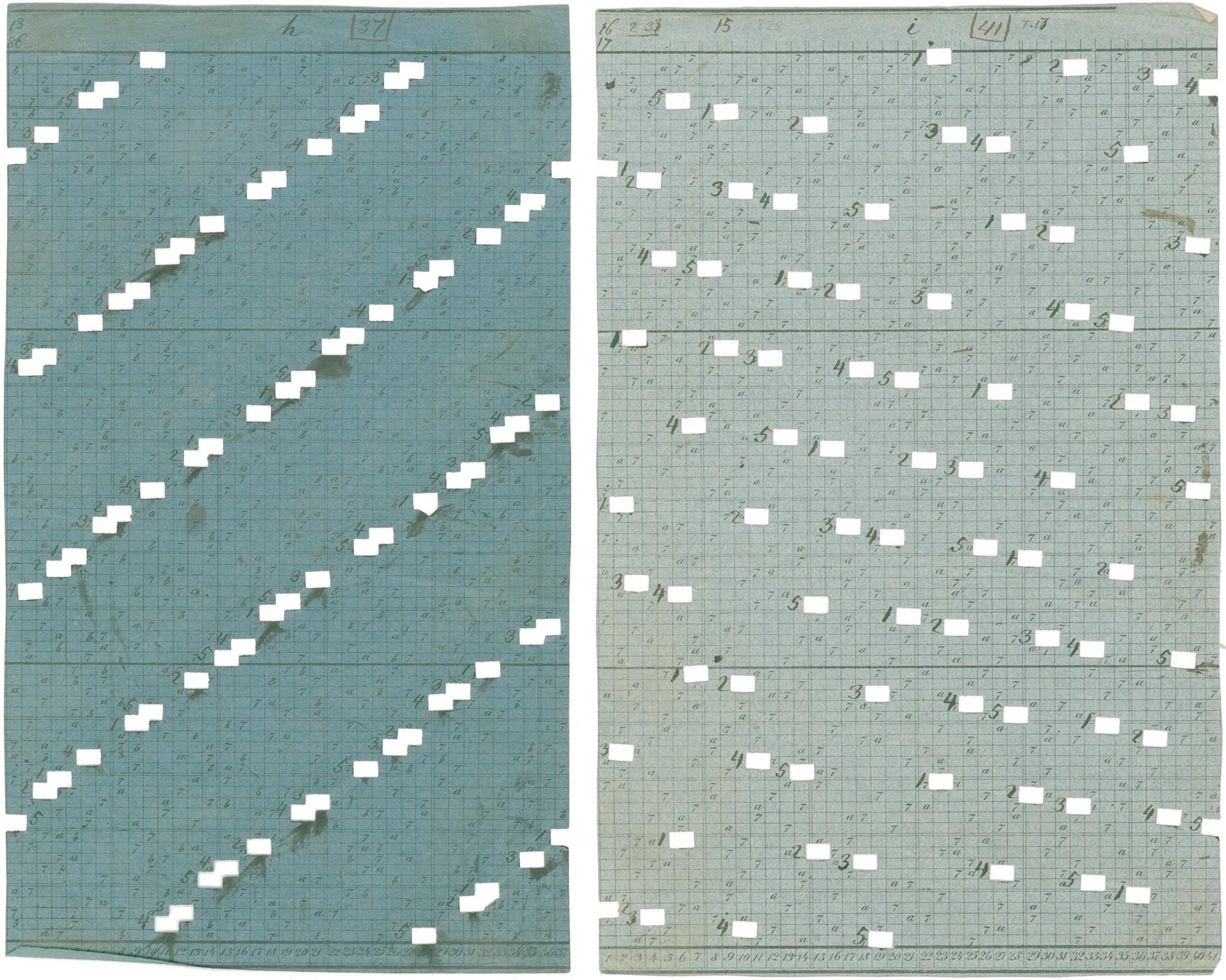


Figure 11: Kulik's matrices for 37 (h) and 41 (i). Each matrix has 80 holes. (AÖAW, Nachlass Kulik, reproduced by the author)

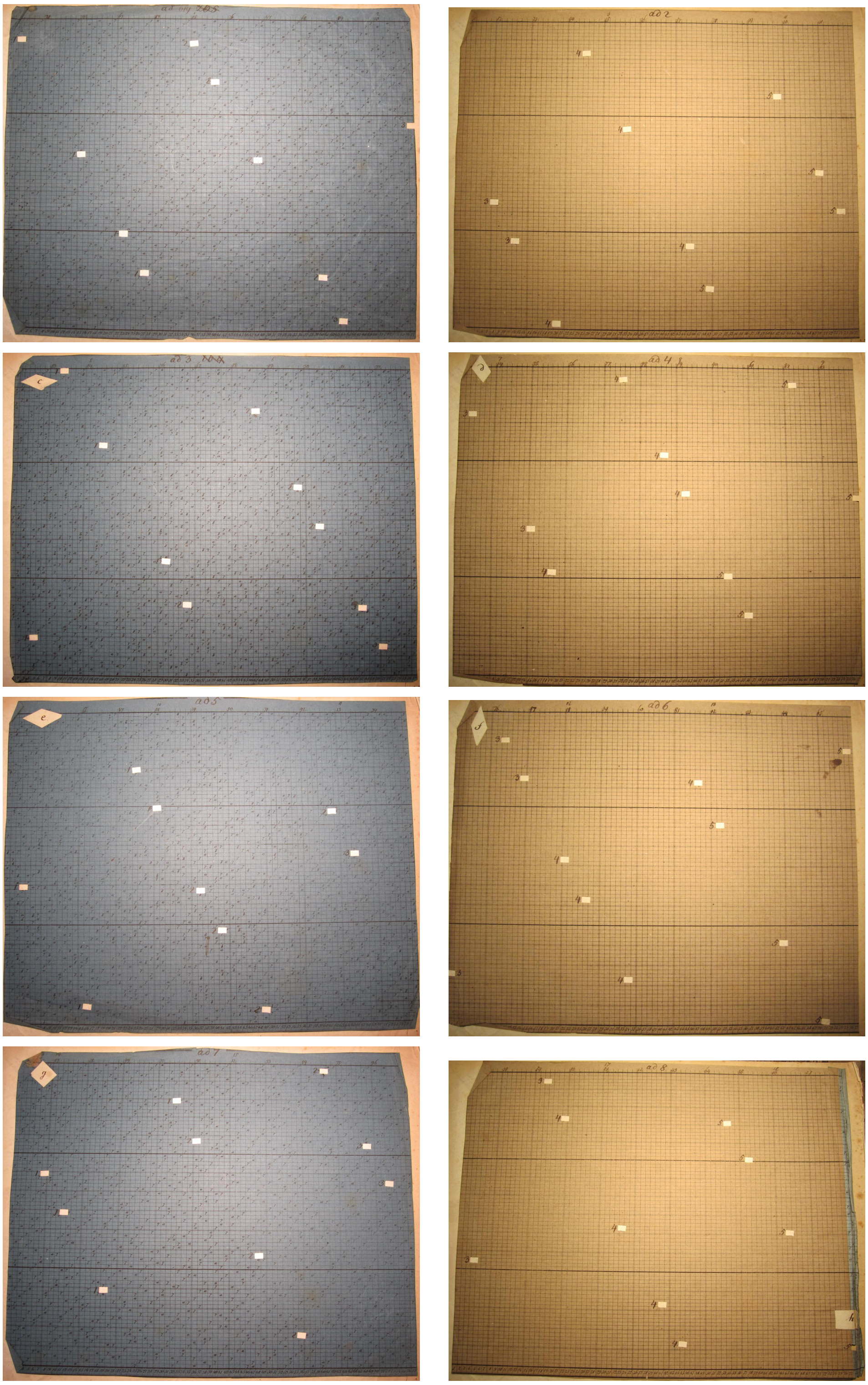


Figure 12: The eight pages of the 619 matrix. The alternate color of the pages may be intentional. A strip of three additional columns (in alternate color) was glued to the last page, so that we have $77 \times 8 + 3 = 619$ columns. The last six pages are labeled ‘c’ to ’h’, and the first two may have been labeled ‘a’ and ’b’. There are exactly 80 holes, one per line. (AÖAW, Nachlass Kulik, reproduced by the author)

3 033 001 – 3 056 100

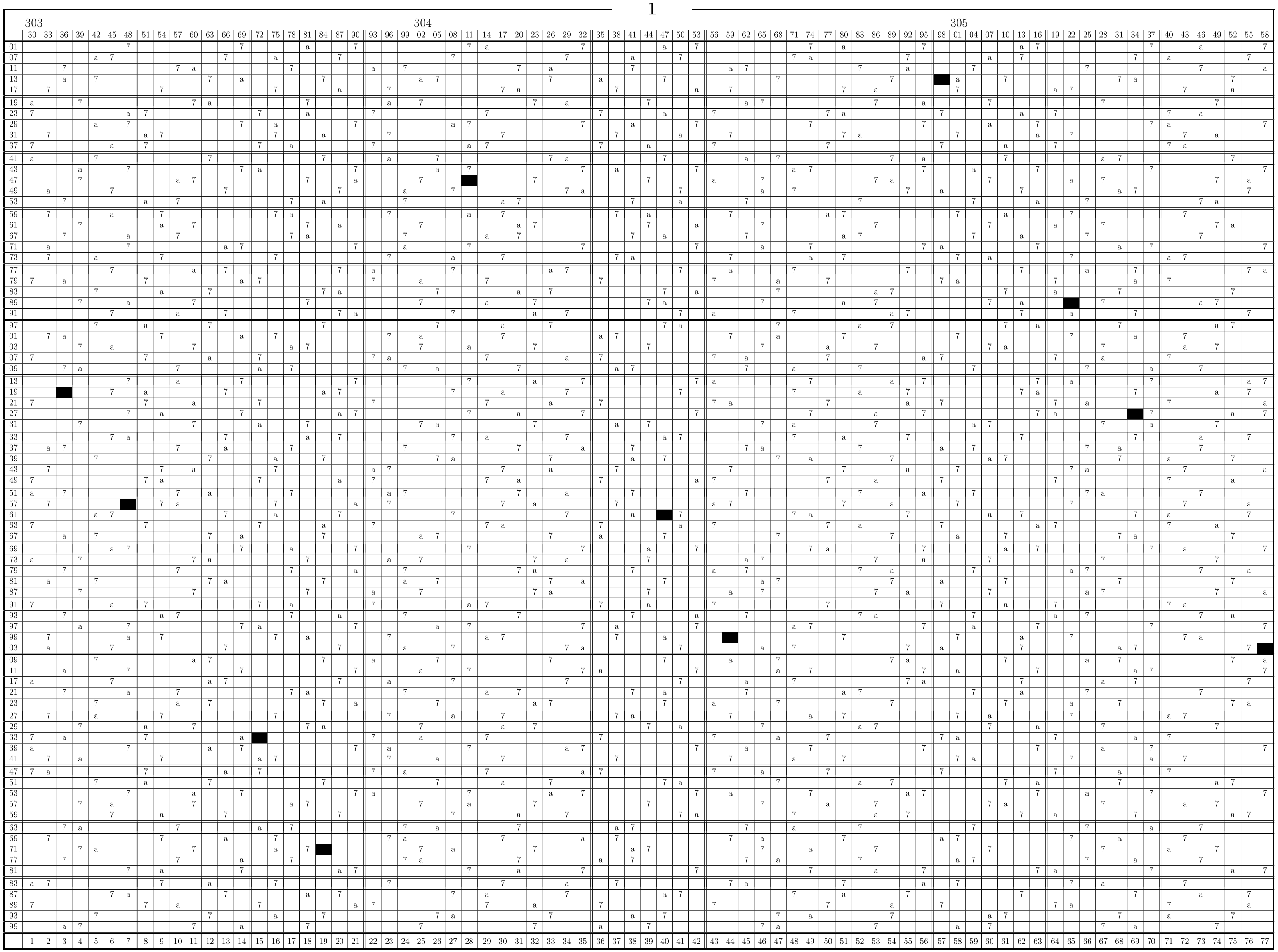


Figure 13: The multiples of 619 on the first page. The symbols “7” and “a” were preprinted.

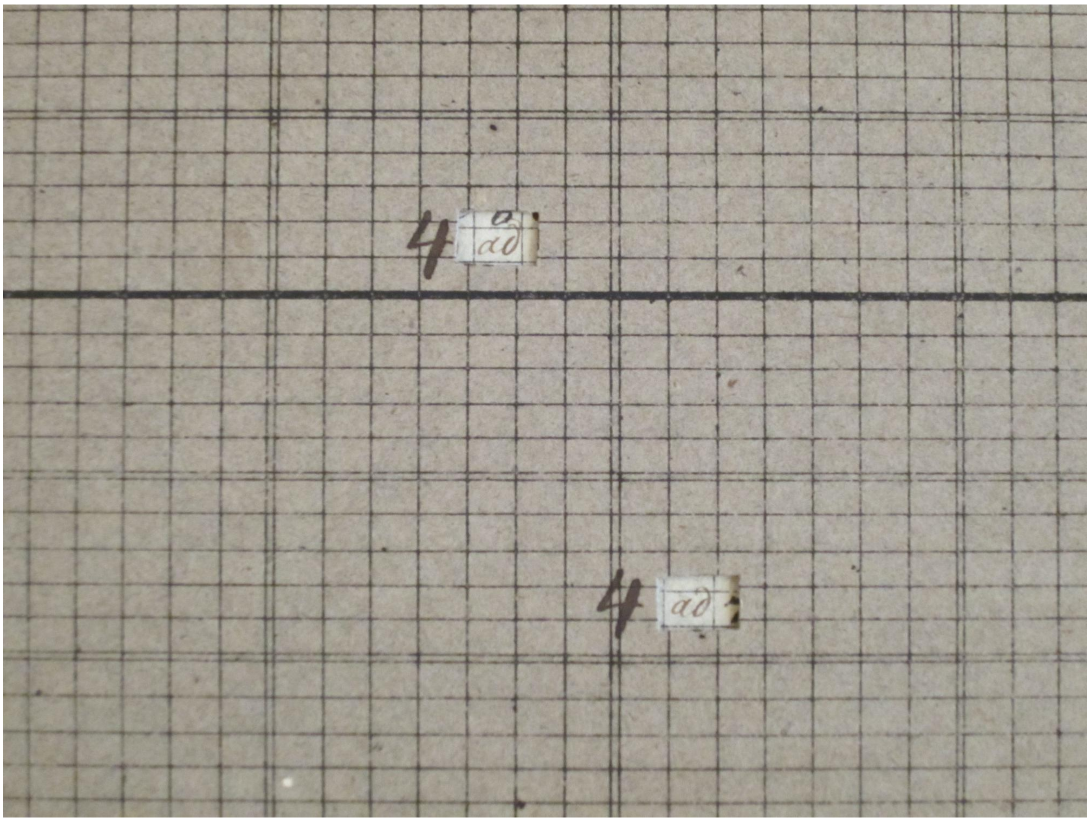


Figure 14: Two holes of the matrix for “ad” (619). For this matrix, pages with no preprinted values were used. Both holes appear on the fourth lines in the groups of five lines separated by double lines. (AÖAW, Nachlass Kulik, reproduced by the author)

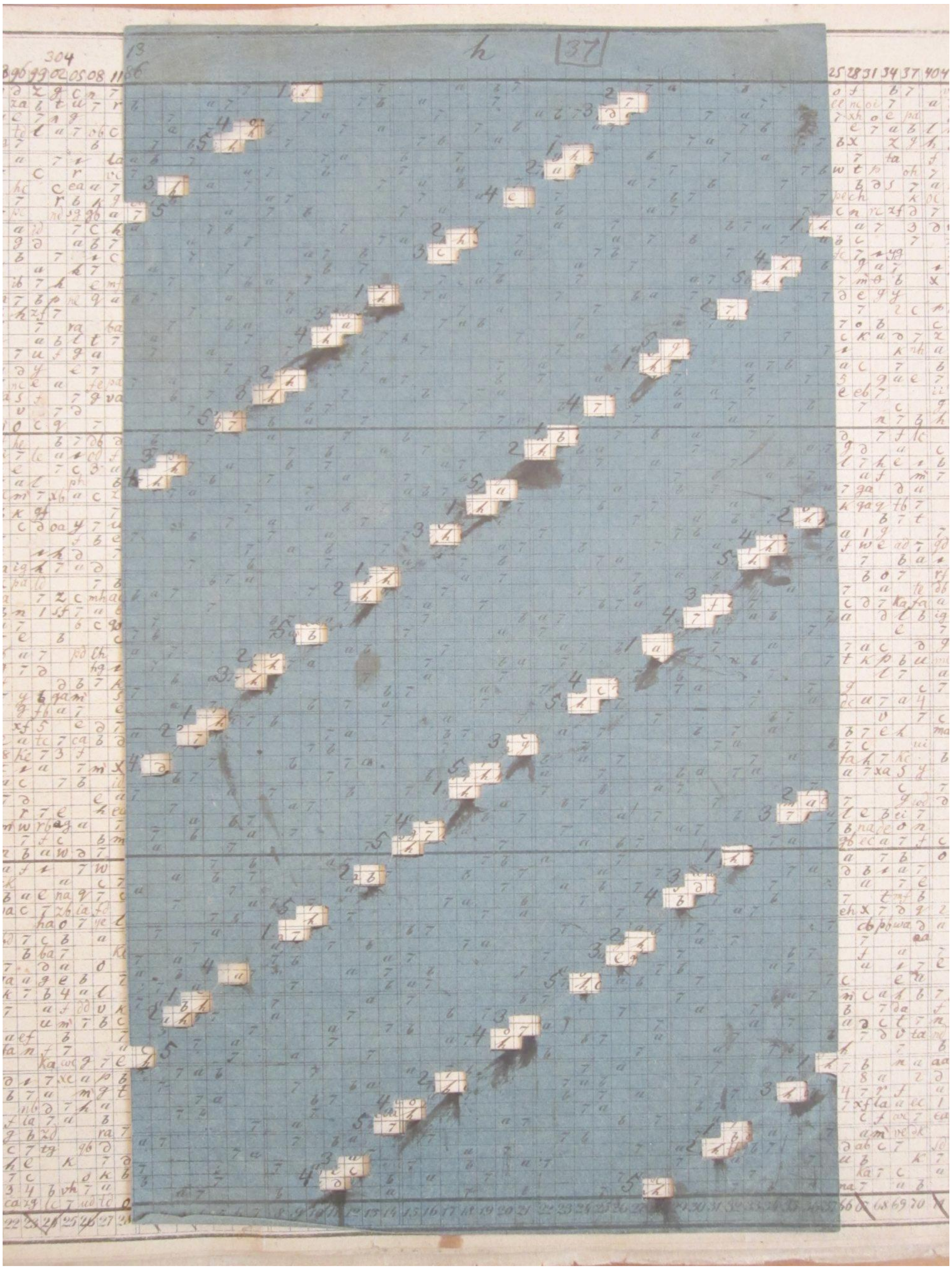


Figure 15: The 37 matrix superimposed on a part of the table. The only factors seen through the holes are factors equal or smaller than 37. When 37 is the smallest factor, the symbol 'h' is written. Note that this page contains preprinted values of the factors 7, 'a', and 'b'. The holes of the matrix are numbered by their positions (1 to 5) in the groups of five lines. (AÖAW, Nachlass Kulik, reproduced by the author)

Figure 16: The 41 matrix superimposed on a part of the table. The only factors seen through the holes are factors equal or smaller than 41. When 41 is the smallest factor, the symbol ‘i’ is written. Note that this page contains only preprinted values of the factors 7, and ‘a’. The holes of the matrix are numbered by their positions (1 to 5) in the groups of five lines. (AÖAW, Nachlass Kulik, reproduced by the author)

Figure 17: The stack of matrices. (AÖAW, Nachlass Kulik, reproduced by the author)

5.2 Enumeration of multiples

Starting with 1009, Kulik seems to have abandonned the use of matrices which were certainly too cumbersome. Kulik then built auxiliary tables of multiples of primes equal or greater than 1009, by primes equal or greater than these numbers.[17] This is the so-called "multiple method." For instance, the first table gives the multiples of 1009 by 1013, 1019, 1021, etc. The multiples of these primes had to be computed until 100 millions, but this limit was in fact never reached [84].

Once these tables were completed, they could be traversed and the multiples of 1009 put in place, *then* those of 1013, and so on. It should be observed that these multiples are not multiples that are obtained by the first method, and therefore these multiples can be put in place totally independently from the completion of the matrix stage.

The tables of multiples of primes are grouped in files, which are small booklets.[18] Each sequence of nine primes starting with 1009 spans several booklets. The primes 1009 to 1051 are contained in six booklets named 1.1 to 1.6. The next sequence is contained in booklets 2.1 to 2.7, and so on. None of the series of multiples is complete and some go farther than others. For large primes, almost no multiple was computed.[19]

There is at least one grouping error, in that the sequence 51 (4649 to 4723) contains 10 primes.

As this example shows, completing this auxiliary table requires adding even multiples of 1009. In order to facilitate these additions, Kulik used small tables of even multiples of these primes.[20] At the same time, this made it necessary to use a triangular arrangement

[17] There is however also a table giving multiples of the primes 233 to 601 in the same manner, but the multiples have not been computed very far, with only one page per group of primes. It is not clear if this table was used, or if it was merely a preliminary experiment.

[18] As observed by Nový, the date "27 Juli 1857" appears at the beginning of booklet 15.1 [84, p. 342]. This seems to indicate a not very advanced stage of computation.

[19] The sequences of primes are the following: 1009 to 1051 (1.1 to 1.6), 1061 to 1109 (2.1 to 2.7), 1117 to 1187 (3.1 to 3.6), 1193 to 1249 (4.1 to 4.5), 1259 to 1303 (5.1 to 5.5), 1307 to 1399 (6.1 to 6.6), 1409 to 1453 (7.1 to 7.5), 1459 to 1511 (8.1 to 8.5), 1523 to 1579 (9.1 to 9.5), 1583 to 1627 (10.1 to 10.5), 1637 to 1709 (11.1 to 11.4), 1721 to 1783 (12.1 to 12.4), 1787 to 1867 (13.1 to 13.3), 1871 to 1931 (14.1 to 14.3), 1933 to 1999 (15.1 to 15.3), 2003 to 2069 (16.1 to 16.3), 2081 to 2131 (17.1 to 17.3), 2137 to 2213 (18.1 to 18.3), 2221 to 2281 (19.1 to 19.3), 2287 to 2347 (20.1 to 20.2), 2351 to 2399 (21.1 to 21.2), 2411 to 2473 (22.1 to 22.2), 2477 to 2557 (23.1 to 23.2), 2579 to 2657 (23bis.1 to 23bis.2), 2659 to 2699 (24.1 to 24.2), 2707 to 2753 (25.1 to 25.2), 2767 to 2833 (26.1 to 26.2), 2837 to 2903 (27.1 to 27.2), 2909 to 2971 (28.1 to 28.2), 2999 to 3061 (29.1 to 29.2), 3067 to 3163 (30), 3167 to 3221 (31), 3229 to 3307 (32), 3313 to 3361 (33), 3371 to 3457 (34), 3461 to 3527 (35), 3529 to 3581 (36), 3583 to 3643 (37), 3659 to 3719 (38), 3727 to 3797 (39), 3803 to 3877 (40), 3881 to 3931 (41), 3943 to 4019 (42), 4021 to 4093 (43), 4099 to 4159 (44), 4177 to 4243 (45), 4253 to 4327 (46), 4337 to 4409 (47), 4421 to 4483 (48), 4493 to 4561 (49), 4567 to 4643 (50), 4649 to 4723 (51), 4729 to 4799 (52), 4801 to 4903 (53), 4909 to 4967 (54), 4969 to 5021 (55), 5023 to 5101 (56), 5107 to 5189 (57), 5197 to 5279 (58), 5281 to 5381 (59), 5387 to 5437 (60), 5441 to 5503 (61), 5507 to 5573 (62), 5581 to 5657 (63), 5659 to 5737 (64), 5741 to 5813 (65), 5821 to 5867 (66), 5869 to 5953 (67), 5981 to 6053 (68), 6067 to 6131 (69), 6133 to 6211 (70), 6217 to 6277 (71), 6287 to 6343 (72), 6353 to 6421 (73), 6427 to 6529 (74), 6547 to 6599 (75), 6607 to 6689 (76), 6691 to 6763 (77), 6779 to 6833 (78), 6841 to 6911 (79), 6917 to 6983 (80), 6991 to 7057 (81), 7069 to 7159 (82), 7177 to 7237 (83), 7243 to 7331 (84), 7333 to 7451 (85), 7457 to 7517 (86), 7523 to 7573 (87), 7577 to 7643 (88), 7649 to 7717 (89), 7723 to 7817 (90), 7823 to 7883 (91), 7901 to 7963 (92), 7993 to 8081 (93), 8087 to 8161 (94), 8167 to 8233 (95), 8237 to 8297 (96), and 8311 to 8389 (97). It is likely that the sequence 2579 to 2657 was initially forgotten and only inserted later.

[20] Burckhardt also used the same method [19], and so did Glaisher, with auxiliary tables of even

of the table of multiples: 1013^2 is on the same line as 1009×1013, 1019^2 is on the same line as 1009×1019 and 1013×1019, etc. Consequently, the difference of two consecutive values in each column, and for a given line, is always the same multiple of the prime corresponding to this column.

These tables were not computed until 100 millions, but the first ones went to about 75 or 80 millions. The multiples of 1009, for instance, were computed until $1009 \times 75211 = 75887899$, those of 1193 until $1193 \times 62057 = 74034001$, those of 1637 until $1637 \times 49727 = 81403099$, and those of 1721 until $1721 \times 49681 = 85501001$. Nový observed that beginning with prime numbers 2221, the extent of the calculations quickly diminished [84, p. 341].

On the back of the table for the even multiples of the primes 1259 to 1303, the name "Tichy Vaclav" appears, and he was obviously the author of that table. This was however the only case of another computer's name.[21]

In such auxiliary tables, some lines can be computed in advance, and this provides a means to check for errors.

Once such an auxiliary table was completed for 1009, the symbol for 1009, namely "ff", could be written for each multiple.

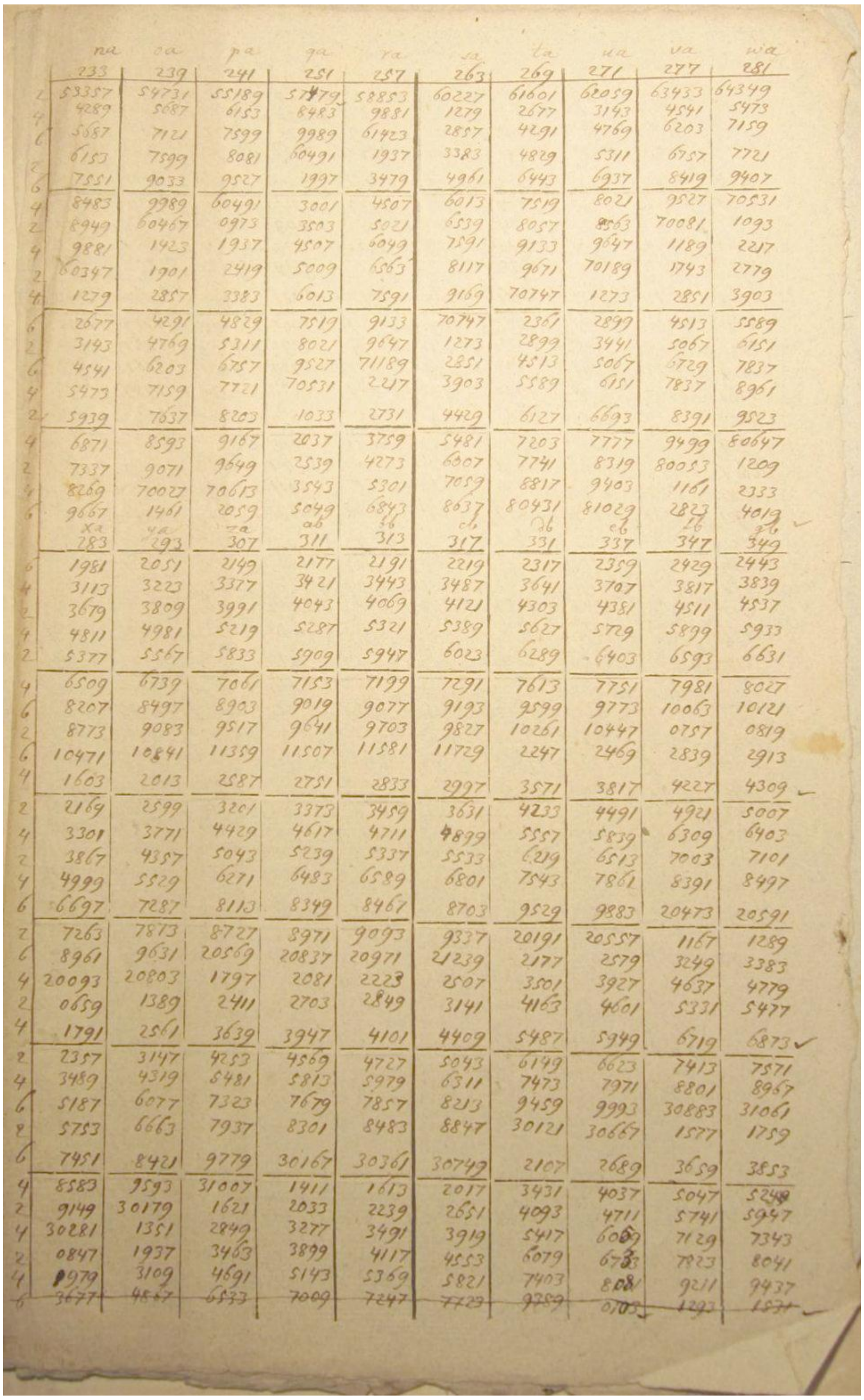
Unfortunately, these auxiliary tables were never completed, and only some of the numbers computed were copied in the main table.[22] According to Nový, the multiples of 1009 to 8589 have been computed at least up to 20 millions.

The auxiliary tables are not always written in the same hand, and it is likely, as assumed by Nový, that Kulik paid other calculators to do some of the calculations.

multiples of primes [43, pp. 132–133]. In some cases, Kulik gave also odd multiples, but it is not clear when and how these multiples were used.

[21]We stress that we haven't gone through each and every page of the tables. It is possible that some detached pages are inserted within some pages in the main tables. In fact, Lehmer withdrew the isolated items in the first volume and put them in an envelope, which is now part of the *Nachlass*. We have also only looked for the auxiliary multiplication tables in the first volume of such tables, and there are two more volumes that should be examined.

[22]Nový gave the limits of computation of some of the booklets, but they should be systematically recorded for each of the 883 primes in the tables.



	na	oa	pa	qa	ra	sa	ta	ua	va	wa
	233	239	241	251	257	263	269	271	277	281
2	53357	54731	55189	57479	58853	60227	61601	62059	63433	64349
4	4289	5687	6153	8483	9881	1279	2677	3143	4541	5473
6	5687	7121	7599	9989	61423	2857	4291	4769	6203	7159
2	6153	7599	8081	60491	1937	3383	4829	5311	6757	7721
6	7551	9033	9527	1997	3479	4961	6443	6937	8419	9407
4	8483	9989	60491	3001	4507	6013	7519	8021	9527	70531
2	8949	60467	0973	3503	5021	6539	8057	8563	70081	1093
4	9881	1423	1937	4507	6049	7591	9133	9647	1189	2217
2	60347	1901	2419	5009	6563	8117	9671	70189	1743	2779
4	1279	2857	3383	6013	7591	9169	70747	1273	2851	3903
6	2677	4291	4829	7519	9133	70747	2361	2899	4513	5589
2	3143	4769	5311	8021	9647	1273	2899	3441	5067	6151
6	4541	6203	6757	9527	71189	2851	4513	5067	6729	7837
4	5473	7159	7721	70531	2217	3903	5589	6151	7837	8961
2	5939	7637	8203	1033	2731	4429	6127	6693	8391	9523
4	6871	8593	9167	2037	3759	5481	7203	7777	9499	80647
2	7337	9071	9649	2539	4273	6007	7741	8319	80053	1209
4	8269	70027	70613	3543	5301	7059	8817	9403	1161	2333
6	9667	1461	2059	5049	6843	8637	80431	81029	2823	4019
	xa	ya	za	ab	bb	cb	db	eb	fb	gb
	283	293	307	311	313	317	331	337	347	349
6	1981	2051	2149	2177	2191	2219	2317	2359	2429	2443
4	3113	3223	3377	3421	3443	3487	3641	3707	3817	3839
2	3679	3809	3991	4043	4069	4121	4303	4381	4511	4537
4	4811	4981	5219	5287	5321	5389	5627	5729	5899	5933
2	5377	5567	5833	5909	5947	6023	6289	6403	6593	6631
4	6509	6739	7061	7153	7199	7291	7613	7751	7981	8027
6	8207	8497	8903	9019	9077	9193	9599	9773	10063	10121
2	8773	9083	9517	9641	9703	9827	10261	10447	0757	0819
6	10471	10841	11359	11507	11581	11729	2247	2469	2839	2913
4	1603	2013	2587	2751	2833	2997	3571	3817	4227	4309
2	2169	2599	3201	3373	3459	3631	4233	4491	4921	5007
4	3301	3771	4429	4617	4711	4899	5557	5839	6309	6403
2	3867	4357	5043	5239	5337	5533	6219	6513	7003	7101
4	4999	5529	6271	6483	6589	6801	7543	7861	8391	8497
6	6697	7287	8113	8349	8467	8703	9529	9883	20473	20591
2	7263	7873	8727	8971	9093	9337	20191	20557	1167	1289
6	8961	9631	20569	20837	20971	21239	2177	2579	3249	3383
4	20093	20803	1797	2081	2223	2507	3501	3927	4637	4779
2	0659	1389	2411	2703	2849	3141	4163	4601	5331	5477
4	1791	2561	3639	3947	4101	4409	5487	5949	6719	6873
2	2357	3147	4253	4569	4727	5043	6149	6623	7413	7571
4	3489	4319	5481	5813	5979	6311	7473	7971	8801	8967
6	5187	6077	7323	7679	7857	8213	9459	9993	30883	31061
2	5753	6663	7937	8301	8483	8847	30121	30667	1577	1759
6	7451	8421	9779	30167	30361	30749	2107	2689	3659	3853
4	8583	9593	31007	1411	1613	2017	3431	4037	5047	5249
2	9149	30179	1621	2033	2239	2651	4093	4711	5741	5947
4	30281	1351	2849	3277	3491	3919	5417	6069	7129	7343
2	0847	1937	3463	3899	4117	4553	6079	6733	7823	8041
4	1979	3109	4691	5143	5369	5821	7403	8081	9211	9437
6	~~3677~~	~~4867~~	~~6533~~	~~7009~~	~~7247~~	~~7723~~	~~9359~~	~~0703~~	~~1293~~	~~1571~~

Figure 18: Multiples of primes 233 to 281. (AÖAW, Nachlass Kulik, reproduced by the author)

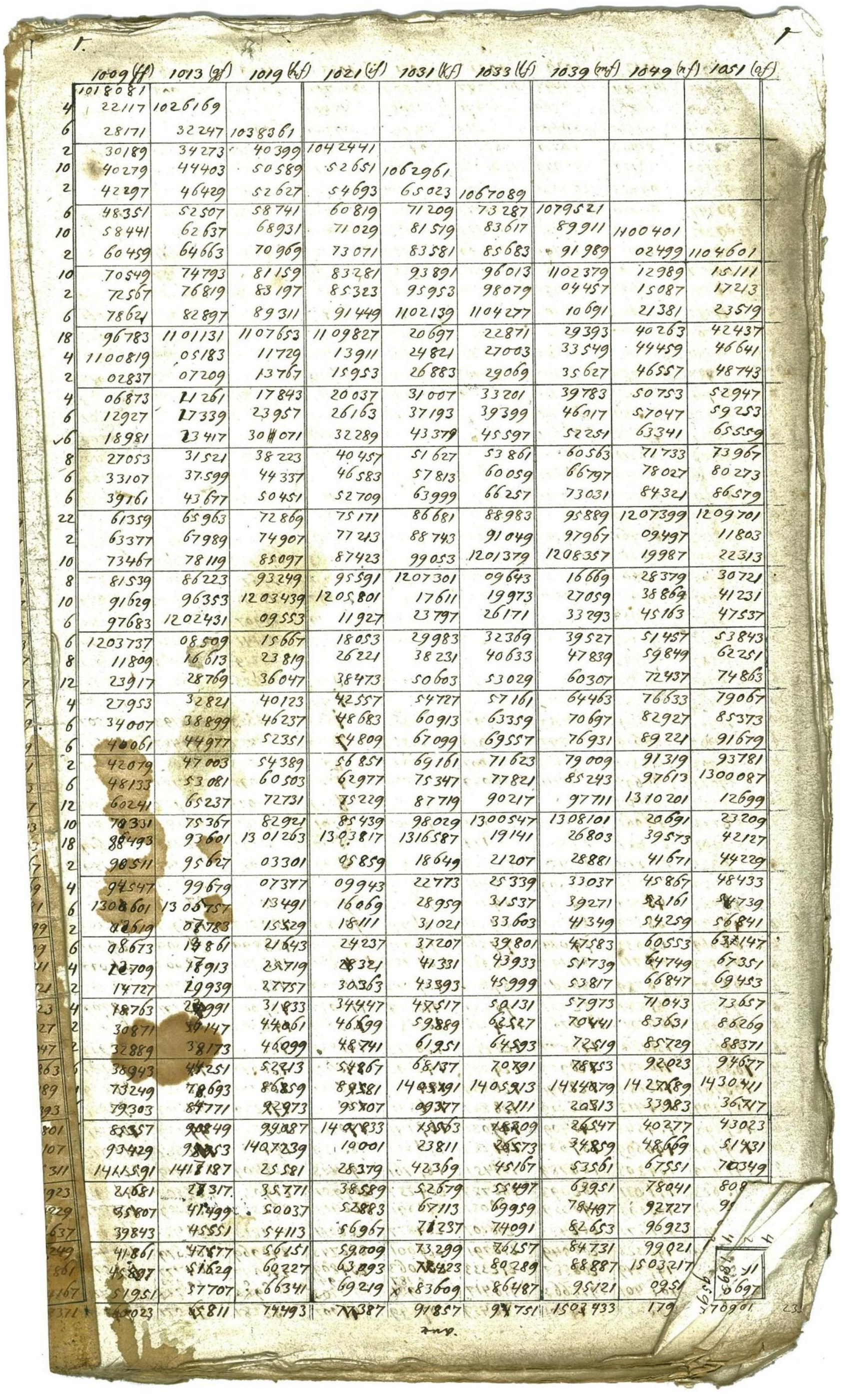


Figure 19: The first multiples of the primes 1009 (ff), 1013 (gf), 1019 (hf), 1021 (if), 1031 (kf), 1033 (lf), 1039 (mf), 1049 (nf), and 1051 (of) by primes equal or greater than them. The first value at the top left is $1009^2 = 1018081$. Below is $1009 \times 1013 = 1022117$, etc. (AÖAW, Nachlass Kulik, reproduced by the author)

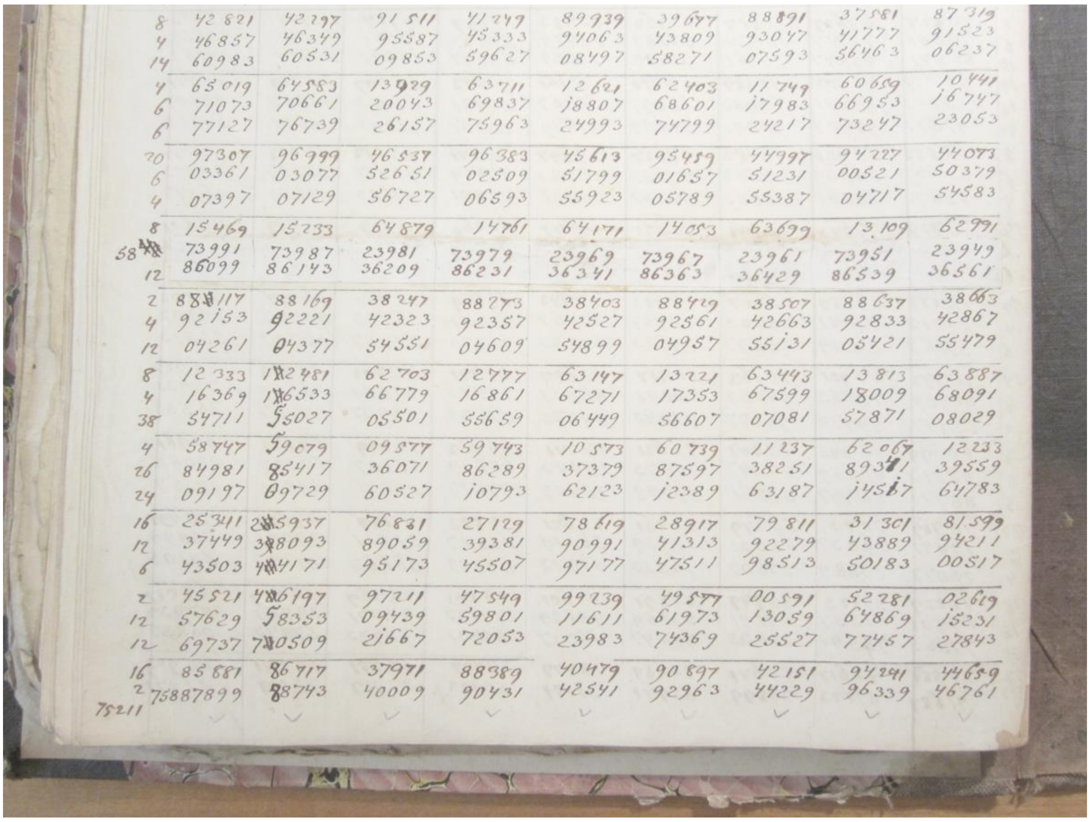
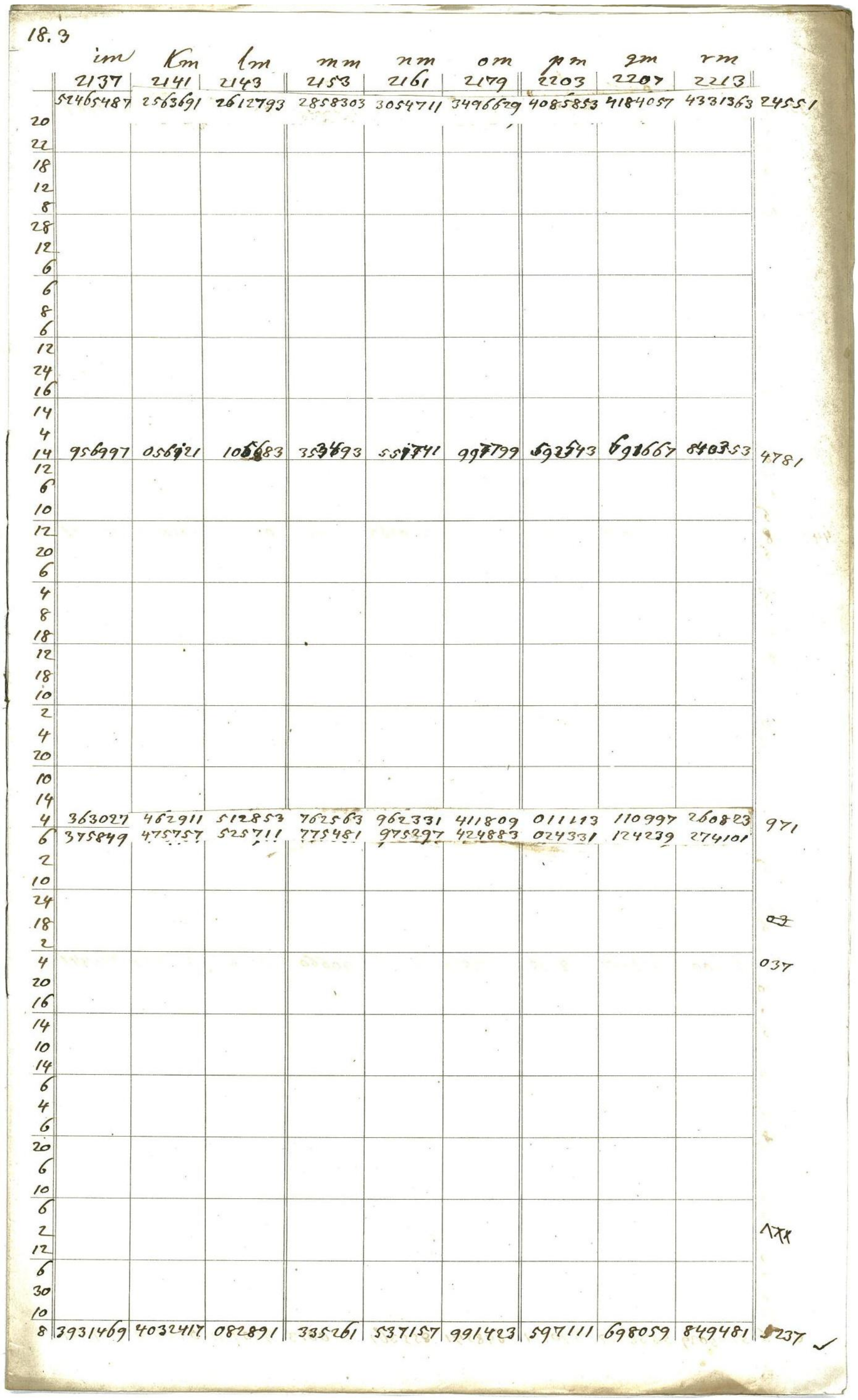


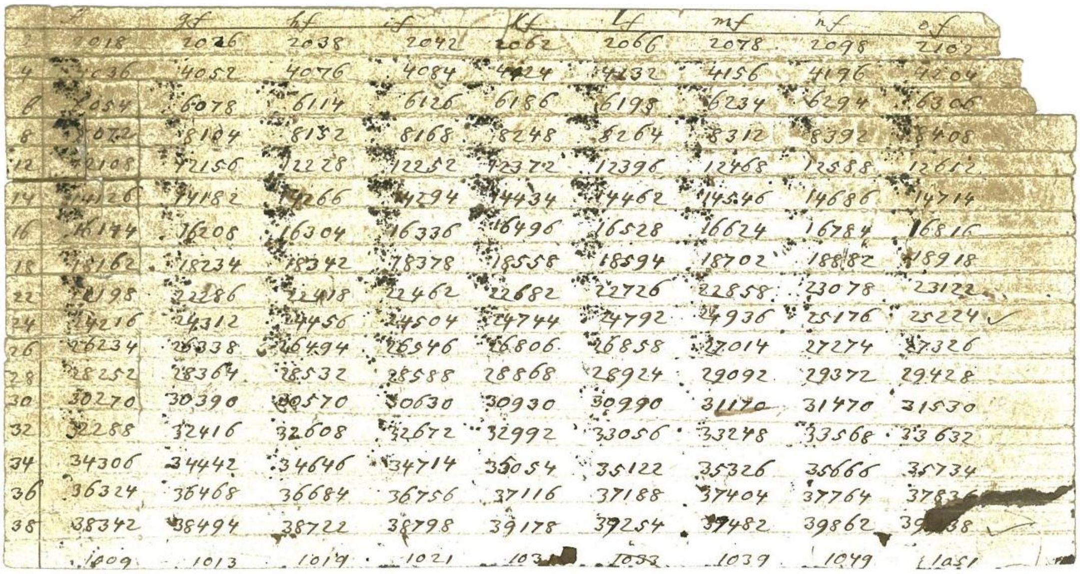
Figure 20: The end of the list of multiples of 1009 to 1051. The multiples of 1009 end with 75887899. The last value in the column left is the multiplier, since 75887899 = 1009 × 75211, 76188743 = 1013 × 75211, etc. (AÖAW, Nachlass Kulik, reproduced by the author)



18. 3

	im	Km	lm	mm	nm	om	pm	qm	rm	
	2137	2141	2143	2153	2161	2179	2203	2207	2213	
	52465487	2563691	2612793	2858303	3054711	3496629	4085853	4184057	4331363	24551
14	956997	056921	105683	353493	559741	997799	592543	691667	840353	4781
4	363027	462911	512853	762563	962331	411809	011113	110997	260823	971
6	375849	475757	525711	775481	975297	424883	024331	124239	274101	
										[illegible]
										037
										[illegible]
8	3931469	4032417	082891	335261	537157	991423	597111	698059	849481	5237

Figure 21: A page for the multiples of the primes 2137, 2141, . . . , 2213 by primes equal or greater than them, but which is only incompletely filled. Some of the lines have been computed in advance, and will serve to check for errors introduced in the additions. The values in the right margin are the multipliers: on the first line, we have 2137 × 24551 = 52465487, 2141 × 24551 = 52563691, etc. (AÖAW, Nachlass Kulik, reproduced by the author)



	f	gf	hf	if	kf	lf	mf	nf	of
2	2018	2026	2038	2042	2062	2066	2078	2098	2102
4	4036	4052	4076	4084	4124	4132	4156	4196	4204
6	6054	6078	6114	6126	6186	6198	6234	6294	6306
8	8072	8104	8152	8168	8248	8264	8312	8392	8408
12	12108	12156	12228	12252	12372	12396	12468	12588	12612
14	14126	14182	14266	14294	14434	14462	14546	14686	14714
16	16144	16208	16304	16336	16496	16528	16624	16784	16816
18	18162	18234	18342	18378	18558	18594	18702	18882	18918
22	22198	22286	22418	22462	22682	22726	22858	23078	23122
24	24216	24312	24456	24504	24744	24792	24936	25176	25224 ✓
26	26234	26338	26494	26546	26806	26858	27014	27274	27326
28	28252	28364	28532	28588	28868	28924	29092	29372	29428
30	30270	30390	30570	30630	30930	30990	31170	31470	31530
32	32288	32416	32608	32672	32992	33056	33248	33568	33632
34	34306	34442	34646	34714	35054	35122	35326	35666	35734
36	36324	36468	36684	36756	37116	37188	37404	37764	3783[illegible]
38	38342	38494	38722	38798	39178	39254	39482	39862	39[illegible]38 ✓
	1009	1013	1019	1021	103[illegible]	1033	1039	1049	1051

Figure 22: An auxiliary table for the even multiples of the primes 1009, 1013, 1019, 1021, 1031, 1033, 1039, 1049, and 1051. The values on the first line are 2018 (1009 $\times$ 2), 2026, 2038, ..., 2102. The multipliers are given in the left column. This table is worn out and each line was folded on and off several times to help for the additions when placed over the main multiplication tables. (AÖAW, Nachlass Kulik, reproduced by the author)

	pf	qf	rf	sf	tf	uf	vf	wf	xf
2	2122	2126	2128	2174	2182	2186	2194	2206	2218
4	4244	4252	4276	4348	4364	4372	4388	4412	4436
6	6366	6378	6414	6522	6546	6558	6582	6618	6654
8	8488	8504	8552	8696	8728	8744	8776	8824	8872
10	10610	10630	10690	10870	10910	10930	10970	11030	11090
12	12732	12756	12828	13044	13092	13116	13164	13236	13308
14	14854	14882	14966	15218	15274	15302	15358	15442	15526
16	16976	17008	17104	17392	17456	17488	17552	17648	17744
18	19098	19134	19242	19566	19638	19674	19746	19854	19962
22	23342	23386	23518	23914	24002	24046	24134	24266	24398
24	25464	25512	25656	26088	26184	26232	26328	26472	26616
26	27586	27638	27794	28262	28366	28418	28522	28678	28834
28	29708	29764	29932	30436	30548	30604	30716	30884	31052
30	31830	31890	32070	32610	32730	32790	32910	33090	33270
32	33952	34016	34208	34784	34912	34976	35104	35296	35488
34	36074	36142	36346	36958	37094	37162	37298	37502	37706
36	38196	38268	38484	39132	39276	39348	39492	39708	39924
38	40318	40394	40622	41306	41458	41534	41686	41914	42172
42	44562	44646	44898	45654	45822	45906	46074	46326	46578
44	46684	46772	47036	47828	48004	48092	48268	48532	48796
46	48806	48898	49174	50002	50186	50278	50462	50738	51014
48	50928	51024	51312	52176	52368	52464	52656	52944	53232
50	53050	53150	53450	54350	54550	54650	54850	55150	55450
52	55172	55276	55588	56524	56732	56836	57044	57356	57668
54	57294	57402	57726	58698	58914	59022	59238	59562	59886
56	59416	59528	59864	60872	61096	61208	61432	61768	62104
58	61538	61654	62002	63046	63278	63394	63626	63974	64222

Figure 23: An auxiliary table for the even multiples of the primes 1061, 1063, …, 1109. (AÖAW, Nachlass Kulik, reproduced by the author)

6 The accuracy of Kulik's tables

6.1 The errors found by Lehmer

Lehmer seems to have gone through the 10th million of Kulik's table and the pages have red checkmarks, as well as some corrections. Lehmer gave a list of 229 errors in the introduction of his table of factors (table 2).[23] Lehmer concluded that Kulik's table was not accurate enough for publication.

It is interesting to analyze the errors found by Lehmer. Even a cursory analysis shows that there are error patterns. For instance, there are often pairs of errors, such as those for 9198221 and 9198281 (see also figure 24). Such errors come in pairs because one factor has been misplaced, causing the appearance of a spurious prime. In this case, the factor 1091 (coded by "tf") was put in the cell for 9198281, but it should have been written in 9198221. Consequently, 9198221 appeared as prime, although it is not. This error was either caused by a computation error in the table of multiples, or by a transfer error from the table of multiples to the final page.[24]

Another type of common error is that of isolated numbers given as primes, but which are not. Such errors do not come in pairs and may either be due to a forgotten entry, for instance when a matrix entry is mistakenly skipped, or to the unfinished state of using the matrices or transferring the list of multiples. Either way, a closer analysis should be able to shed more light on these errors.

In some cases, as observed by Lehmer, the factor given for a number is not the smallest. This occurs for 9454943, 9457547, 9524527, 9602347, 9757847, 9846919, and 9877867. These errors can easily be explained by a hole skipped in a matrix, and the cell was then only filled at a later stage. These errors naturally only occur for primes smaller than 1000.

In several cases, the wrong factor is given in a cell, but without another matching error. This is for instance the case for 9696389 which has 1163 in its cell, but should have 1327. For 9735293, we find 2503, but the correct entry is 2699. There are several similar errors, and they can also easily be explained by looking at the table of primes. In each case, the two factors are neighbors, or almost neighbors, on the same line of the table. It is therefore likely that the person who filled the table was distracted and took the entry in the wrong column. On the other hand, these errors only occur for primes above 1000, and the list of multiples have the symbol of the prime at the top of the lists, so that in principle it is not necessary to use the table of primes at this point.

This short typology of errors shows that Kulik did not have any means to enforce correctness, and even though some errors could be detected in the table of multiples, others could have crept in between, or could have arisen during the transfer from the tables of multiples to the main table, or during the application of the matrices. Another way to increase the accuracy of the tables would have been to compute them twice by independent computers, and, if possible, by independent methods. Such a procedure was followed by Prony in his tables of logarithms, but even that did not prevent the appearance of many errors [99].

[23] Porubský wrote instead that Lehmer found 226 errors in the first 10 millions [93].

[24] We have not checked the exact cause of these errors, but this is something that can easily be done using our table 2 and the list of multiples in the *Nachlass*.

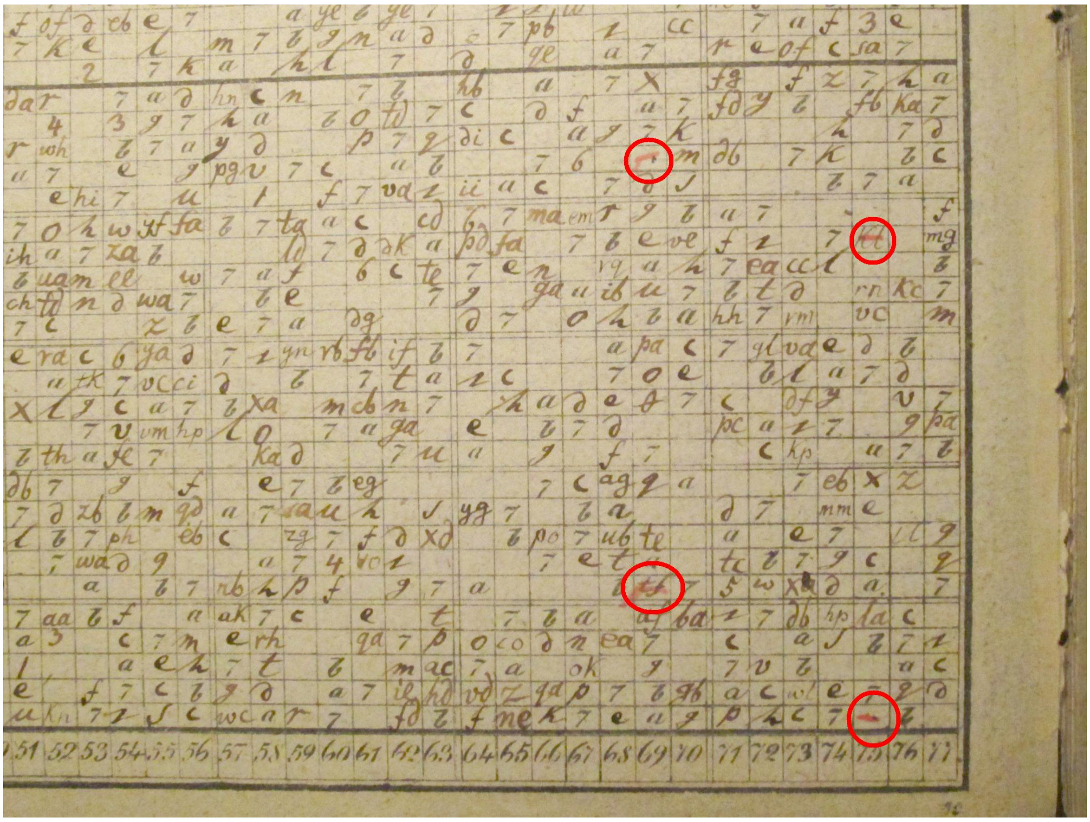


Figure 24: Some of Lehmer's corrections on page 267. The circles are ours. Note that the "b"s are preprinted on this page and that the "10" at the bottom identifies the layout of these "b"s. The four entries circled are 9198221 (top left), 9198281 (bottom left), 9200029 (top right) and 9200099 (bottom right). This example shows two pairs of related errors. The entry for 9198281 should have been put for 9198221 and the entry for 9200029 should have been put for 9200099. Two initial errors caused the appearance of two spurious primes. (AÖAW, Nachlass Kulik, reproduced by the author)

Number	Cor.	K	Number	Cor.	K	Number	Cor.	K	Number	Cor.	K	Number	Cor.	K	Number	Cor.	K	Number	Cor.	K
9009703	281	P	9197789	2341	P	9315179	2243	P	9430901	797	P	9569083	1123	P	9740309	P	1931	9824699	349	P
9011599	211	P	9198221	1091	P	9316583	199	P	9431159	P	31	9569093	P	1123	9745999	P	13	9832357	P	2699
9012841	P	109	9198281	P	1091	9317603	1307	1129	9431203	281	P	9572867	1151	P	9748339	P	1447	9832457	2699	P
9015031	2731	P	9200029	P	1973	9334361	191	P	9436769	1289	P	9585337	1433	P	9748439	1447	P	9846919	71	331
9019873	2341	P	9200099	1973	P	9335231	479	P	9437069	P	1289	9585437	P	1433	9749893	2269	P	9870293	383	P
9019973	P	2341	9200773	181	P	9341099	271	P	9441323	2897	P	9586679	1151	P	9754229	467	P	9874607	P	2671
9021079	1879	P	9204947	709	479	9354067	691	P	9454637	1049	P	9592883	1327	P	9755953	1187	P	9874687	2671	P
9021179	P	1879	9206149	179	P	9354781	P	2447	9454943	37	47	9592889	P	1327	9755983	P	1187	9877867	17	241
9031483	1427	P	9207431	1117	P	9354881	2447	P	9457547	11	499	9598009	1889	P	9757847	11	17	9894011	347	P
9031489	P	1427	9208181	137	P	9366151	P	1249	9476557	2131	P	9598109	P	1889	9758423	59	19	9900073	1699	P
9033533	P	2699	9216563	167	P	9366251	1249	P	9476567	P	2131	9602347	73	199	9767701	1601	P	9907361	191	P
9035553	2699	P	9216817	887	P	9376751	P	1319	9477697	353	P	9615367	353	P	9767711	P	1601	9910073	P	1699
9073379	7	11	9218611	1511	P	9376771	1319	P	9490993	1543	P	9616963	P	2273	9768403	281	P	9915439	349	97
9073381	P	1667	9219103	647	P	9380983	1663	P	9491093	P	1543	9617063	2273	P	9773593	P	2029	9931487	347	P
9073481	1667	P	9219761	191	P	9384787	1669	1663	9495191	359	P	9634073	1499	P	9773693	2029	P	9933569	347	P
9085003	797	P	9245917	2999	P	9384821	1511	P	9499337	1091	P	9637073	P	1499	9779041	419	P	9936967	683	P
9094003	281	P	9255319	P	2549	9385799	373	P	9501847	383	P	9643069	P	1931	9780229	491	P	9937003	281	P
9106351	P	1361	9255419	2549	P	9387223	P	1277	9502501	419	P	9654247	P	1867	9784651	1151	P	9940459	593	P
9106451	1361	P	9262657	1427	P	9387227	1277	P	9502903	P	2347	9654257	1867	P	9791207	1597	1163	9959233	P	1871
9113443	883	P	9265657	P	1427	9391661	191	P	9503003	2347	P	9655253	P	2053	9791209	P	1597	9959333	1871	P
9128419	P	1427	9268813	P	1279	9393887	1511	P	9512827	131	P	9655259	2053	P	9791297	1163	P	9962179	P	1399
9128519	1427	P	9268913	1279	P	9393919	1823	P	9515503	281	P	9658009	1733	P	9792871	11	P	9962279	1399	P
9133169	353	P	9270133	353	P	9399931	1511	P	9522157	P	2017	9658109	P	1733	9794231	337	P	9967261	P	1487
9134171	307	2957	9277061	191	P	9406883	2393	P	9522257	2017	P	9675079	P	2141	9794611	P	1151	9967361	1487	P
9134173	2957	P	9279791	359	P	9406889	P	2393	9524353	523	P	9675179	2141	P	9798079	P	2543	9985039	1483	P
9153427	2969	P	9291823	1523	P	9407227	P	1907	9524527	97	149	9678161	191	P	9798179	2543	P	9985139	P	1483
9153527	P	2969	9291991	1889	P	9407231	1907	P	9537397	P	2311	9681209	1291	P	9798583	197	P	9991481	1213	P
9157247	11	P	9295211	433	P	9410363	P	2609	9537497	2311	P	9696389	1327	1163	9798977	197	P	9991489	P	1213
9159131	P	503	9301823	P	1523	9410663	2609	P	9555913	101	P	9730309	1931	P	9800159	197	P	10009079	P	139
9159517	859	P	9301991	P	1889	9424819	3061	P	9557447	109	P	9734269	P	1433	9801689	P	1433	10015903	P	1481
9164017	2087	1889	9306937	1873	1489	9429307	797	P	9562951	P	2459	9734369	1433	P	9809893	P	2269	10016003	1481	P
9178459	P	353	9310583	1861	P	9430063	1171	P	9563051	2459	P	9735293	2699	2503	9814669	P	3019			
9181483	2819	P	9315079	P	2243	9430163	P	1171	9568259	P	79	9735461	191	P	9814769	3019	P			

Table 2: The 229 errors given by Lehmer for the 10th million and beyond [73, pp. XIII–XIV]. The correct entries are indicated by "Cor." and the entries given by Kulik are indicated by "K." The entries marked "P" stand for primes.

6.2 The incompleteness of the manuscript

The remaining part of volume 1, as well as the other volumes probably also contain errors, but the main problem with these volumes is that they are incomplete. This seems to have been first observed in print by Nový in 1963. More recently, Porubský observed that the numbers 64713907 and 64713923 were given as primes, when they actually were not, but he does not seem to have noticed that the cells had actually not yet been filled [93, p. 324].

It may seem surprising that so few people noticed that Kulik's tables were unfinished, but one reason is certainly that the tables give an appearance of completeness, as testified by the last page shown earlier (figure 6). Only a close examination reveals that this is not so, and this examination is made more difficult by the symbols used by Kulik, and by the absence of tables of similar layout and extent.

Another reason is Kulik's statement that his table was nearly complete [62], and Petzval's repetition thereafter [89, 90, 91]. This statement was not changed after Lehmer's first book [73], because Lehmer had only had volume 1 in his hands, and volume 1 is nearly complete.

The completeness of each volume varies, and earlier volumes are more complete than later ones. In the following excerpts of pages 859, 867, 1963, 2792, 3068, 3353, 3925, and 4212, if the factor is made of two symbols, the second symbol seems always to be "a," indicating that the factors did not extend 307. Larger factors have only been given in volume 1, and probably also in volume 2.

The excerpt of page 859 is somewhat misleading, because, as noticed by Nový [84, p. 340], there are actually symbols for the primes 1009 to 1061, that is from "ff" to "pf" on that page, but not on the excerpt shown in figure 26. For instance, Kulik gives the factor "gf" for 22860371 (see figure 5). It is indeed particularly strange that factors 1009 to 1061 are written on this page, but not factors between 307 and 1009. One might be tempted to view this as an error, and an error which by chance didn't have consequences, because on this page whenever a number is a multiple of a prime between 1009 and 1061, this prime is the smallest factor, or the smallest factor has a one-symbol representation and is already on that page.

In fact, the gap between about 307 and 1009 is not an error, because only the first sieving process needs to be performed in order. The "multiple method" can be performed in parallel, because the multiples are computed in such a way that they *all* have to be written down. Page 859, and perhaps other pages, is therefore interesting, because it is an example of a page where the two methods were applied in parallel, and none of the two was complete.

According to Nový, factors exceeding 257 are only given up to approximately page 1000, and towards the end of volume 3, only factors up to 211 are recorded [84, p. 340]. We have not checked these limits in detail, but they appear quite plausible. If this is true, this means that checking the extent of the table only requires checking how far all primes up to approximately 307 have been written.

Kulik's table was one of the tables used to produce a list of the primes of the 11th million [13]. Beeger writes that many of Kulik's symbols could not be interpreted during the construction of this list. There is unfortunately no example of such a symbol, and this assertion is therefore difficult to check. It may just be that the handwriting was difficult to read.

The 13th million in volume 1 has also been studied more accurately. Indeed, in 1953, Palamà and Poletti published a list of primes at the beginning of the 13th million [87] and Sexton then compared it with Kulik's list as made available on microfilm by the Carnegie Institute in 1948 [5].[25] He found some discrepancies [125]. Sexton gave in particular a number of integers shown as prime by Kulik, but which are not. Interestingly, several of these integers are multiples of 601, and it thus seems that the 601 matrix was already not applied starting with the 13th million.[26]

Although volume 2 is lost, it is still possible to set an upper limit to its content, that is, to say something about how much it could be complete. Indeed, if we assume that no table of multiples was lost, we can find a greatest multiple for every prime between 1009 and 8389. When this multiple is in the range of volume 2, we can be sure that a specific list of multiples was not filled.

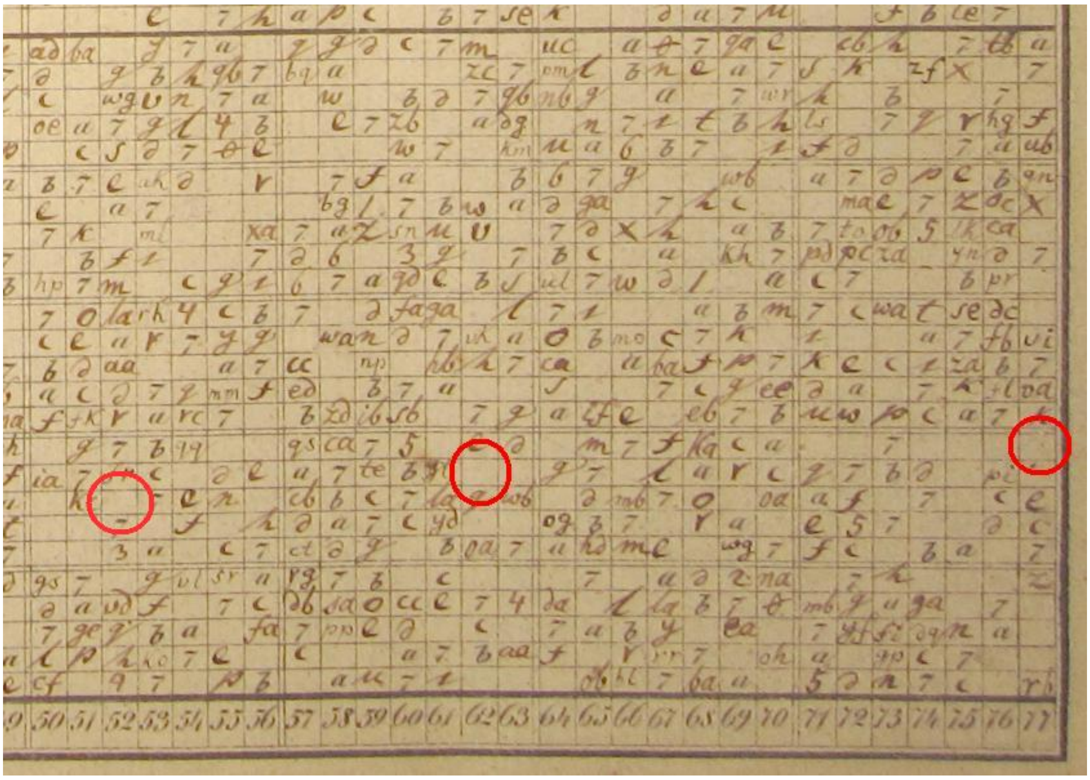


Figure 25: Excerpt of page 416, the last page of volume 1. We have marked three factors that were forgotten, probably when holes in a matrix were mistakenly skipped. The symbols "ai" should have been written in column 52 and 62, and "rh" in column 77. The previous occurrences of "ai" in column 22 and of "rh" in column 48 are correctly given. It is therefore possible that volume 1 is complete, minus some errors or omissions. (AÖAW, Nachlass Kulik, reproduced by the author)

[25]This microfilm contained pages 259 to 416 of Kulik's table, that is all pages from volume 1 giving the factors of numbers beyond the 9th million. It should be noted that upon suggestion by the mathematician George Birkhoff, this part of Kulik's table had already been sent by the Carnegie Institution to the Harvard University Library in 1925, for use in the Widener Library.

[26]We have not checked this part of volume 1, but this is one area where further investigation is needed.

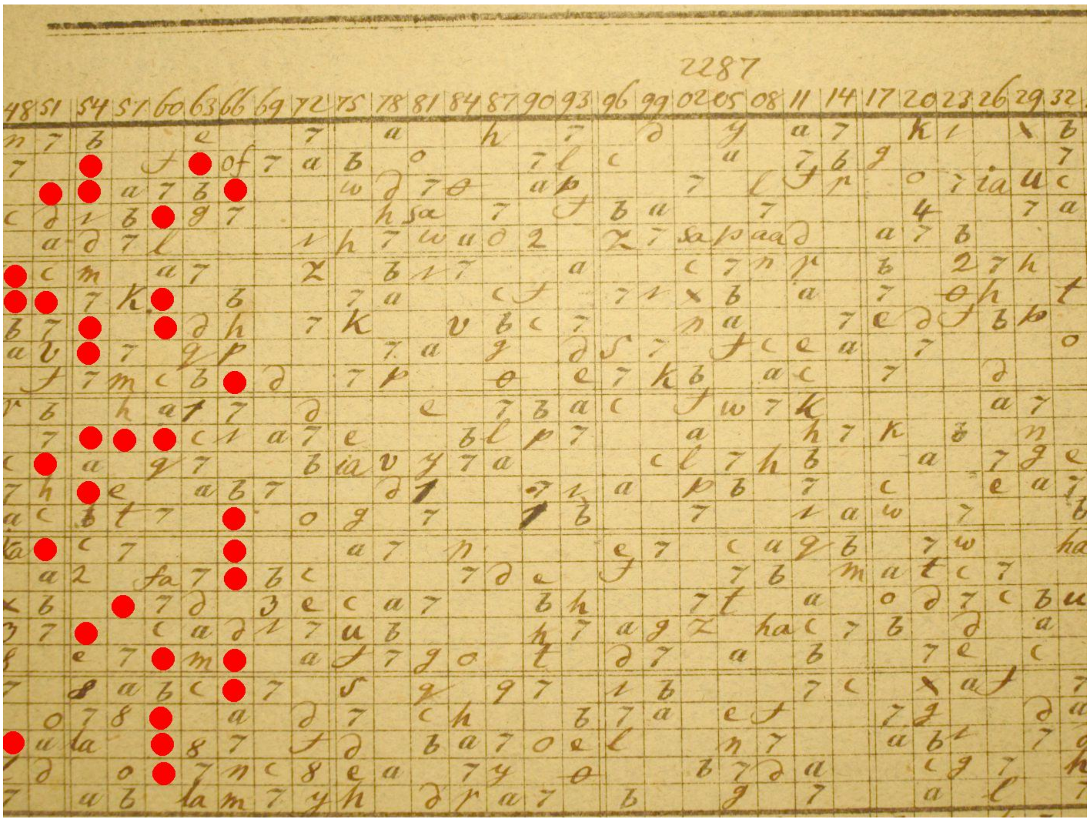


Figure 26: Excerpt of page 859, the first page of the third volume. The circles show the missing values in columns 48 to 66, but there are many missing values in the other columns. (AÖAW, Nachlass Kulik, reproduced by the author)

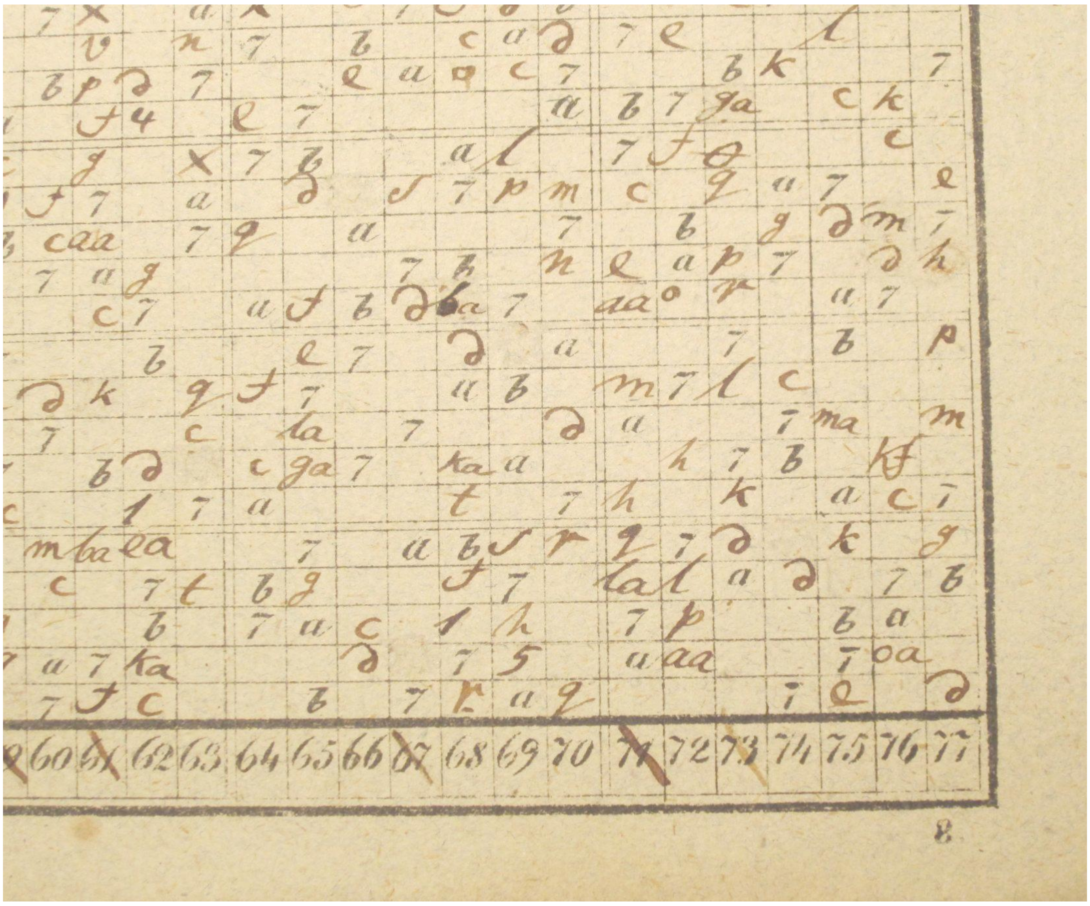


Figure 27: Excerpt of page 867. There are many missing values, and they can be located easily using our reconstructions. Note that a '8' appears at the bottom of the page, probably correlated with the positions of the 'b's. (AÖAW, Nachlass Kulik, reproduced by the author)

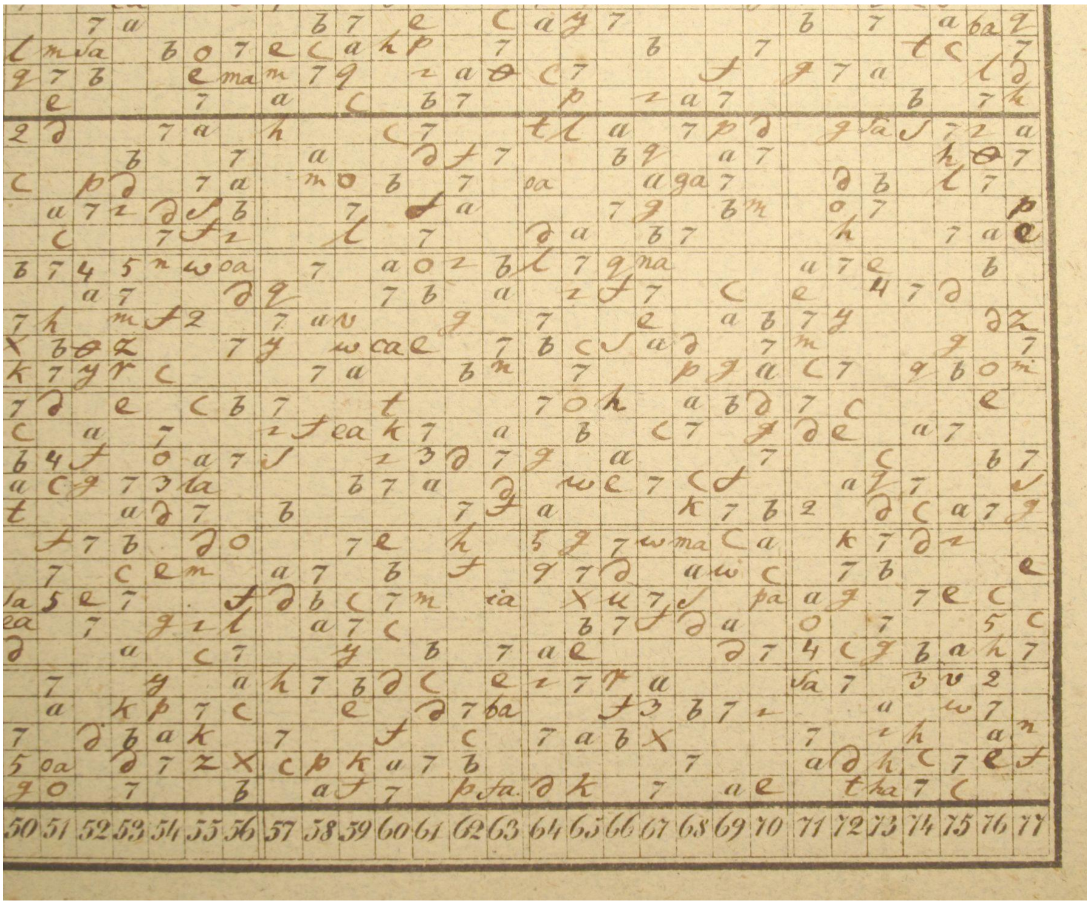


Figure 28: Excerpt of page 1963. There are many missing values, and they can be located easily using our reconstructions. (AÖAW, Nachlass Kulik, reproduced by the author)

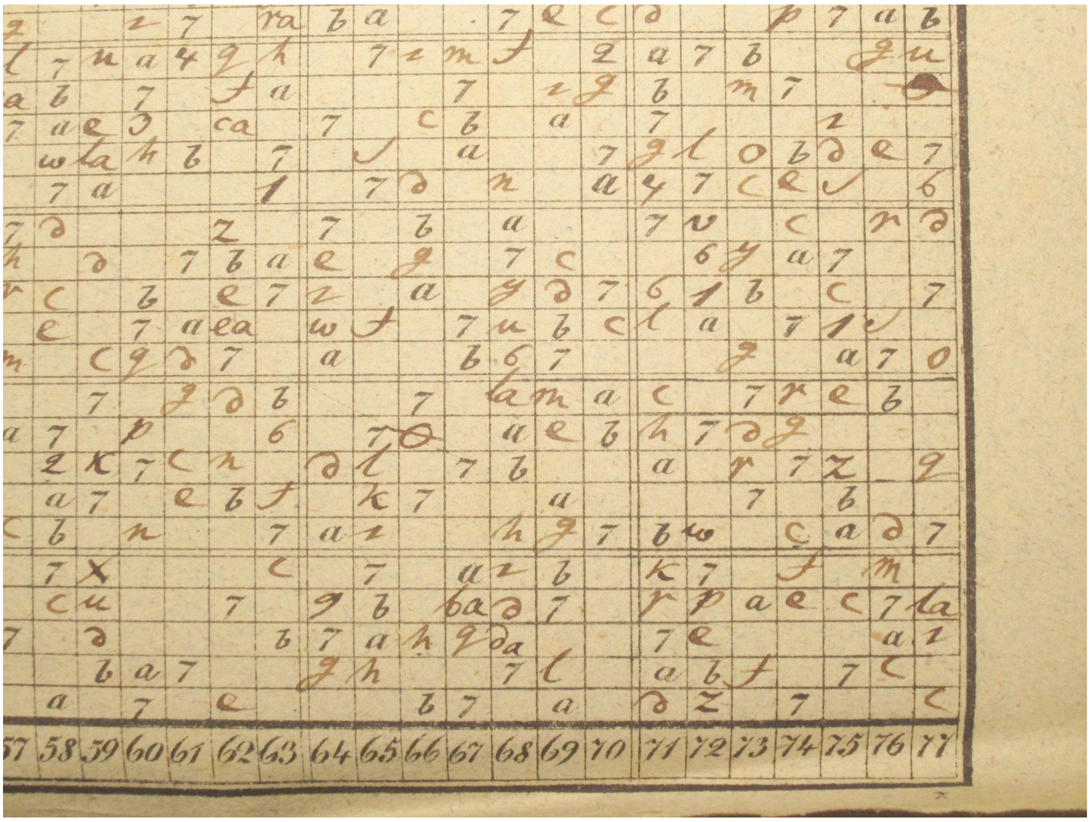


Figure 29: Excerpt of page 2792. There are many missing values, and they can be located easily using our reconstructions. (AÖAW, Nachlass Kulik, reproduced by the author)

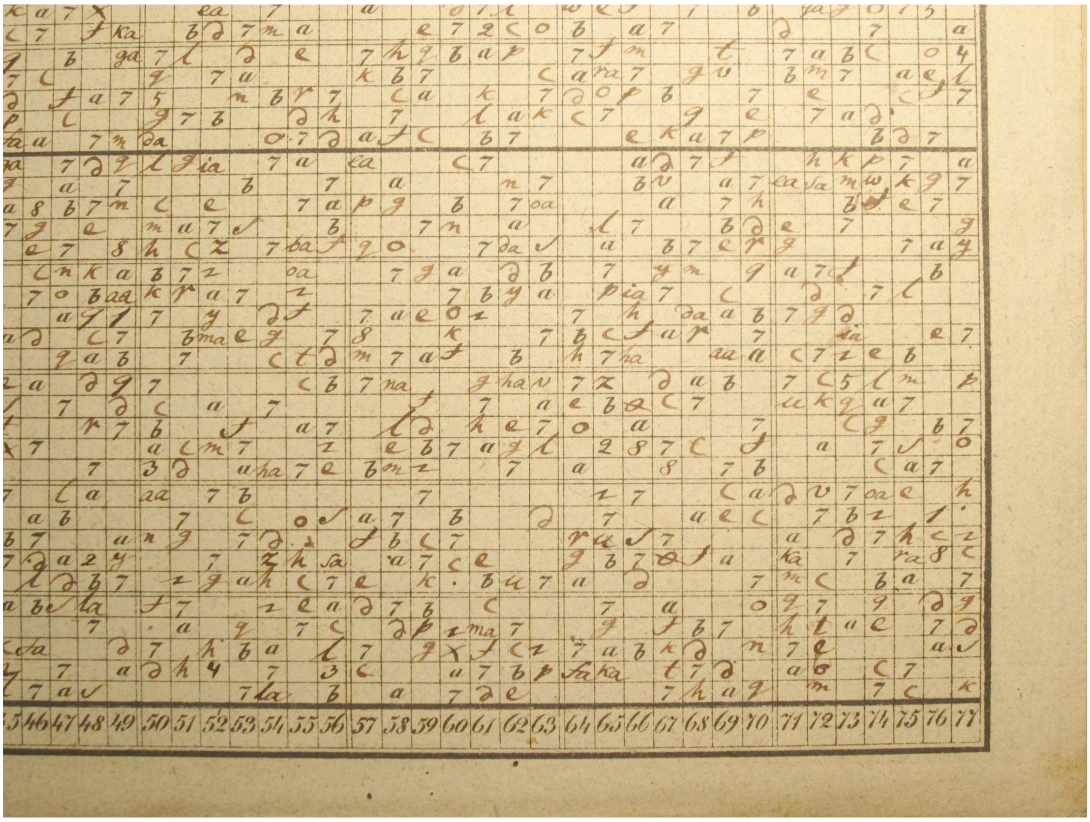


Figure 30: Excerpt of page 3068. There are many missing values, and they can be located easily using our reconstructions. (AÖAW, Nachlass Kulik, reproduced by the author)

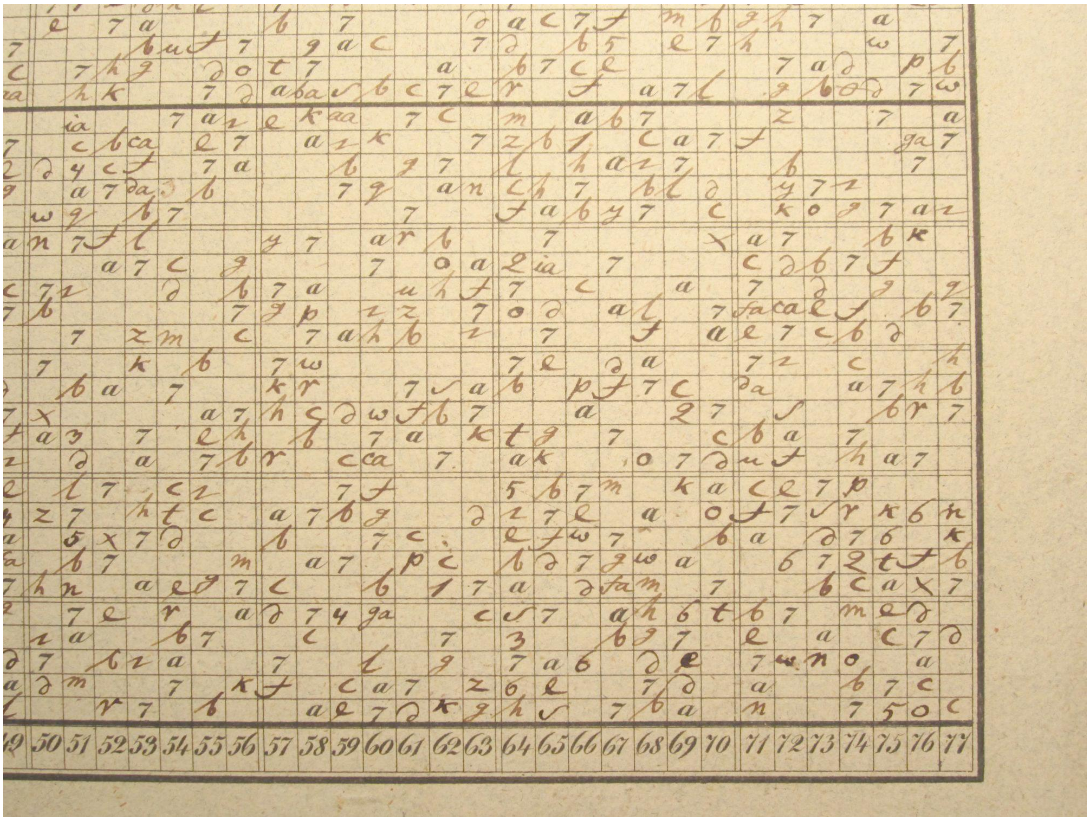


Figure 31: Excerpt of page 3353. There are many missing values, and they can be located easily using our reconstructions. (AÖAW, Nachlass Kulik, reproduced by the author)

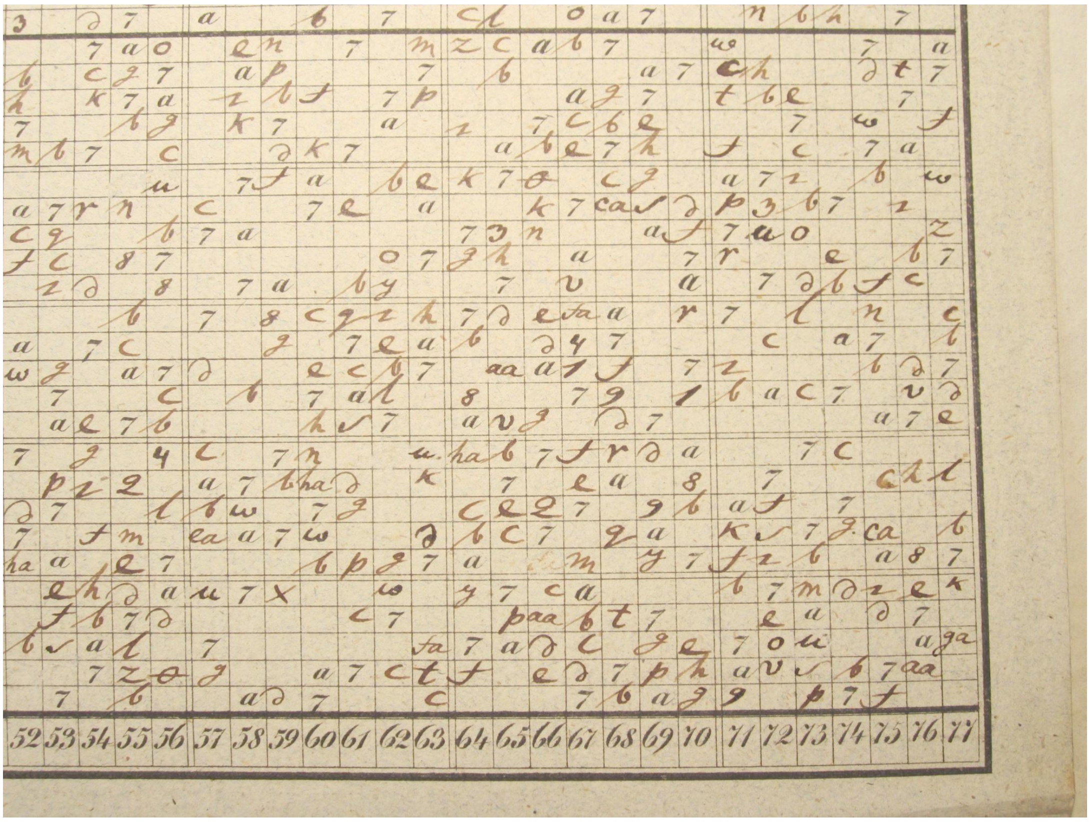


Figure 32: Excerpt of page 3925. There are many missing values, and they can be located easily using our reconstructions. (AÖAW, Nachlass Kulik, reproduced by the author)

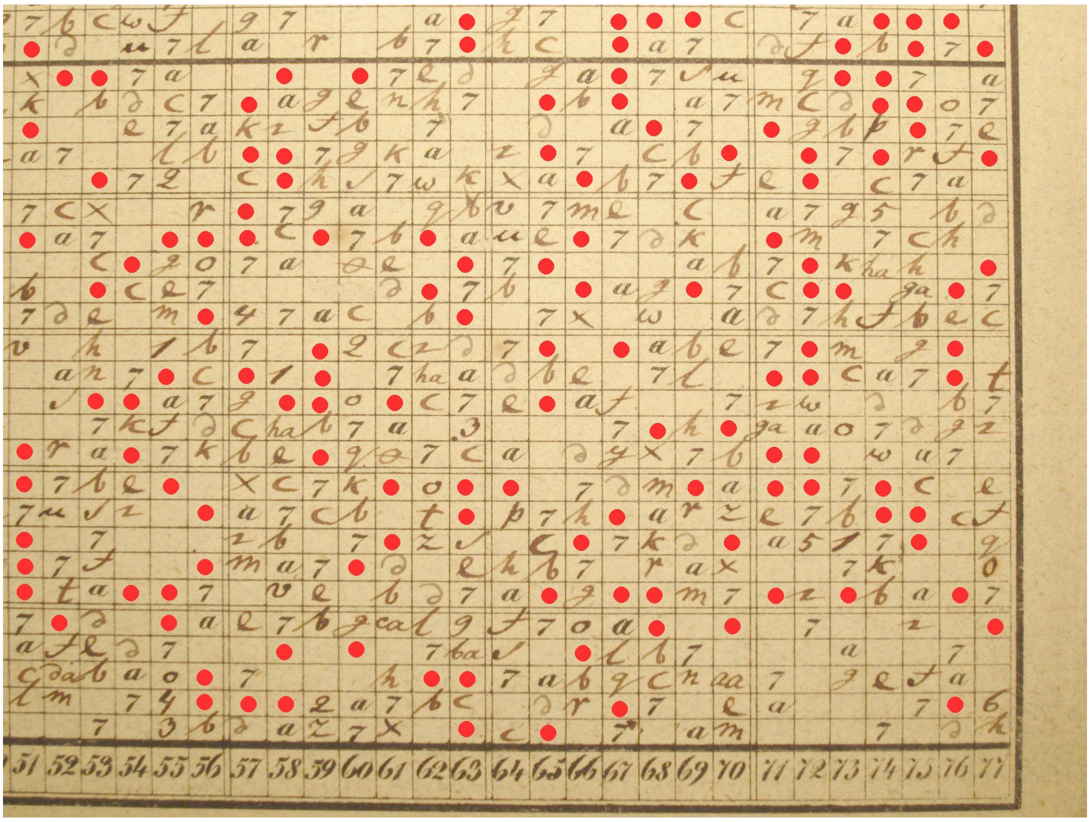


Figure 33: Excerpt of page 4212, the last page of the tables. The circles indicate the missing factors. Note that the "b"s are not preprinted here. (AÖAW, Nachlass Kulik, reproduced by the author)

7 The fate of Kulik's tables

7.1 The deposit of the tables and Petzval's report (1863–1866)

Kulik died in February 1863, and on the meeting of the mathematical and natural sciences committee on March 12, 1863, Josef Petzval informed the committee that Kulik's manuscripts do still exist. Professor Reuss[27] was assigned the task to investigate this matter.[28]

On the meeting which took place on April 16, 1863, Professor Reuss reported that Kulik's heirs were willing to let the Academy examine Kulik's manuscripts.[29] In May 1863, Kulik's son Justin then gave Kulik's tables and manuscripts to the Academy of sciences.[30]

On the meeting which took place on May 15, 1863, it was reported that Kulik's manuscripts had been acquired. Petzval was assigned the task to check the correction of the manuscripts and to report on it during a future meeting.[31]

The next mention of Kulik occured on the meeting which took place on March 8, 1866, when a letter by Justin Kulik dated from February 22 was mentioned, in which he asked about the results of the analysis of the tables.[32] On the meeting which took place on March 15, 1866,[33] Petzval reported on Kulik's *Nachlass* and the report was set to appear in the *Sitzungsberichten*.[34] It is not clear if the report was prompted by Justin Kulik's letter.

In his report, Petzval announced that Kulik's *Nachlass* had been given to the Academy of sciences in Vienna [89, 90, 91] in May 1863. The table was said to give the smallest factors of all numbers not divisible by 2, 3, and 5 from 3 to 100 millions. Petzval wrote that the tables occupy six volumes, but this might be a typographical error. Petzval mentioned the work of Burckhardt, as well as Crelle's unpublished tables deposited at the Academy of sciences in Berlin, and Dase's seventh and eighth millions.[35] Petzval observed that Dase needed one volume per million, but that Kulik fits 97 millions in six (sic) volumes, and he attributed this reduction to the use of symbols instead of numbers.[36] He regreted however that the size of the table was still too big for printing.

Petzval suggested that Kulik's naming scheme could be iterated, and that the two-letter symbols could be replaced by specially designed symbols, which he called a "prime

[27]Probably August Emanuel von Reuss (1811–1873) who was an Austrian doctor, geologist, and palaeontologist.

[28]AÖAW, Sitzungsprotokoll der Mathematisch-Naturwissenschaftlichen Klasse, B 454.

[29]AÖAW, SP mn-Klasse, B 457.

[30]AÖAW, Allg. Akt. 668/1905. The Kulik file in the Archives of the Academy of sciences contains several letters by Justin Kulik, but these letters have not been analyzed for our study.

[31]AÖAW, SP mn-Klasse, B 460.

[32]AÖAW, SP mn-Klasse, B 543.

[33]AÖAW, SP mn-Klasse, B 544.

[34]The published report is [90] and appeared appended to the reports for the meeting of 22 March 1866, although Kulik is not mentioned in the corresponding *Sitzungsprotokoll* 545.

[35]Petzval seems to have been unaware of Dase's 9th million which had been published in 1865.

[36]Nový wrote that Petzval's description was not accurate [84, p. 329], but this is not our feeling. Petzval does not go into the details, but apart from the number of volumes, his description is not really wrong.

number stenography." He set the task of finding simple symbols for all primes having up to four digits. These symbols should take little horizontal and vertical space, and should not risk to be confused with each other. He suggested the use of certain features to help identify whether the prime has 2, 3, or 4 digits. Petzval wrote that the symbols should give a pleasing appearance when they are printed on the page.[37] Petzval thought that this would make it possible to reduce the size by a quarter, and to fit 25 millions in one volume.

On April 1st, 1866, Justin Kulik wrote to the Academy in order to thank for the publication of Petzval's notice [3, 4]. On the meeting which took place on April 12, 1866, mention was made of Justin Kulik's letter.[38]

7.2 Lehmer's request to view the tables (1905–1914)

Kulik's tables do not seem to have raised any attention between 1866 and 1905. It was in 1905 that the mathematician Derrick Norman Lehmer (1867–1938) first contacted the Academy of sciences in Vienna.[39] Lehmer was then preparing a table of factors for the first ten millions, and he could only compare the first nine millions with the tables of Burckhardt, Dase and Glaisher. No table covering the tenth million had been published, although Dase computed it partially. This tenth million had been deposited at the Academy of sciences in Berlin, but when Lehmer needed it, it seemed to have vanished and could not be located [73, p. VIII]. Lehmer's attention was then drawn to Kulik's work.

Lehmer's letter dated June 7, 1905 (figure 34) mentioned Petzval's 1866 description [90] and asked if the manuscript could be sent to California for a comparison with the table Lehmer was computing.

On the meeting of the Academy which took place on June 23, 1905, Lehmer's request was reported.[40] On the meeting which took place on July 6, 1905, the deposit of Kulik's manuscripts was recounted, as well as Petzval's hope that someone could use these tables in the future. Lehmer's request did fit this hope and the Academy therefore agreed, but first decided to contact Kulik's heirs for their permission.[41]

A letter from the Academy to the *Landesgericht* (court) in Prague and dated 12 July 1905 stated that the Academy would be happy to oblige, but wished that the work of Kulik be fully recognized and requested details about how to proceed in this case, fully aware that this was an opportunity for recognition of Kulik. The Academy also asked if Kulik's son Justin, or other heirs, were still alive.

On the same day, the Academy replied to Lehmer that the manuscripts were filling a crate, and that the Academy did not object to sending the manuscripts, but that it had first to request permission from the heirs, because the manuscripts were only given in deposit, not donated to the Academy. The Academy also stressed that in case that the

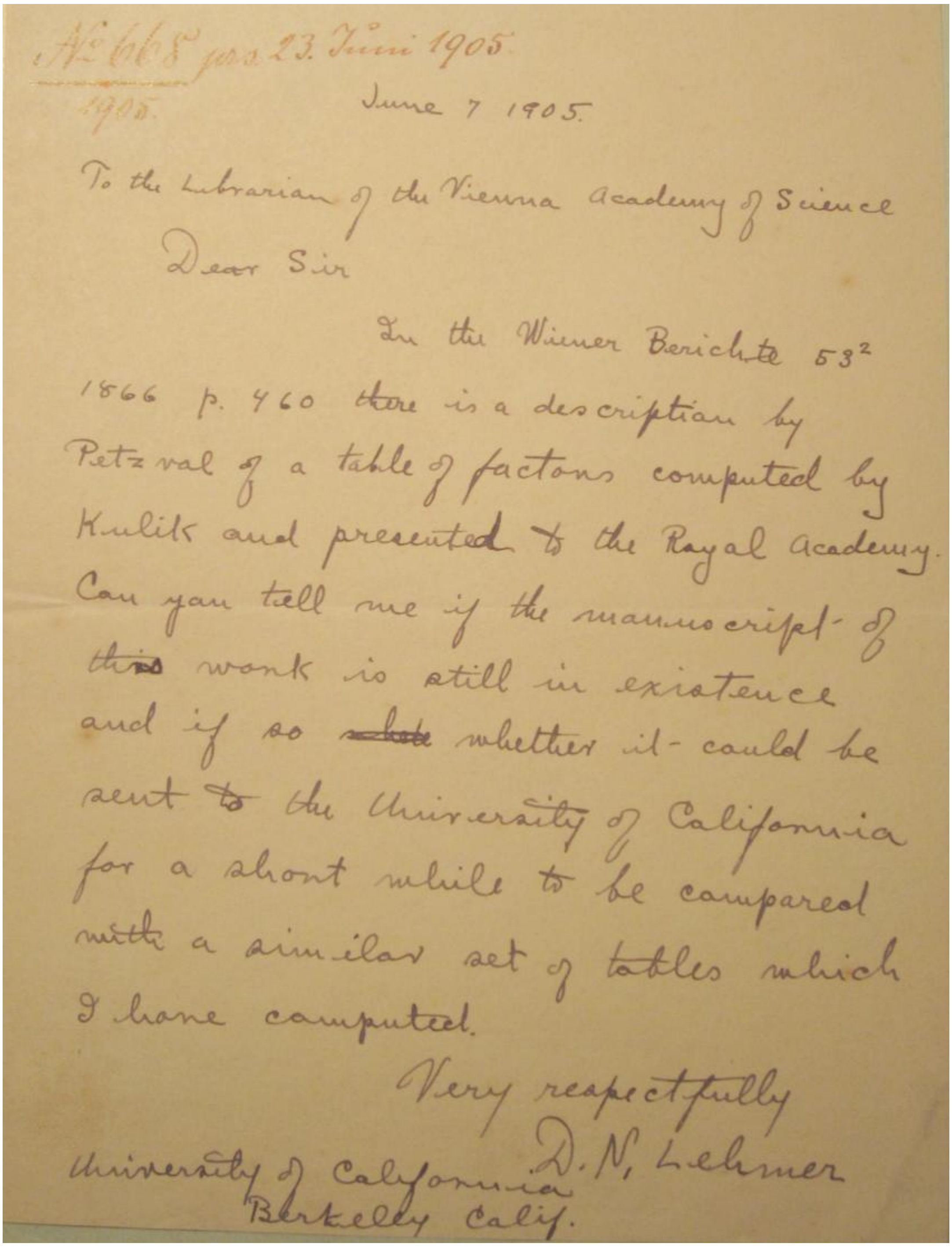
[37] One readily solution that comes to mind is to use the number notation developed in the late 13th century by Cistercian monks and described by David King [49].

[38] AÖAW, SP mn-Klasse, B 546.

[39] AÖAW, Allg. Akt. 668/1905. The letters concerning the interaction with Lehmer are all contained in this file, and we will not repeat this reference.

[40] AÖAW, SP mn-Klasse, B 1674.

[41] AÖAW, SP mn-Klasse, B 1675.



No. 668 pr. 23. Juni 1905.
1905.

June 7 1905.

To the Librarian of the Vienna Academy of Science

Dear Sir

In the Wiener Berichte 53² 1866 p. 460 there is a description by Petzval of a table of factors computed by Kulik and presented to the Royal Academy. Can you tell me if the manuscript of this work is still in existence and if so ~~what~~ whether it could be sent to the University of California for a short while to be compared with a similar set of tables which I have computed.

Very respectfully
D. N. Lehmer

University of California
Berkeley Calif.

Figure 34: Lehmer's first letter to the Academy of sciences in Vienna, dated June 7, 1905. (AÖAW, Allg. Akt. 668/1905, reproduced by the author)

manuscripts were sent, the costs of shipping would be Lehmer's. In the eventuality of a publication, Lehmer was also expected to recognize Kulik's work.

Kulik's heirs, Justin Kulik in Prague (1837–1915), and Angela von Randa in Dobřichovice (1841–1925, wife of Antonin Randa, 1834–1914),[42] gave their permission dated 26 July 1905 through the court in Prague. Kulik's heirs stated that they agreed with sending all or parts of the manuscripts, as this fulfilled Kulik's wishes that his tables serve to facilitate the work of others.

A letter dated 11 August 1905 from the Academy to Kulik's heirs acknowledged their permission to send the manuscripts of Kulik to Lehmer, thanked them for granting the Academy the rights on the manuscripts, let them know that the work of their father would be recognized, and promised them a copy of Lehmer's work based on Kulik's tables. The truth, of course, is that Kulik's table mainly served as a check of Lehmer's table, and that Lehmer's work was not really based on that of Kulik.

On August 11, 1905, the Academy also wrote to the court in Prague to thank it for its assistance.

The Academy then informed Lehmer of the permission on the same day. However, Lehmer was also informed that the Academy could not send the manuscripts to a private person but only to an organization and it requested his suggestions. The Academy suggested for instance to go through the direction of the University of Berkeley or the Smithsonian Institution in Washington. The Academy also wished that Lehmer send a copy of his work to the heirs. It also asked if Lehmer wished the entire collection of manuscripts, or only a part of it, stressing that the manuscripts were contained in a large crate and sending them all would be expensive.

Lehmer answered on 31 August 1905 and informed the Academy that his work would be published by the Carnegie Institution of Washington which will cover the cost of shipping. In his letter, Lehmer wrote that he needed to examine the eight volumes of tables, the two volumes of matrices, the three volumes of multiples, the booklet on factoring, and the table of periodic fractions.

On the 12th of October 1905, the Academy wrote to the Carnegie Institution of Washington, summarizing the current state of affairs, and indicating that it had not yet received news from the Institution. It therefore asked if the Carnegie Institution was willing to cover the cost of shipping.

On October 27, 1905, Robert Simpson Woodward, the president of the Carnegie Institution wrote to Eduard Suess, the president of the Academy of sciences in Vienna, acknowledging receipt of the letter from October 12, and confirming that it would pay for the entire cost of shipping and insurance. Furthermore, Woodward stated that the manuscripts would be put in a fire proof building.

It must have been in November that the Academy discovered that some of the volumes were missing, and on November 20, the Academy wrote to Ludwig Erményi about this loss. Erményi had written an article on the life and work of Josef Petzval (1807–1891) in 1902 [33] and the Academy thought that it was perhaps Petzval who had taken Kulik's manuscripts home. Erményi replied on November 22, 1905 that he had gone through Petzval's *Nachlass*, and hadn't noticed any manuscript by Kulik, but that some of Petzval's papers had been lost.

[42] Kulik, his wife, children, and Antonin Randa are all buried in the same place in Prague.

The Academy then wrote on November 25, 1905 to the Carnegie Institution that a closer inspection of the manuscripts had revealed that they were not totally complete. Volume 2, as well as one of the volumes of matrices, were reported missing. The cubic volume of the manuscripts was estimated to be about 0.7m $\times$ 0.54m $\times$ 0.45m.

The president of the Carnegie Institution wrote on January 2, 1906, in order to ask that the first volume of Kulik's tables, as well as the three volumes of multiples, and the volume of matrices be sent. The fact that not all volumes were requested was certainly a consequence of the shipping cost and the fact that Lehmer didn't really need all of Kulik's tables for his own table.

The Academy wrote on January 27, 1906 to the Carnegie Institution to confirm that the volumes have been sent, and a letter from the Carnegie Institution of March 22, 1906, acknowledged receipt of the five requested volumes.

The Academy wrote on 11 April 1906 to the Carnegie Institution about the payment of the cost of shipping and insurance, amounting to 112 Kronen and 53 Heller (112.53 Kronen), and a voucher was sent by the Carnegie Institution on April 30, 1906 for payment.

It is interesting to note that the Academy of sciences considered that Petzval's hope for a printing of the tables could perhaps eventually be filled by Lehmer.

In his 1909 book, Lehmer concluded that Kulik's table was not accurate enough for publication [73, pp. IX–X]. Lehmer acknowledged that Kulik's naming scheme was unsuitable for comparison with other tables. But he also observed that Kulik's scheme could be combined with Lehmer's layout and make it possible to produce a table of factors to 100 millions which would only occupy about 5 volumes like Lehmer's table of factors. If such a work were undertaken, says Lehmer, "the importance of the Kulik manuscripts would be inestimable."

Some time around 1910–1913, Lehmer then went to Vienna and examined all volumes, except the second one, of course. But he apparently failed to see that they were not complete.

Lehmer was instrumental in maintaining the impression that Kulik's tables were complete, albeit with errors.

In 1914, Lehmer wrote that Kulik's table would serve as a good check for a list of primes [74, pp. IX–X]. Such a check would not require the manipulation of Kulik's symbols.

7.3 More recent research

After Lehmer's publications, Kulik's tables began to attract other researchers, although some researchers may have discovered Kulik's work accidentally through Petzval's report.

In 1927, Kuno Schaefer from Danzig had the project of constructing a table of factors, and he therefore contacted the Academy of sciences about the extent of Kulik's tables in order to avoid duplicate work. The Academy of sciences asked the mathematician Wilhelm Wirtinger to give an answer.[43]

In 1946, S. A. Joffe wrote to the Academy of sciences, inquiring about the exact structure of Kulik's table, which resulted in a short note correcting Lehmer's statements [48].

[43] AÖAW, Allg. Akt. 668/1905.

Luboš Nový must have come to Vienna in the early 1960s and he subsequently gave the first detailed description of Kulik's *Nachlass* [84].

Then the American Mathematical Society wrote on 20 June 1969 to the Academy of sciences in order to compile an index of unpublished mathematical tables (UMT).[44] It therefore wished to obtain either Kulik's tables, or a copy of the tables. A first answer was sent on 11 August 1969, with the details of the seven volumes. In addition, R. Biebl answered on 24 August 1969 that no volume of the table could be sent, but that photocopies could be made for storage in Washington. The American Mathematical Society replied on September 23, 1969. It is not clear if copies were made for the UMT file.

Various other researchers or interested persons have consulted the tables until the beginning of the 21st century, but often without publishing on the tables.[45] Among those who have either examined the tables or wrote on them recently, we can cite Edmund Hlawka, Paulo Ribenboim [98], Christa Binder, and Štefan Porubský.

8 Reconstruction

The greatest part of our reconstruction was completed in March 2011, based on the fairly accurate descriptions given by Lehmer and Nový. The original tables were examined in August 2011, and our reconstruction was subsequently slightly improved. We had initially made the assumption that Kulik had followed Burckhardt's layout, where a page was separated in three parts with three headers, and this proved to be wrong. Kulik's layout makes it somewhat more difficult to locate a number in the lower parts of the tables.

We have added the interval of numbers at the top of each page, as did Lehmer later in his tables, but this feature does not appear in Kulik's tables.

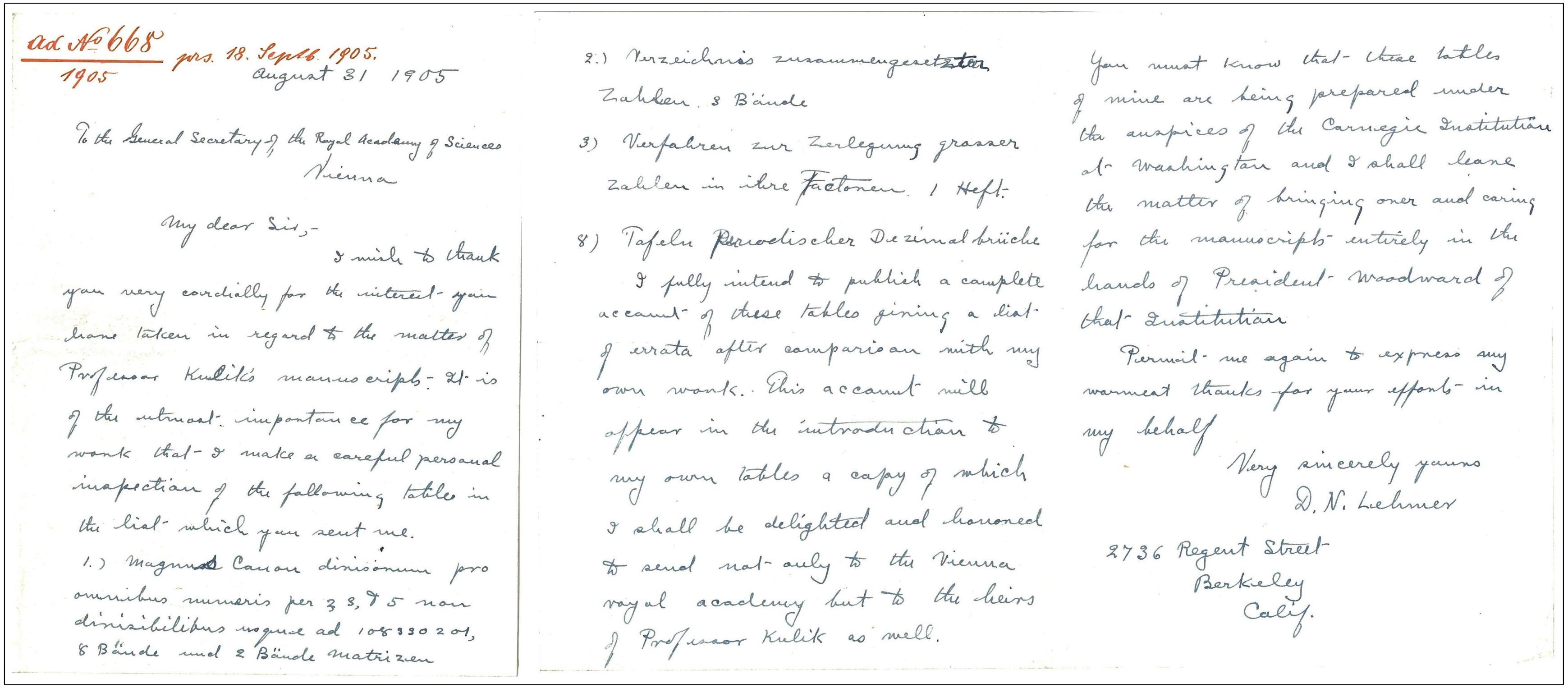
For the symbols, we have decided to use only roman and gothical letters, but no italics, as the italics would have conflicted with the gothical characters.

In addition to the "standard" volumes, we have also produced a "flat" version in 16 volumes, where all sequences run continuously, but where front and back parts have been separated. These volumes are named X-1 to X-16 (table 3).

[44] AÖAW, Allg. Akt. 668/1905.

[45] Among the persons who examined Kulik's tables, we can mention Fenwick Wesencraft (1921–2008), a retired British banker. He visited the Academy of sciences in 1973 and observed that the list of primes did not extend beyond 8059 and he set out to check several numbers in volume 8 whose smallest factors are greater than 8059. Wesencraft's case is probably typical of many researchers, in that a first examination of the manuscript was not sufficient, and was followed by some computations at home, and then new questions. The lack of preparation has always made it necessary to postpone some investigations, and we hope that our work will make it easier for researchers to prepare their examination of Kulik's tables and papers.

In that case, the curator Klaus Wundsam answered to Wesencraft that there was a gap in the table and that the numbers that Wesencraft sought were not to be found. In fact, Wundsam was confused because volume 8 contains two parts, and the continuation of the first part is on the back of volume 7 (see table 1). Wundsam eventually sent photocopies of several pages and Wesencraft observed that Kulik had only inserted factors up to 211 in these pages.



ad No 668 / 1905 pr. 18. Sept. 1905.

August 31 1905

To the General Secretary of the Royal Academy of Sciences
Vienna

My dear Sir,-

I wish to thank you very cordially for the interest you have taken in regard to the matter of Professor Kulik's manuscript. It is of the utmost importance for my work that I make a careful personal inspection of the following tables in the list which you sent me.

1.) ~~Magnus~~ Magnus Canon divisorum pro omnibus numeris per 2, 3, & 5 non divisibilibus usque ad 108330201, 8 Bände und 2 Bände Matrizen

2.) Verzeichnis ~~zusammengesetzten~~ zusammengesetzter Zahlen. 3 Bände

3) Verfahren zur Zerlegung grosser Zahlen in ihre Factoren. 1 Heft.

8) Tafeln periodischer Dezimalbrüche

I fully intend to publish a complete account of these tables giving a list of errata after comparison with my own work. This account will appear in the introduction to my own tables a copy of which I shall be delighted and honored to send not only to the Vienna royal academy but to the heirs of Professor Kulik as well.

You must know that these tables of mine are being prepared under the auspices of the Carnegie Institution at Washington and I shall leave the matter of bringing over and caring for the manuscript entirely in the hands of President Woodward of that Institution.

Permit me again to express my warmest thanks for your efforts in my behalf.

Very sincerely yours
D. N. Lehmer

2736 Regent Street
Berkeley
Calif.

Figure 35: Lehmer's letter from August 31, 1905 to the Academy of sciences in Vienna. (AÖAW, Allg. Akt. 668/1905, reproduced by the author)

volume	first page	last page	first number	last number
X- 1	1	208	3 033 001	7 837 799
X- 2	209	416	7 837 801	12 642 599
X- 3	417	637	12 642 601	17 747 699
X- 4	638	858	17 747 701	22 852 799
X- 5	859	1135	22 852 801	29 251 499
X- 6	1136	1411	29 251 501	35 627 099
X- 7	1412	1687	35 627 101	42 002 699
X- 8	1688	1963	42 002 701	48 378 299
X- 9	1964	2240	48 378 301	54 776 999
X-10	2241	2516	54 777 001	61 152 599
X-11	2517	2792	61 152 601	67 528 199
X-12	2793	3068	67 528 201	73 903 799
X-13	3069	3353	73 903 801	80 487 299
X-14	3354	3640	80 487 301	87 116 999
X-15	3641	3925	87 117 001	93 700 499
X-16	3926	4212	93 700 501	100 330 199

Table 3: Structure of the 16 “flat” volumes of Kulik’s tables.

9 Kulik’s *Nachlass*

9.1 Manuscripts deposited in 1863

After Kulik’s death in February 1863, Kulik’s son Justin brought to the Academy a crate containing following manuscripts:[46]

1. *Magnus Canon divisorum pro omnibus numeris per 2, 3 & 5 non divisibilibus usque ad 108330201. 8 Bände und 2 Bände Matrizen.*
   [8 volumes and 2 volumes of matrices]
2. *Verzeichniß zusammengesetzer Zahlen. 3 Bände.*
   [Index of added numbers. 3 volumes.]
3. *Verfahren zur Zerlegung großer Zahlen in ihre Factoren. 1 Heft.*
   [Procedure for splitting large numbers into their factors. One booklet.]
4. *Natürliche Logarithmen der Sekanten für jedes halbe hunderteldes Grades in 12 decimalstellen.*
   [Natural logarithms of the secants for for each half hundredth of a degree to 12 decimal places.]
5. *Vorarbeiten goniometrischer Tafeln.*
   [Preparatory work for goniometric tables.]

[46]AÖAW, Allg. Akt. 668/1905. The note dated 6 May 1863 and signed Reuss (probably August Emanuel von Reuss) states that Kulik’s son has *just* deposited a box of manuscripts. At that time, the exact conditions of deposit were not made totally clear.

6. *Logarithmisch-trigonometrische Tafeln zu 10 Decimalstellen.*
   [Logarithmic-trigonometric tables to 10 decimal places.]
7. *Tafeln zur Berechnung briggischer Logarithmen.*
   [Tables for the computation of Briggian logarithms.]
8. *Tafeln periodischer Decimalbrüche.*
   [Tables of periodic decimal fractions.]
9. *Längen elliptischer Quadranten in 10 Stellen.*
   [Lengths of elliptic quadrants to 10 places.]

and the description adds

> *und mehrere andere Tafeln deren Bedeutung dem Übersender unbekannt ist.*

In other words, there were also some tables whose purpose Justin Kulik did not know.

9.2 The *Nachlass* in 2011

As was already stated, the second volume was lost, sometime before Lehmer was sent some volumes. It is quite likely that the second volume of matrices was also lost at the same time. The two were perhaps together, and it is not impossible that they were borrowed by someone who wished to check the list of primes with the matrices. These volumes may even still exist somewhere. One person who was thought to have these manuscripts was Josef Petzval, but Kulik's manuscripts were not found in his *Nachlass*.[47]

Apart from these two losses, items 3 and 8 above seem also missing, but the remaining part seems still part of the current *Nachlass*. We have gone through the whole *Nachlass* in August 2011, but not page by page. The only prior published description of Kulik's *Nachlass* seems to be that of Nový [84]. The items have been organized by the curator Klaus Wundsam in the 1970s in seven groups, as follows:[48]

1. Works on mathematical tables.
   1. final digits of decimal logarithms of numbers 1000 to 20000; lengths of elliptical quadrants; "Toasir"-tables; tables for the resolution of cubic equations;
   2. preparatory works for goniometric tables: decimal logarithms of cosines from 0 to 45 degrees with 14 places; miscellaneous tables;
   3. preparatory works for goniometric tables: natural sines and cosines to 11 places and natural tangents to 10 places;
2.
   1. computation of natural logarithms of sines and tangents; computation of decimal logarithms of sines and tangents to 12 places;
   2. hyperbolic sectors;
3. trigonometric tables;

[47] See above, letter by Erményi, 22 November 1905.

[48] The numbering given here is the current numbering of the *Nachlass*, so that the second volume of tables of multiples, for instance, has the signature 5.2.

4. trigonometric tables;
5. tables of multiples:
    1. primes 1009 to 1627;
    2. primes 1637 to 3069;
    3. primes 3061 to 8369;
6. matrices;
7. Magnus Canon divisorum: seven volumes (volume 2 missing).

In its current state, the *Nachlass* does not seem to contain the tables of decimal periods, but an inventory dated 1969 does mention them, and they do probably still exist.

10 Open questions

The analysis of Kulik's tables is not finished, and there are still a number of unanswered questions and things to check.

Although we know that Kulik's *Magnus Canon Divisorum* is far from complete, we do not know *how far* it is from being complete. The precise extent of Kulik's calculations should therefore be appraised. For every prime greater or equal to 1009, the greatest multiple computed should be recorded (to the exclusion of values computed in advance). Then, for every prime, both greater and smaller than 1009, the limit of inscription in the main table should be recorded. For primes smaller than 1000, these limits should decrease as the primes increase, because of the necessity to write the multiples in increasing order with the matrices. These data should make it possible to evaluate the amount of computation and to determine precisely the time that would have been needed to complete the table.

Some practical details need also to be clarified. For instance, Kulik appears to have used two different types of forms. The basic form only bears preprinted values of 7 and 11 (a), and is valid for every page. But there are also some forms with preprinted 13 (b). It seems that there are 13 different such forms, but a closer inspection of the table should reveal when these forms were used. Were they used throughout volumes 1–6, or were there gaps? Were they also used in some places of volumes 7 and 8?

Volumes 5.2 and 5.3 of the *Nachlass* probably contain auxiliary multiplication tables, similar to the ones in figures 22 and 23 and they should be located.

All the matrix pages should be checked, and in particular their backs.

More generally, the whole set of seven volumes of tables should be examined page by page, at least to be sure that nothing is hiding between the pages.

We have currently only a sketchy idea of the accuracy of Kulik's tables, and more samples should be taken to get a good idea of the table's accuracy. Are there wrong values in some cells? Or are the wrong values factors which are not the smallest ones? Can this be explained by cells that were mistakenly not filled at some stage and were later filled by a factor which was not the smallest? As we have seen earlier, some of the errors found by Lehmer are of this type, but this analysis should be extended to all the errors found by Lehmer and also to other parts of the table.

It would also be useful to evaluate the computation time and the organization of the computations. And they should in turn be compared with the computation of the first table to 30 millions, whether or not it was completed.

11 Conclusion

In this study, we have tried to gather all the material available on Kulik's tables. Kulik's methods are now reasonably well understood and we have a good idea of the condition of his tables. Our analysis will hopefully pave the way for more detailed investigations.

But our study leaves an important question unanswered: why did Kulik fail? He was a great calculator, he worked on factor tables, on tables of multiplication, on tables of squares and cubes, on trigonometrical tables, on tables of logarithms, etc. Many of these tables were completed and published. Why was he not able to complete the *Magnus Canon Divisorum*?

We believe that there are several reasons for this failure. It is possible that Kulik did not plan well the amount of work, and perhaps did not organize the work in a sufficient rational way. He may also have suffered from a shortage of people to carry the calculations. If the calculations had been more advanced, the problem of the sheets wearing out through repeated manipulation would probably have come into consideration, but we really think that the culprit is the organization. Kulik does not seem to have organized the work as Prony did it for his table of logarithms [99].

Even though Kulik's tables are incomplete, they are still probably more complete than any other table computed before him. He went beyond the unpublished tables of Ulbrich, Felkel, Hindenburg, Schenmark, and others, as well as the published ones by Burckhardt and others who came after him.

The first complete tables of factors and primes up to 10 millions were published in 1909 and 1914 by Lehmer [73, 74]. Lehmer's table of factors was extended to 11 millions in 1951 [13] and in 1959, Charles L. Baker and Fred Joseph Gruenberger computed the list of primes up to 104 395 301 and made them available on microcards [72], about a century after Kulik's death.

12 Acknowledgements

It is our pleasure to acknowledge the help of Štefan Porubský who kindly sent us his articles on Kulik. They were especially useful for the reconstruction of Kulik's tables of multiplication [60] and of squares and cubes [59].

It was only very recently that we decided to go to Vienna and examine Kulik's table and his other manuscripts. We had already reconstructed the entire *Canon*, almost faithful to the original table. But in order to avoid producing only a "tentative" version of the reconstruction, we contacted the Archives of the Academy of sciences and Dr. Stefan Sienell immediately welcomed our visit. During our work at the Archives, it was Mrs. Petra Aigner who was most helpful. She provided every possible support for our

research and tried to locate every document relevant to Kulik's *Nachlass*. Her support and interest were invaluable.

References

The following list covers the most important references[49] related to Kulik's tables. Not all items of this list are mentioned in the text, and the sources which have not been seen are marked so. We have added notes about the contents of the articles in certain cases.

[1] Anonymous. Kulik, Jacob Philipp. In Constant von Wurzbach, editor, *Biographisches Lexikon des Kaiserthums Oesterreich*, volume 13, pages 356–359. Wien: k.k. Hof- und Staatsdruckerei, 1865.

[2] Anonymous. Kulik, Jakub Filip. In František Ladislav Rieger and Jakub Josef Dominik Malý, editors, *Slovník naučný*, volume 4, page 1054. Prague: I. L. Kober, 1865.

[3] Anonymous. (Justin Kulik's letter to the Academy). *Sitzungsberichte der mathematisch-naturwissenschaftlichen Classe der kaiserlichen Akademie der Wissenschaften*, 53(1):311, 1866.

[4] Anonymous. (Justin Kulik's letter to the Academy). *Sitzungsberichte der mathematisch-naturwissenschaftlichen Classe der kaiserlichen Akademie der Wissenschaften*, 53(2):465, 1866.

[5] Anonymous. A film of part of Kulik's Magnus Canon for sale. *Mathematical Tables and other Aids to Computation*, 3(23):222, July 1948.

[6] Raymond Clare Archibald. A volume of tables by Kulik. *Mathematical Tables and other Aids to Computation*, 2(13):59–60, 1946.

[7] Raymond Clare Archibald. New information concerning Isaac Wolfram's life and calculations. *Mathematical Tables and other Aids to Computation*, 4(32):185–200, 1950. [pages 195–196 concern Kulik's work on tables of natural logarithms]

[8] Raymond Clare Archibald. Giuseppe Palamà & L. Poletti: "Tavola dei numeri primi dell'intervallo 12 012 000 — 12 072 060" (review). *Mathematical Tables and other Aids to Computation*, 7(43):173, July 1953.

[9] Nicolaas George Wijnand Henri Beeger. On a scarce factor table. *Mathematical Tables and other Aids to Computation*, 2(20):326–330, 1947.

[49]**Note on the titles of the works:** Original titles come with many idiosyncrasies and features (line splitting, size, fonts, etc.) which can often not be reproduced in a list of references. It has therefore seemed pointless to capitalize works according to conventions which not only have no relation with the original work, but also do not restore the title entirely. In the following list of references, most title words (except in German) will therefore be left uncapitalized. The names of the authors have also been homogenized and initials expanded, as much as possible.

The reader should keep in mind that this list is not meant as a facsimile of the original works. The original style information could no doubt have been added as a note, but we have not done it here.

[10] Nicolaas George Wijnand Henri Beeger. A committee on factor tables. *Mathematical Tables and other Aids to Computation*, 4(30):121, 1950.

[11] Nicolaas George Wijnand Henri Beeger. Giuseppe Palamà and L. Poletti, "Tavola dei numeri primi dell'intervallo 12 012 000 — 12 072 060" (errata). *Mathematical Tables and other Aids to Computation*, 10(53):54, January 1956.

[12] Nicolaas George Wijnand Henri Beeger. Table of the least factor of the numbers that are not divisible by 2, 3, 5 of the eleventh million (review). *Mathematical Tables and other Aids to Computation*, 10(53):36, 1956.

[13] Nicolaas George Wijnand Henri Beeger, Albert Gloden, Luigi Poletti, and Robert James Porter. *Liste des nombres premiers du onzième million (plus précisément de 10 006 741 à 10 999 997) d'après les tables manuscrites de J. Ph. Kulik, L. Poletti et R. J. Porter*. Amsterdam: imprimerie WERTO, 1951. [reconstructed in [100]]

[14] Karl Christian Bruhns. Burckhardt, Johann Karl. In Historische Kommission bei der Bayerischen Akademie der Wissenschaften, editor, *Allgemeine Deutsche Biographie*, volume 3, pages 571–572. Leipzig: Duncker & Humblot, 1876.

[15] Maarten Bullynck. Factor tables 1657–1817, with notes on the birth of number theory. *Revue d'histoire des mathématiques*, 16(2):133–216, 2010.

[16] Johann Karl Burckhardt. *Table des diviseurs pour tous les nombres du deuxième million, etc.* Paris: Vve Courcier, 1814. [reconstructed in [103]]

[17] Johann Karl Burckhardt. *Table des diviseurs pour tous les nombres du troisième million, etc.* Paris: Vve Courcier, 1816. [reconstructed in [104]]

[18] Johann Karl Burckhardt. *Table des diviseurs pour tous les nombres des 1er, 2e et 3e million, etc.* Paris: Vve Courcier, 1817.

[19] Johann Karl Burckhardt. *Table des diviseurs pour tous les nombres du premier million, etc.* Paris: Vve Courcier, 1817. [reconstructed in [102]]

[20] Moritz Cantor. Crelle, August Leopold. In Historische Kommission bei der Bayerischen Akademie der Wissenschaften, editor, *Allgemeine Deutsche Biographie*, volume 4, pages 589–590. Leipzig: Duncker & Humblot, 1876.

[21] Moritz Cantor. Dase, Johann Martin Zacharias. In Historische Kommission bei der Bayerischen Akademie der Wissenschaften, editor, *Allgemeine Deutsche Biographie*, volume 4, page 759. Leipzig: Duncker & Humblot, 1876.

[22] Moritz Cantor. Felkel, Anton. In Historische Kommission bei der Bayerischen Akademie der Wissenschaften, editor, *Allgemeine Deutsche Biographie*, volume 6, page 612. Leipzig: Duncker & Humblot, 1877.

[23] Ladislaus Chernac. *Cribrum arithmeticum sive, tabula continens numeros primos, a compositis segregatos, occurrentes in serie numerorum ab unitate progredientium, usque ad decies centena millia, et ultra haec, ad viginti millia (1020000). Numeris compositis, per 2, 3, 5 non dividuis, adscripti sunt divisores simplices, non minimi tantum, sed omnino omnes*. Deventer: J. H. de Lange, 1811. [reconstructed in [105]]

[24] August Leopold Crelle. Auszug der Primzahlen in den ersten 6 Millionen aus den vorhandenen Factoren-Tafeln, 1842. [manuscript at the Archives of the Berlin Academy of Sciences, Nachlass Crelle, number 55, reconstructed in [122]]

[25] August Leopold Crelle. Abhandlung über die Mittel, eine Tafel der Primfactoren der Zahlen bis zu beliebiger Höhe möglichst leicht und sicher aufzustellen. *Bericht über die zur Bekanntmachung geeigneten Verhandlungen der Königl. Preuss. Akademie der Wissenschaften zu Berlin*, pages 272–279, 1853. [summary of [26]]

[26] August Leopold Crelle. Wie eine Tafel der untheilbaren Factoren der Zahlen bis zu beliebiger Höhe möglichst leicht und sicher aufzustellen sei. *Journal für die reine und angewandte Mathematik*, 51(1):61–99, 1855. [article supplemented by five plates; a summary is given in [25]]

[27] August Leopold Crelle. Factor tables for the fourth, fifth, and sixth million, ca. 1840. 3 manuscript volumes, Berlin

[28] Johann Martin Zacharias Dase. *Factoren-tafeln für alle Zahlen der siebenten Million etc.* Hamburg: Perthes-Besser & Mauke, 1862. [reconstructed in [108]]

[29] Johann Martin Zacharias Dase. *Factoren-tafeln für alle Zahlen der achten Million etc.* Hamburg: Perthes-Besser & Mauke, 1863. [reconstructed in [106]]

[30] Johann Martin Zacharias Dase and H. Rosenberg. *Factoren-tafeln für alle Zahlen der neunten Million etc.* Hamburg: Perthes-Besser & Mauke, 1865. [reconstructed in [107]]

[31] Harold T. Davis. *Tables of the higher mathematical functions*, volume 1. Bloomington, Indiana: The Principia Press, Inc., 1933. [Kulik's *Magnus Canon* is mentioned on page 165, but as having six volumes only]

[32] Иван Яковлевич (I. Ya.) Депман (Depman). Замечательные славянские вычислители Г. Вега и Я.Ф. Кулик (The remarkable slavic calculators G. Vega and J. P. Kulik). *Историко-математические исследования (Istorikomatematicheskie Issledovaniya)*, 6:573–608, 1953.

[33] Ludwig Erményi. Dr. Josef Petzval's Leben und wissenschaftliche Verdienste. *Photographisches Centralblatt*, 8(12):247–277, 1902. [not seen, an extended version was published in Hungarian in 1906]

[34] Anton Felkel. *Tafel aller einfachen Factoren der durch 2, 3, 5 nicht theilbaren Zahlen von 1 bis 10 000 000. I. Theil. Enthaltend die Factoren von 1 bis 144000.*

Wien: von Ehelenschen, 1776. [There is also a Latin edition [35] of this first part.] [reconstructed in [109]]

[35] Anton Felkel. *Tabula omnium factorum simplicum numerorum per 2, 3, 5 non divisibilium, ab 1 usque 10 000 000. Pars I. Exhibens factores ab 1 usque 144000.* Wien: A. Gheleniana, 1777. [Latin version of [34].] [not seen] [reconstructed in [109]]

[36] Anton Felkel. *Tabula factorum. Pars II. Exhibens factores numerorum ab 144001 usque 336000.* Wien: A. Gheleniana, 1777? [reconstructed in [109]]

[37] Anton Felkel. *Tabula factorum. Pars III. Exhibens factores numerorum ab 336001 usque 408000.* Wien: A. Gheleniana, 1777? [reconstructed in [109]]

[38] Jaroslav Folta. Social conditions and the founding of scientific schools: An attempt at an analysis on the example of the Czech geometric school. *Acta historiae rerum naturalium necnon technicarum*, special issue 10:81–179, 1977. [see pp. 89–90 for Kulik]

[39] James Glaisher. *Factor table for the fourth million etc.* London: Taylor and Francis, 1879. [reconstructed in [111]]

[40] James Glaisher. *Factor table for the fifth million etc.* London: Taylor and Francis, 1880. [reconstructed in [110]]

[41] James Glaisher. *Factor table for the sixth million etc.* London: Taylor and Francis, 1883. [reconstructed in [112]]

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Kulik's auxiliary table of primes (reconstruction, D. Roegel, 2011)

	a	b	c	d	e	f	g	h	i	k	l	m	n	o	p	q
a	167	311	461	619	797	971	1129	1307	1499	1693	1879	2083	2281	2459	2683	2851
b	173	313	463	631	809	977	1151	1319	1511	1697	1889	2087	2287	2467	2687	2857
c	179	317	467	641	811	983	1153	1321	1523	1699	1901	2089	2293	2473	2689	2861
d	181	331	479	643	821	991	1163	1327	1531	1709	1907	2099	2297	2477	2693	2879
e	191	337	487	647	823	997	1171	1361	1543	1721	1913	2111	2309	2503	2699	2887
f	193	347	491	653	827	1009	1181	1367	1549	1723	1931	2113	2311	2521	2707	2897
g	197	349	499	659	829	1013	1187	1373	1553	1733	1933	2129	2333	2531	2711	2903
h	199	353	503	661	839	1019	1193	1381	1559	1741	1949	2131	2339	2539	2713	2909
i	211	359	509	673	853	1021	1201	1399	1567	1747	1951	2137	2341	2543	2719	2917
k	223	367	521	677	857	1031	1213	1409	1571	1753	1973	2141	2347	2549	2729	2927
l	227	373	523	683	859	1033	1217	1423	1579	1759	1979	2143	2351	2551	2731	2939
m	229	379	541	691	863	1039	1223	1427	1583	1777	1987	2153	2357	2557	2741	2953
n	233	383	547	701	877	1049	1229	1429	1597	1783	1993	2161	2371	2579	2749	2957
o	239	389	557	709	881	1051	1231	1433	1601	1787	1997	2179	2377	2591	2753	2963
p	241	397	563	719	883	1061	1237	1439	1607	1789	1999	2203	2381	2593	2767	2969
q	251	401	569	727	887	1063	1249	1447	1609	1801	2003	2207	2383	2609	2777	2971
r	257	409	571	733	907	1069	1259	1451	1613	1811	2011	2213	2389	2617	2789	2999
s	263	419	577	739	911	1087	1277	1453	1619	1823	2017	2221	2393	2621	2791	3001
t	269	421	587	743	919	1091	1279	1459	1621	1831	2027	2237	2399	2633	2797	3011
u	271	431	593	751	929	1093	1283	1471	1627	1847	2029	2239	2411	2647	2801	3019
v	277	433	599	757	937	1097	1289	1481	1637	1861	2039	2243	2417	2657	2803	3023
w	281	439	601	761	941	1103	1291	1483	1657	1867	2053	2251	2423	2659	2819	3037
x	283	443	607	769	947	1109	1297	1487	1663	1871	2063	2267	2437	2663	2833	3041
y	293	449	613	773	953	1117	1301	1489	1667	1873	2069	2269	2441	2671	2837	3049
z	307	457	617	787	967	1123	1303	1493	1669	1877	2081	2273	2447	2677	2843	3061

Kulik's auxiliary table of primes (reconstruction, D. Roegel, 2011)

	r	s	t	u	v	w	x	y	z	𝔞	𝔟	𝔠	𝔡	𝔢	𝔣	𝔤
a	3067	3301	3499	3677	3889	4093	4289	4519	4733	4967	5171	5413	5623	5821	6047	6263
b	3079	3307	3511	3691	3907	4099	4297	4523	4751	4969	5179	5417	5639	5827	6053	6269
c	3083	3313	3517	3697	3911	4111	4327	4547	4759	4973	5189	5419	5641	5839	6067	6271
d	3089	3319	3527	3701	3917	4127	4337	4549	4783	4987	5197	5431	5647	5843	6073	6277
e	3109	3323	3529	3709	3919	4129	4339	4561	4787	4993	5209	5437	5651	5849	6079	6287
f	3119	3329	3533	3719	3923	4133	4349	4567	4789	4999	5227	5441	5653	5851	6089	6299
g	3121	3331	3539	3727	3929	4139	4357	4583	4793	5003	5231	5443	5657	5857	6091	6301
h	3137	3343	3541	3733	3931	4153	4363	4591	4799	5009	5233	5449	5659	5861	6101	6311
i	3163	3347	3547	3739	3943	4157	4373	4597	4801	5011	5237	5471	5669	5867	6113	6317
k	3167	3359	3557	3761	3947	4159	4391	4603	4813	5021	5261	5477	5683	5869	6121	6323
l	3169	3361	3559	3767	3967	4177	4397	4621	4817	5023	5273	5479	5689	5879	6131	6329
m	3181	3371	3571	3769	3989	4201	4409	4637	4831	5039	5279	5483	5693	5881	6133	6337
n	3187	3373	3581	3779	4001	4211	4421	4639	4861	5051	5281	5501	5701	5897	6143	6343
o	3191	3389	3583	3793	4003	4217	4423	4643	4871	5059	5297	5503	5711	5903	6151	6353
p	3203	3391	3593	3797	4007	4219	4441	4649	4877	5077	5303	5507	5717	5923	6163	6359
q	3209	3407	3607	3803	4013	4229	4447	4651	4889	5081	5309	5519	5737	5927	6173	6361
r	3217	3413	3613	3821	4019	4231	4451	4657	4903	5087	5323	5521	5741	5939	6197	6367
s	3221	3433	3617	3823	4021	4241	4457	4663	4909	5099	5333	5527	5743	5953	6199	6373
t	3229	3449	3623	3833	4027	4243	4463	4673	4919	5101	5347	5531	5749	5981	6203	6379
u	3251	3457	3631	3847	4049	4253	4481	4679	4931	5107	5351	5557	5779	5987	6211	6389
v	3253	3461	3637	3851	4051	4259	4483	4691	4933	5113	5381	5563	5783	6007	6217	6397
w	3257	3463	3643	3853	4057	4261	4493	4703	4937	5119	5387	5569	5791	6011	6221	6421
x	3259	3467	3659	3863	4073	4271	4507	4721	4943	5147	5393	5573	5801	6029	6229	6427
y	3271	3469	3671	3877	4079	4273	4513	4723	4951	5153	5399	5581	5807	6037	6247	6449
z	3299	3491	3673	3881	4091	4283	4517	4729	4957	5167	5407	5591	5813	6043	6257	6451

Kulik's auxiliary table of primes (reconstruction, D. Roegel, 2011)

	$\mathfrak{h}$	$\mathfrak{i}$	$\mathfrak{k}$	$\mathfrak{l}$	$\mathfrak{m}$	$\mathfrak{n}$	$\mathfrak{o}$	$\mathfrak{p}$	$\mathfrak{q}$	$\mathfrak{r}$	$\mathfrak{s}$	$\mathfrak{t}$	$\mathfrak{u}$	$\mathfrak{v}$	$\mathfrak{w}$	$\mathfrak{x}$
a	6469	6701	6911	7129	7393	7589	7823	8069	8287	8537	8737	8963	9187	9413	9629	9839
b	6473	6703	6917	7151	7411	7591	7829	8081	8291	8539	8741	8969	9199	9419	9631	9851
c	6481	6709	6947	7159	7417	7603	7841	8087	8293	8543	8747	8971	9203	9421	9643	9857
d	6491	6719	6949	7177	7433	7607	7853	8089	8297	8563	8753	8999	9209	9431	9649	9859
e	6521	6733	6959	7187	7451	7621	7867	8093	8311	8573	8761	9001	9221	9433	9661	9871
f	6529	6737	6961	7193	7457	7639	7873	8101	8317	8581	8779	9007	9227	9437	9677	9883
g	6547	6761	6967	7207	7459	7643	7877	8111	8329	8597	8783	9011	9239	9439	9679	9887
h	6551	6763	6971	7211	7477	7649	7879	8117	8353	8599	8803	9013	9241	9461	9689	9901
i	6553	6779	6977	7213	7481	7669	7883	8123	8363	8609	8807	9029	9257	9463	9697	9907
k	6563	6781	6983	7219	7487	7673	7901	8147	8369	8623	8819	9041	9277	9467	9719	9923
l	6569	6791	6991	7229	7489	7681	7907	8161	8377	8627	8821	9043	9281	9473	9721	9929
m	6571	6793	6997	7237	7499	7687	7919	8167	8387	8629	8831	9049	9283	9479	9733	9931
n	6577	6803	7001	7243	7507	7691	7927	8171	8389	8641	8837	9059	9293	9491	9739	9941
o	6581	6823	7013	7247	7517	7699	7933	8179	8419	8647	8839	9067	9311	9497	9743	9949
p	6599	6827	7019	7253	7523	7703	7937	8191	8423	8663	8849	9091	9319	9511	9749	9967
q	6607	6829	7027	7283	7529	7717	7949	8209	8429	8669	8861	9103	9323	9521	9767	9973
r	6619	6833	7039	7297	7537	7723	7951	8219	8431	8677	8863	9109	9337	9533	9769	10007
s	6637	6841	7043	7307	7541	7727	7963	8221	8443	8681	8867	9127	9341	9539	9781	10009
t	6653	6857	7057	7309	7547	7741	7993	8231	8447	8689	8887	9133	9343	9547	9787	10037
u	6659	6863	7069	7321	7549	7753	8009	8233	8461	8693	8893	9137	9349	9551	9791	10039
v	6661	6869	7079	7331	7559	7757	8011	8237	8467	8699	8923	9151	9371	9587	9803	10061
w	6673	6871	7103	7333	7561	7759	8017	8243	8501	8707	8929	9157	9377	9601	9811	10067
x	6679	6883	7109	7349	7573	7789	8039	8263	8513	8713	8933	9161	9391	9613	9817	10069
y	6689	6899	7121	7351	7577	7793	8053	8269	8521	8719	8941	9173	9397	9619	9829	10079
z	6691	6907	7127	7369	7583	7817	8059	8273	8527	8731	8951	9181	9403	9623	9833	10091