A reconstruction of
Krause’s table of factors
(1804)

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1  Krause (1781–1832)

Karl Christian Friedrich Krause (1781–1832) was a German philosopher. He studied philosophy and mathematics at the University of Jena. He later went to Berlin, Göttingen and Munich. In Göttingen, he was one of Schopenhauer’s teachers.

2  Krause’s table of factors and primes

One of Krause’s first books was his table of factors and primes, published in 1804 [9]. In this book, Krause gave a number of lists in a very compact and unusual way. Here, we will only focus on two of his tables: the table of primes and the table of factors.

In his introduction, Krause mentions the tables of Lambert [10] and Felkel [2, 3, 4, 5], and he understands the objections towards the use of letters for numbers which was favored by Felkel. He nevertheless decided to provide his two short tables, in order to show that letters can be used more conveniently than in Felkel’s table. The main tables in Krause’s book, however, do not use letters.

Krause wrote that his tables were computed (neuberechnet), and presumably not copied, although he certainly compared his tables with earlier ones. He also announced an extension of his table of factors to one million (with only the smallest factors), as well as an extension of the table of primes to two millions. These tables have never been published.

2.1  Krause’s table of primes

In this table, Krause gave a list of all primes from 1 to 100000. Each prime is encoded by a letter and the whole table fits on two pages. The first five primes are given explicitly, and they are followed by the list “efghmnprstwyzbeghmnop 2eklmoqrfusvwxabeghmnop ...

The first sequence gives the primes up to 97. The symbols are to be read in the following table:

We have therefore the primes 11 (e), 13 (f), 17 (g), etc. The symbols from “a” to “p” can refer to two different numbers, but the sequences above remove any ambiguity. The last symbol of the first sequence, namely “o”, represents 97, and not 33, because the numbers represented by the symbols appear in increasing order.

After 97, we have a new sequence, starting with “1”. This sequence gives all prime numbers between 100 and 200. The first digit is “1” and the two other digits are given by the symbol, using the encoding given above. Each sequence therefore corresponds to an interval of 100 integers. Every ten such sequences, the prefix is given in isolation: 10, 20, etc. Every hundred such sequences, the prefix (or rather the number of thousands) is given in the margin.

One might wonder about the ambiguities which could arise given that some symbols are duplicated in the encoding list. It appears that there is only one ambiguous case in
the range of the table, namely for the interval 31400 to 31500. In this interval, the first prime is 31469, and the first alphabet is not at all used, causing possible confusion. In that case, Krause wrote the sequence “4cfl” instead of “4cfl”, and he drew the attention of the reader to this special case at the end of his list of primes.

In our reconstruction, we have faithfully followed Krause’s scheme and used exactly the same line and page breaks as he did. During the reconstruction, we have found one error in Krause’s table, namely that he gave 16587 and 16593 as prime, the correct values being 16567 and 16573. This was possibly a typographic error at some point, as the two errors seem to be related. There may be other errors, but at least the values at the line breaks should otherwise be correct.

2.2 Krause’s table of factors

In this table, Krause gave the list of all factors for all numbers not divisible by 2, 3, and 5 until 10000. Each number is also encoded by a letter, but the factors are always given explicitly. Moreover, there is now no ambiguity in the symbols, since Krause uses latin and gothical alphabets.

Krause intended to give all factors, and not merely the simple factors. The composite numbers are given with prefixes and almost the same symbols as above. The only difference, apart from the use of a second alphabet, are that the latin “o” is written “ϕ,” presumably in order to avoid a confusion with the digit “0,” although such a confusion normally does not arise.

The list of composite numbers starts with

\[ \text{u 7.7 f 7.11 m 7.13 - 1 - h 7.17 i 11.11 ϕ 7.19 s 11.13 z 7.23 c 13.13 f 11.17 - 2 - \ldots} \]

The first “u” stands for 49, then “f” is for 77, and so on. Hundreds are separated like in the first table, so that the first “h” stands for the number 119.

Each composite number is followed by the list of its simple factors, separated by dots. When a simple factor has a multiplicity greater than 1, it is repeated, as for \( 9n = 931 = 7 \times 7 \times 19 \):

\[ \ldots - 9 - a 17.53 f 11.83 g 7.131 k 13.71 n 7.7.19.133.49 s 23.41 \ldots \]

This example also shows that the list of simple factors given in increasing order is followed by the list of composite factors given in decreasing order. 931 has two such composite factors (other than itself), namely 133 and 49.

Although Krause intended to give all factors, the lists of composite factors are often incomplete. One of the last numbers of the table is 9947 and Krause omits the factors 203 and 49. His list of factors should have been instead

\[ t 7.7.7.29.1421.343.203.49 \]

Since Krause forgot a number of composite factors, we have decided not to follow his line breaks in our reconstruction. It should nevertheless be useful, first in order to convey Krause’s scheme, and second in order to provide a tool for checking Krause’s table.
### Figure 1: The first page of Krause's table of primes.  
(source: Staats- und Universitätsbibliothek Dresden)
Figure 2: The second page of Krause’s table of primes. (source: Staats- und Universitätsbibliothek Dresden)
Figure 4: The second page of Krause’s table of factors. (source: Staats- und Universitätsbibliothek Dresden)
| a | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p | q | r | s | t | u | v | w | x | y | z |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |

**II. Anhang**

Ordnen der Größen nach ihrer eigenen Größe (oder allgemeiner der kleineren Einheit), die sich mit den Zahlen 1 bis 35 schreiben lassen.
References

The following list covers the most important references related to Krause’s table. Not all items of this list are mentioned in the text, and the sources which have not been seen are marked so. We have added notes about the contents of the articles in certain cases.


**Note on the titles of the works:** Original titles come with many idiosyncrasies and features (line splitting, size, fonts, etc.) which can often not be reproduced in a list of references. It has therefore seemed pointless to capitalize works according to conventions which not only have no relation with the original work, but also do not restore the title entirely. In the following list of references, most title words (except in German) will therefore be left uncapitalized. The names of the authors have also been homogenized and initials expanded, as much as possible.

The reader should keep in mind that this list is not meant as a facsimile of the original works. The original style information could no doubt have been added as a note, but we have not done it here.


[12] Denis Roegel. A reconstruction of Felkel’s tables of primes and factors (1776). Technical report, LORIA, 2011. [This is a reconstruction and an extension of Felkel’s tables [2, 3, 4, 5].]


[14] Denis Roegel. A reconstruction of Lambert’s table of factors (1770). Technical report, LORIA, Nancy, 2011. [This is a reconstruction of the table of factors in [10].]


Krause’s table of factors (1804) (reconstruction, D. Roegel, 2011)
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